Applications of Particle Filtering to Communications: A Review*

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ABSTRACT

In the past few years, particle filtering has emerged as a powerful methodology for solving problems in communications. Many such problems can be represented as dynamic state-space models that involve nonlinear functions and non-Gaussian noise. These problems are very difficult to solve with classical methods, and as an alternative approach, particle filtering is particularly attractive. In this paper we review the literature of particle filtering that emphasizes applications to blind equalization, blind detection over frequency non-selective dispersive channels, multiuser detection, and estimation and detection of space-time trellis codes.

1 Introduction

Particle filtering has become an important methodology in the field of statistical signal processing [8]. Although its beginnings can be traced back to the fifties and its important advances to the seventies, the signal processing community embraced it in the early nineties. One of the main applications of this methodology has been in object tracking, but very recently, it has also been used for solving difficult problems in communications. The main objective of this paper is to review some of the literature on the application of particle filtering in communications. First we provide some general comments about particle filtering and its use in communications, and then we discuss more specifically its application to blind equalization, blind detection over frequency non-selective dispersive channels, multiuser detection, and estimation and detection of space-time trellis codes. This review is by no means complete. We do not address other problems in communications where particle filtering is also used such as tracking of nonstationary behavior of communication networks [7], implementation of various types of nonlinear filters in digital receivers [1], localization and tracking of mobiles [23], and FM demodulation [14].

2 Particle filtering in communications - general comments

Particle filtering is used for processing of data modeled by dynamic state-space models. In particular, a dynamic state-space representation of data is given by a state equation defined at time instants $t = 0, 1, 2, \cdots$ by

$$\mathbf{x}_t = \mathbf{f}_t(\mathbf{x}_{t-1}, \mathbf{u}_t) \tag{1}$$

and an observation equation,

$$\mathbf{y}_t = \mathbf{g}_t(\mathbf{x}_t, \mathbf{v}_t) \tag{2}$$

where $\mathbf{x}_t \in \mathbb{R}^{d_x}$ is a state vector of the model, $\mathbf{y}_t \in \mathbb{R}^{d_y}$ is a vector of observations, $\mathbf{u}_t \in \mathbb{R}^{d_u}$ and $\mathbf{v}_t \in \mathbb{R}^{d_v}$ are noise vectors, $\mathbf{f}_t : \mathbb{R}^{d_x} \times \mathbb{R}^{d_u} \mapsto \mathbb{R}^{d_x}$ is a system transition function, and $\mathbf{g}_t : \mathbb{R}^{d_x} \times \mathbb{R}^{d_v} \mapsto \mathbb{R}^{d_y}$ is a measurement function. Given the observations $\mathbf{y}_{1:t} = \{\mathbf{y}_1 \ \mathbf{y}_2 \ \cdots \mathbf{y}_t\}$, the objective is to estimate *sequentially* the unobserved states $\mathbf{x}_t$.

There are three densities that play critical role in sequential signal processing. They are

1. the *filtering density*, $p(\mathbf{x}_t|\mathbf{y}_{1:t})$,
2. the *predictive density*, $p(\mathbf{x}_{t+l}|\mathbf{y}_{1:t})$, $l \geq 1$, and
3. the *smoothing density*, $p(\mathbf{x}_t|\mathbf{y}_{1:T})$, where $T > t$.

All the information about $\mathbf{x}_t$ regarding filtering, prediction or smoothing is captured by these densities, respectively.

The objective of tracking the above densities recursively is often very difficult because analytical expression for updating, for example, $p(\mathbf{x}_t|\mathbf{y}_{1:t})$ from $p(\mathbf{x}_{t-1}|\mathbf{y}_{1:t-1})$ do not exist. Particle filtering, which is based on the theory of sequential importance sampling, accomplishes this by approximating densities with discrete random measures defined by particles $\mathbf{x}_t^{(m)}$ and their weights $w_t^{(m)}$, $m = 1, 2, \cdots, M$, so that, for instance,

$$p(\mathbf{x}_t|\mathbf{y}_{1:t}) \simeq \sum_{m=1}^{M} w_t^{(m)} \delta(\mathbf{x}_t - \mathbf{x}_t^{(m)}).$$

Then the statistics of any function of $\mathbf{x}_t$ can easily be computed from $\{\mathbf{x}_t^{(m)}, w_t^{(m)}\}_{m=1}^{M}$. Excellent references on particle filtering are [3, 8, 9, 21].

Typically, the first step in applying particle filtering to communications is the representation of the problem in a form given generically by (1) and (2). To that end, it is important to note that many processes in communications can easily be modeled as Markovian processes as in (1). Examples include the time variation of the coefficients of flat-fading or frequency selective channels and symbols with memory in coded systems. For instance, mobile channels are in general Rayleigh fading (time-varying) channels, and the time variation of the fading processes of the channels are usually modeled as ARMA or AR processes. The coefficients of the ARMA (AR) processes are selected to match the physical characteristics of the channel, and the equations that describe the variation of the channels can be given in a state-space form. Also, the representation of single user

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systems can often be extended to incorporate multiple input multiple output systems such as CDMA and OFDM systems.

Once the model is defined, one resorts to applications of particle filtering. The application itself requires careful consideration of various issues specific to the problem that is addressed. Note also that the index of the observations and the states is not always time, and that it can be, for example, frequency or CDMA user. The states are most often the unknown channels and the transmitted symbols.

In the applications of particle filtering in communications, a special emphasis is given to the operation of smoothing. Namely, the estimation of the transmitted symbols can often be carried out with some delay, which allows for making inference from the smoothing density $p(\mathbf{x}_t|\mathbf{y}_{1:T})$, $t < T$ [27, 28]. Procedures that rest on smoothing should have better performance, that is, they should yield smaller BERs, especially in cases when the signals are channel-coded or when the noise in the communication system is colored.

3 Blind equalization

When digital symbols $\{b_t\}$ are transmitted over a dispersive channel, intersymbol interference occurs and the received sampled signal can be represented as

$$y_t = \sum_{k=0}^{L-1} b_{t-k} h_{t,k} + v_t = \mathbf{h}_t^\top \mathbf{b}_t + v_t \tag{3}$$

where y_t is the received signal at time instant t, $\mathbf{b}_t^\top = [b_t \; b_{t-1} \cdots b_{t-L+1}]$, $\mathbf{h}_t^\top = [h_{t,0} \; h_{t,1} \cdots \; h_{t,L-1}]$ are the coefficients of the unknown FIR channel impulse response, L is the length of the channel and v_t is an additive noise which is mostly considered as a zero mean Gaussian process with a known variance σ^2. The state $\mathbf{x}_t$ here is composed of the unknown symbols $\mathbf{b}_t$ and the channel coefficients $\mathbf{h}_t$.

The objective of equalization is to estimate the transmitted symbols $\{b_t\}$ in presence of intersymbol interference with or without determination of the channel coefficients. In a particle filtering context, one is interested to determine the marginal posterior distribution of b_t given all the observations up to the current time t, $p(b_t|y_{1:t})$. This distribution is represented by a set of particles with their corresponding weights, and the symbol with the marginal maximum a posterior (MMAP) probability is taken as the estimated symbol.

Several authors have addressed the problem of blind equalization using particle filtering. Liu and Chen [20] considered time-invariant channels and described the problem as blind deconvolution. The channel coefficients are assumed to have a Gaussian prior distribution. This formulation permits the channel parameters to be analytically marginalized, which allows for a direct drawing of samples from $p(b_{t+1}|b_{1:t}, y_{1:t+1})$ as well as evaluation of $p(y_{t+1}|b_{1:t}, y_{1:t})$ needed for the weight updates. The procedure is repeated M times and the M particles with their corresponding weights approximate the posterior distribution $p(b_{t+1} \mid y_{1:t+1})$. The MAP estimate of the symbol b_{t+1} are then readily obtained from the particles and the weights. It is to be noted that the posterior distribution of the channel $p(\mathbf{h}_t|b_{1:t}, y_{1:t})$ is Gaussian with a mean and covariance that are recursively updated and whose estimates are obtained by weighted averages. A similar problem has also been addressed in [5, 6], where the emphasis is on the methods for fixed-lag blind equalization and the interest is in obtaining the smoothing distribution $p(b_{t-l} \mid y_{1:t})$, where l is the fixed lag.

When the additive noise of the channel is Gaussian, data detection and channel estimation can be performed jointly following the concept of mixture Kalman filtering (MKF) [4]. It is to be noted that given the transmitted symbols, the state space model becomes a linear Gaussian system and the posterior distribution of the channel $p(\mathbf{h}_t|b_{1:t}, y_{1:t})$ remains Gaussian for all t. This permits tracking of the channel by Kalman filtering while particle filtering is applied to the data detection part [10].

A strength of particle filtering is that unlike other methods it can easily be extended to non-Gaussian noises. In [24], the additive complex noise is a mixture of K zero mean Gaussians. There, a latent variable z_t is defined to indicate the distribution of v_t. The procedure draws particles from an importance function $p(b_t, z_t|b_{1:t-1}, z_{1:t-1}, y_{1:t})$ that are used for approximation of the joint posterior distribution $p(b_t, z_t|y_{1:t})$ from which the MAP estimates of the symbols are obtained.

Recently, a similar treatment has been extended to OFDM systems over frequency selective channels [28]. One important difference with the above treatment is that the received signal y_t is considered to be an observation in frequency domain where the index t represents the different subcarriers. In such systems the observed signal for all the subcarriers is simultaneously received. In [22], the blind equilization of a frequency selective channel is addressed for a single carrier communication system. An important feature of the approach, shared with the MKF [4] and the OFDM receiver is that symbol detection is carried out without explicit channel estimation.

Blind equalization for satellite communications was considered in [19]. A dynamic state-space model provided a description of a satellite communication system, where the state equations are nonlinear and reflect a cascade of linear filters and a memoryless nonlinear traveling wave tube amplifier, and the observation consists of the state variable embedded in additive Gaussian noise. A generic particle filtering detector employing the prior importance function was proposed to combat the nonlinear distortion of the channel.

4 Blind detection over frequency non-selective dispersive channels

Most of the applications of particle filtering in mobile communications are in processing of signals transmitted over Rayleigh fading channels. In a dynamic state space modeling of the problem, h_t denotes the channel coefficient, and the state equation reflects the statistics of the underlying Rayleigh fading channels which are usually described by the Jakes' model. Moreover, since distortion in flat fading channels is multiplicative in nature, the observation is represented by

$$y_t = h_t b_t + v_t.$$

A typical problem is concerned with sequential detection of symbols without knowledge of h_t.

In [16], a linear model (i.e. an AR or ARMA model) for the channel variation was adopted, known model coefficients were assumed, and a generic particle filtering solution with a prior importance function was proposed. A more efficient implementation using MKF was reported in [4]. It was used for systems with Gaussian and impulsive noises and it was demonstrated to work for uncoded and coded systems. At the same time, in [24, 25], an almost identical algorithm to MKF was developed using the Rao-Blackwellization concept. Blind detection in the presence of impulsive noise by a Gaussian sum particle filtering detector was proposed in [18]. The above algorithms assume known channel model coefficients. When the model coefficients are also unknown, a hybrid algorithm was first presented in [15]. The proposed method uses particle filtering in conjunction with the recursive least square algorithm for estimation of the coefficients. However, pilot signals are required for proper channel tracking. A fully blind particle filtering detector was reported in [12]. To achieve efficient implementation, the detector employs a hybrid importance function [13], an MKF, and an auxiliary particle filter with a smoothing kernel. Further, some prior information about the underlying communication system is used to allow for full blind detection.

In addition to linear models, alternative modeling of fading channels may be preferred, especially if one wants to capture the nonlinearities of channels. A wavelet-based modeling of fading channels was used in [11], and a blind receiver employing MKF was proposed. The blind receiver requires no channel statistics, and it can determine the number of desired wavelets dynamically.

Apart from the Rayleigh fading channels, detection in impulsive channels was addressed in [17]. The models of such scenarios resembles that of the Rayleigh fading channels except that the impulsive channels are described by an AR process driven by a mixture Gaussian. To achieve blind detection, a Gaussian particle filtering scheme was developed.

5 Particle filtering for multiuser detection

Optimum multiuser detection (MUD) with or without known channel state information (CSI) has a complexity that is exponential to the number of users. Numerous approximate detectors have been developed in the past to reduce this complexity. Usually these detectors are based on interim hard decisions and are therefore prone to error propagation. As a result, their performance is not near optimum.

The earliest application of particle filtering to joint channel estimation and MUD appeared in [2] and subsequently in [26]. There, the Rayleigh flat fading channel is modeled using a state space equation,

$$\mathbf{h}_t = \mathbf{F}_t \mathbf{h}_{t-1} + \mathbf{G}\mathbf{u}_t \tag{4}$$

where $\mathbf{h}_t$ is the channel state vector approximated using an ARMA model, and $\mathbf{F}$ and $\mathbf{G}$ are known matrices with coefficients chosen to fit the spectrum of the fading process. The observation sampled at the system chip rate is represented as

$$y_t = \mathbf{b}_t^\top \mathbf{S}_t \mathbf{\Theta} \mathbf{h}_t + v_t$$

where $\mathbf{b}_t = [b_{1,t}, \cdots, b_{K,t}]^\top$ denotes the data symbols from all K users, $\mathbf{S}_t$ is a diagonal matrix of spreading codes, and $\mathbf{\Theta}$ is a known coefficient matrix corresponding to the MA part of the ARMA model. Samples of $\mathbf{b}_t$ are taken from an alphabet of size 2^K, and the channel state is integrated out as a nuisance parameter. This is equivalent to the MKF algorithm proposed in [4]. With large number of users, the sample space of $\mathbf{b}_t$ grows exponentially and the calculation of the importance weight becomes computationally very expensive.

Recently in [32] and [31], an alternative state-space representation of CDMA systems based on whitened matched filter (WMF) outputs was proposed. This representation allows for an efficient application of particle filtering with or without perfect CSI. In these papers, the observation is written as

$$\bar{y}_i = \mathbf{c}_k^\top \widetilde{\mathbf{B}}_i \mathbf{\Theta} \mathbf{h}_i + \bar{v}_i$$

where $i = (l-1)K + k$ is derived from the symbol duration index l and user index k, $\bar{y}_i$ is the WMF observation, $\mathbf{c}_k$ is the kth column of $\mathbf{C}$, the lower triangular matrix in the Cholesky decomposition of the signature cross-correlation matrix, and $\widetilde{\mathbf{B}}_i = \mathrm{diag}\{b_{1,l}, \cdots, b_{k,l}, 0, \cdots\}$. The CSI, $\mathbf{h}_i$ is modeled similarly as in (4). Note that $\widetilde{\mathbf{B}}_i$ only takes two valid values given $\widetilde{\mathbf{B}}_{i-1}$ as the particle filtering algorithm evolves according to index i, and therefore the complexity does not grow exponentially with the number of users. Simulations have shown consistent near optimum performance with linear complexity in systems with perfect and unknown CSIs.

6 Estimation and detection of space-time codes in fading channels

Space-time coding is a powerful tool for exploiting spatial and temporal diversities to combat fading in wireless communications. Although space-time trellis codes (STTCs) are deemed to possess the best coding efficiency, they are hard for detection especially when the problem involves unknown time varying fading coefficients. Particle filtering is considered for this problem in [29]. As demonstrated in [4] and [24], it is quite straightforward to represent binary or M-ary convolutional (trellis) coded systems in fading channels using state-space models. In these models, the state variable (usually the state of the encoder), evolves according to (1), and the observation is the product of the state variable and CSI plus noise. and is described in the form of (2). In [29], a space-time representation of the STTC systems is derived that has form similar to that in [4]. Inherent to all joint channel estimation and detection algorithms is the problem of phase ambiguity. While it is possible to employ differential coding or to send pilot signals to remove the ambiguity, it is demonstrated in [29] that one can design STTCs that have decreased phase ambiguity.

7 Final remarks

Particle filtering is becoming an important method for solving difficult problems in communications. With further advances of the method, old problems are resolved more efficiently, and new, even more difficult problems are readily addressed with it. A drawback of the method is the high computational power needed for its implementation. Much of the processing that takes place, however, can be parallelized, which makes the method potentially attractive for practical use. A successful hardware implementation of particle filtering will further boost the research on the theory and practice of particle filters in communications. Also, research oriented towards reducing the complexity of the algorithms will be very beneficial [30].

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