SELECTING THE OPTIMAL TIME–FREQUENCY DISTRIBUTION FOR REAL–LIFE MULTICOMPONENT SIGNALS UNDER GIVEN CONSTRAINTS

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ABSTRACT

Selecting the optimal time–frequency distribution (TFD) for a given real–life signal is one of the major challenges in the field of time–frequency analysis. In this paper, we define a methodology, based on the performance measure for quadratic class of TFDs [1, 2], that allows for such selections to be made for different regions in the time–frequency domain, where specific signal features of interest are located. A TFD which is capable of preserving and enhancing those features, while at the same time it satisfies specific, application–dependent constraints, is selected as the signal optimal TFD for the observed region. The use of this methodology is illustrated on an example of a multicomponent, nonstationary, real–life signal, for which the optimal TFD is required to meet the components' unbiased and efficient instantaneous frequency estimation constraint.

1 INTRODUCTION

Many real–life signals (e.g. speech, biomedical, underwater, seismic) are nonstationary [3]. Therefore, their spectral analysis needs to be carried out using tools which accommodate for the time-varying spectral content of the signals. One of the best known of such tools are the time–frequency distributions (TFDs) [4, 5, 6, 7].

The last two decades have seen a significant contribution to the field of time–frequency signal analysis, with many time–frequency distributions being proposed. Among the most studied and used are the TFDs from the quadratic class of time–frequency distributions [3, 8]:

$$\rho(t,f)=\iiint g(\nu,\tau)z(u+\frac{\tau}{2})z^{*}(u-\frac{\tau}{2})e^{j2\pi(\nu(u-t)-f\tau)}d\nu du d\tau \quad (1)$$

where $z(t)$ is the analytic associate of the real signal under analysis [3, 5], and $g(\nu,\tau)$ is the kernel function which defines the TFD and its properties [3, 5, 6, 7].

Although they all contain the same information, different TFDs may display that information in the time–frequency plane with different amount of details and accuracy. Selecting a TFD which does this in the "optimal way" is a critical factor when applying time–frequency analysis to nonstationary signals, in particular to real–life signals.

In this paper, we define a methodology, based on the TFDs' performance measure P [1, 2, 9], that allows for the selection of the optimal TFD, under application–specific constraints, for real–life, multicomponent signals, in time–frequency regions where the signal features of interest are located. An example of how this methodology is applied in practice is included, in which the optimal TFD, that satisfies the unbiased, efficient, multicomponent instantaneous frequency (IF) estimation constraint, is found for an Australian bird song signal.

2 METHODOLOGY FOR SELECTING THE OPTIMAL TFD FOR REAL–LIFE SIGNALS UNDER GIVEN CONSTRAINTS

In order to choose a TFD which satisfies specific constraints, and is optimal for a given multicomponent, real–life signal, the following methodology can be used:

1. First, represent the signal in the time–frequency domain using the Wigner–Ville distribution (WVD) [3], the spectrogram [5], and the Modified B distribution (MBD) [10]. These three distributions will provide us with indications of the main signal features in the time–frequency plane: the number of components, their relative amplitudes, the components' durations and bandwidths (features obtained from the MBD and spectrogram), as well as the cross–terms locations (obtained from the WVD).

   The WVD is a parameter–free TFD, while the parameters of the spectrogram and MBD (the window type and length, and the parameter β, respectively) need to be initialised. In the case of spectrogram, we use the Hanning window as the initial window type, and from several window lengths (whose values depend on the signal time duration) select as the initial window length the one that results into the most visually appealing time–frequency plot. In the case of MBD, the initial value for β is set to 0.001. The choice of these initial parameter values is based on the fact that the spectrogram with the

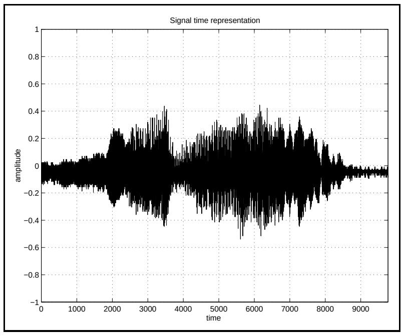

Figure 1: *Time representation of the Noisy Minor song signal*

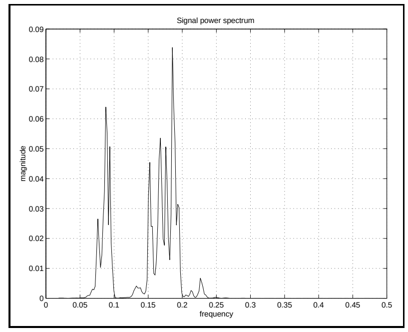

Figure 2: *Frequency representation of the Noisy Minor song signal*

Hanning window and MBD with $\beta \leq 0.001$ have been found to perform very good for the majority of various signals whose TFDs' performance we have analysed [1, 2, 9, 11].

2. Using those TFDs, we then identify regions in the time–frequency domain where certain application–specific signal features (e.g. closely–spaced components, crossing components, sudden change in a component FM law, etc) are located. We call such regions the Regions of Interest (ROIs). The ROIs are the rectangles in the time–frequency plane whose dimensions are the "time of interest" and the "frequency of interest". The time (frequency) of interest is the time interval (frequency band) of the TFD where the signal features we are interested in are located in time (frequency).

3. Different TFDs are now optimised, as described in [1, 2] and summarised in Section 2.1, for each of the selected ROIs, such that the signal characteristics in a particular ROI are represented in the best possible way.

4. Only those TFDs that meet the given, application–specific constraints are then considered, and the one which achieves the best performance among them, as measured by the TFDs' performance measure P (see Section 2.1), in a particular ROI, is selected as the signal optimal TFD in that ROI.

2.1 Performance Measure for TFDs

There are two major factors that affect the performance of quadratic TFDs when used to represent nonstationary, multicomponent signals in the joint time-frequency domain. These factors are commonly referred to as concentration, and resolution [9].

The concentration is measured by the width of components' mainlobes (a.k.a the instantaneous bandwidth [5]) about their respective instantaneous frequency (IF) laws [3]. The resolution, on the other hand, is measured by the minimum frequency separation between the components' mainlobes, for which the individual components' features (mainlobe amplitude and bandwidth) are just preserved [9].

To objectively evaluate the concentration and resolution capabilities of TFDs, the instantaneous (at the TFDs' time instant $t = t_i$) performance measure P_i has been defined [1, 2]:

$$P_i = 1 - \frac{1}{3}\left\{\left|\frac{A_S}{A_M}\right| + \frac{1}{2}\left|\frac{A_X}{A_M}\right| + \frac{B_1 + B_2}{2(f_2 - f_1)}\right\}, \quad (0 \leq P_i \leq 1) \tag{2}$$

where, for a pair of signal components, A_M and A_S are the average amplitudes of the components' mainlobes and sidelobes, respectively, A_X is the cross–term amplitude, and f_1 and f_2 are the components' IFs with their corresponding instantaneous bandwidths B_1 and B_2.

When P_i is close to 1, the TFD is said to have a good performance, while for P_i close to 0, it has a poor performance. By averaging P_is over a range of TFD's time instants, we obtain the "overall" performance measure P_o. The value of the TFD parameter that maximise P_o is chosen as its optimal value, with the corresponding P_o being the TFD's optimal performance measure P. A TFD with the largest P (hence with the best concentration and best resolution) is the optimal time–frequency distribution for the given signal [1, 2].

3 EXAMPLE

Let us illustrate the use of the methodology defined in Section 2 for the "Noisy Minor" (Manorina Melanocephala) bird song signal [12]. The constraint which the best performing TFD needs to satisfy in this example is the unbiased, efficient, multicomponent IF estimation in time–frequency regions where the signal components are closely–spaced.

The time and frequency (power spectral density) plots of the signal are shown in Figures 1 and 2, respectively. From the signal time–domain plot, we can see how its amplitude varies with time, and from Figure 2 what frequencies (with what magnitudes) are present in the signal. However, neither of the two plots can provide us with information on the signal internal structure (number of signal components, their amplitudes, IF laws, time intervals and frequency bands those components

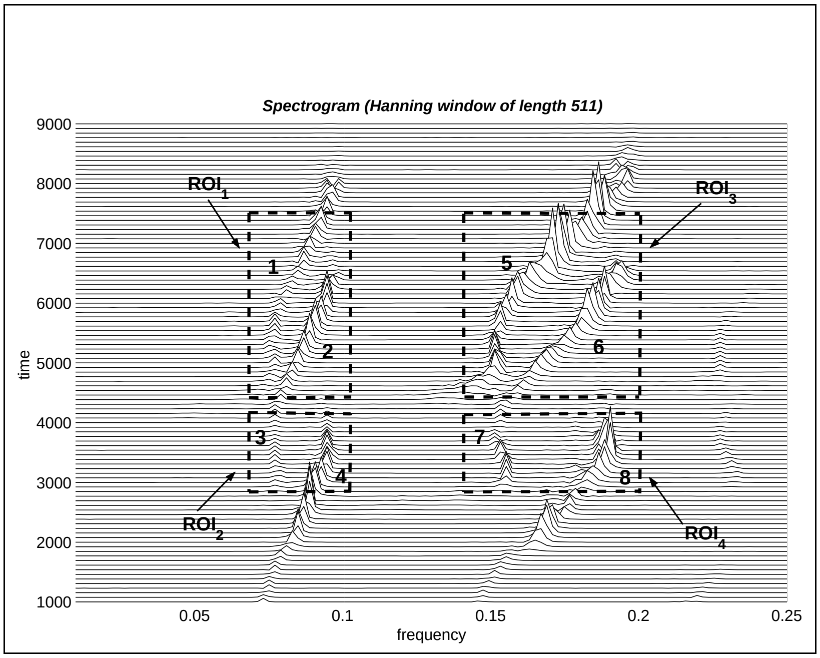

Figure 3: *Regions of Interest (ROIs) for the Noisy Minor song signal's time–frequency distributions*

occupy, etc).

To have a more complete "picture" of this signal, we analyse it in the time–frequency domain. This requires identifying a TFD (among many different TFDs belonging to the quadratic class) that is optimal for the signal. By employing the steps of the previously defined methodology, we can find the regionally optimal TFD(s) for the Noisy Minor song signal as follows.

First, we represent the signal in the time–frequency plane using the WVD, MBD ($\beta = 0.001$), and the spectrogram with the Hanning window of length 511. In selecting the optimal window length for the spectrogram, other lengths (127, 255, 1023) we also tested, but 511 resulted in the most visually appealing time–frequency plot of the signal spectrogram.

By comparing the plots of those TFDs, several dominant ridges (components) have been identified in the signal, and four regions of interest have been defined where the components form closely–spaced pairs in the time–frequency plane (see Figure 3). Our objective is to improve on different TFDs' resolution in the selected ROIs such that the best possible separation of the signal components is achieved. This would then allow one, to more accurately extract the components IFs from their peaks [3].

For each of the four ROIs, we optimise different TFDs using the performance measure P, as described in Section 2.1. The optimisation results are given in Table 1. From Table 1, we can see that the Modified B distribution with $\beta = 5 \times 10^{-5}$, having the largest P among the eight TFDs we have used in this example, achieves the best concentration and resolution of the signal components in ROI_1. It just outperforms the smoothed WVD (with the Hamming window of length 415 chosen as the smoothing window) and the spectrogram (with the Hanning window of length 447). Other TFDs, even after being optimised in ROI_1, still do not achieve as good performance as the MBD does. The Rihaczek distribution, and WVD in particular, perform poorly, as indicated by their P values of 0.5825 and 0.4984.

Similar analysis can be done for ROI_2, ROI_3 and ROI_4. What is interesting to observe is that the spectrogram has the best resolution (and therefore concentration) performance in each of ROI_2, ROI_3 and ROI_4. The spectrogram's performance closely matches that of MBD in ROI_1 too. Note also that the three best performing TFDs for all ROIs considered in this example, with very similar performances, are the spectrogram, the Modified B distribution, and the smoothed WVD. The fact that more than one TFD performs relatively good for a certain signal can be of benefit to the signal analyst, giving him(her) more freedom in selecting the optimal TFD for that signal, when satisfying certain constraints is also required in a particular application.

In this example, apart from the fact that we want to find TFDs that result into the best concentration and resolution of the signal components in the selected ROIs, we also want to use those TFDs to estimate the components' IF laws by extracting the peaks of the components ridges [3] in the same ROIs. Among the three best performing TFDs (spectrogram, smoothed WVD, and MBD), only the MBD is capable of providing us with unbiased and efficient multicomponent FM signals' IF estimates [10]. Therefore, under the given IF estimation constraint, we select the Modified B distribution as the *optimal TFD* of the Noisy Minor song signal, *for all*

TFD	ROI_1		ROI_2		ROI_3		ROI_4	
	P	*parameter*	P	*parameter*	P	*parameter*	P	*parameter*
Born–Jordan [4]	0.7574	N/A	0.9005	N/A	0.8647	N/A	0.8837	N/A
Choi–Williams [4]	0.8755	$\sigma = 3$	0.9323	$\sigma = 0.09$	0.8723	$\sigma = 0.3$	0.9077	$\sigma = 3$
Modified B	0.9198	$\beta = 5\times10^{-5}$	0.9433	$\beta = 6\times10^{-5}$	0.8907	$\beta = 4\times10^{-4}$	0.9602	$\beta = 3\times10^{-5}$
Rihaczek [8]	0.5825	N/A	0.7883	N/A	0.6260	N/A	0.6916	N/A
Smoothed WVD [7]	0.9195	Hamm, 415	0.9420	Bart, 383	0.8879	Bart, 159	0.9534	Rect, 287
Spectrogram	0.9145	Hann, 447	0.9574	Bart, 383	0.9261	Hann, 223	0.9710	Bart, 447
WVD	0.4984	N/A	0.7858	N/A	0.6077	N/A	0.5813	N/A
Zhao–Atlas–Marks [4]	0.7900	$a = 1$	0.8409	$a = 1$	0.7922	$a = 3$	0.8191	$a = 2$

Table 1: *The performance measure P and optimal parameter values of TFDs of the Noisy Minor song signal for four different regions–of–interest (ROIs), as defined in Figure 3*

ROIs. Note that the optimal value of the MBD parameter β varies across the considered ROIs, so that in each of those regions the best components' concentration and resolution is achieved; both of which are required for a robust IF estimation for multicomponent signals [10].

4 CONCLUSION

A methodology for selecting a time–frequency distribution, which, under some application–specific constraints, is optimal for the given real–life signal, has been defined in this paper. The use of the methodology in practice, was illustrated on an Australian bird song signal. We have shown that the only TFD which is capable of enhancing the signal components' concentration and resolution, while at the same time satisfying the unbiased, efficient, components' IF estimation constraint, in all selected time–frequency regions where the signal components were closely–spaced to each other, was the Modified B distribution. The distribution parameter β was optimised for each of those regions, such that the best components' concentration and resolution (and so more robust IF multicomponent estimations) were achieved.

A major advantage in using this methodology is that additional constraints can be easily introduced in the analysis. This only requires, for a particular time–frequency region, identifying those optimised TFDs which meet the given constraints, and selecting the one with the best performance (as measured by the TFDs' performance measure P) among them as the signal optimal TFD in that region.

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