A multi-slot method to estimate fast-varying channels in TD-CDMA systems

Monica Nicoli, Osvaldo Simeone and Umberto Spagnolini
Dip. Elettronica e Informazione, Politecnico di Milano, P.zza L. da Vinci, 32, I-20133 Milano (Italy)
E-mail: Monica.Nicoli@elet.polimi.it, Umberto.Spagnolini@elet.polimi.it

ABSTRACT

In mobile communications, the multipath radio channel is characterized by fast-varying faded amplitudes and slowly-varying delays. These different varying rates can be exploited by estimating the temporal subspace of the channel through a multi-slot averaging. This paper proves that the temporal subspace basis can be calculated with any accuracy (provided that the averaging memory is large enough), so that the mean square error (MSE) on the channel response depends only on the number of fast-varying parameters. The method shows promising benefits in the channel estimation for time slotted system such as the third generation TD-CDMA standards (TDD-UTRA [1] or TD-SCDMA [2]).

1 Introduction

In mobile communication systems, the training sequences used to estimate the channel response are required to be long enough to reduce the estimation error at the expenses of transmission efficiency. Estimation accuracy can be increased in time-slotted systems by merging the information about the slowly varying channel parameters from neighbouring slots or from slots in consecutive frames. In this way, the estimate from the ensemble of slots is expected to have a lower error than the single-slot estimate, as for a virtually longer training sequence but without any loss of transmission efficency. The delay-pattern can be considered as stationary within large time-scales. For instance, for the TDD-UTRA system [1], a terminal with a radial speed of 50 km/h moves approximately 3 m in 200 ms (20 slots), leading to a delay variation of 10^{-2} μs that is an order of magnitude less than the temporal resolution (given by the inverse of the bandwidth, which is equal to 5 MHz).

The multi-slot approach is trivial for a static channel since the faded amplitudes are stationary and the channel estimates can simply be averaged. In order to cope with fast-varying fading channels, the proposed multi-slot technique estimates the slowly-varying (temporal) subspace of the channel without explicitly computing the delays of the multipath. This paper proves analytically that the temporal subspace basis can be calculated with an accuracy that increases with the number of slots (provided that the delay pattern remains the same). The mean square error (MSE) on the channel response depends only on the number of faded amplitudes that are to be calculated on a slot-by-slot basis.

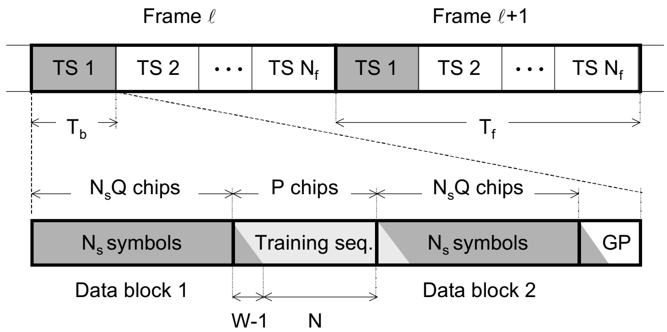

Figure 1: Frame structure of a TD-CDMA system.

In this paper we will consider third generation mobile radio systems based on time slotted CDMA, which uses a combination of TDMA (time division multiple access) and CDMA (code division multiple access) to separate users in the time and/or in the code domain. Channel estimation for the uplink (mobile-base station) is discussed in Sec. 2. Then, the properties of the channel are investigated in Sec. 3. The multi-slot method is in Sec. 4 and the analytical derivation of the MSE bound on the channel estimate is in Sec. 5. The performance analysis is validated by numerical simulations for TDD-UTRA standard and compared with analytical results in Sec. 6.

2 TD-CDMA signal model

The frame structure of a hybrid TD-CDMA system (such as the 3rd generation standards TDD-UTRA [1] and TD-SCDMA [2]) is depicted in Fig. 1. The frame has duration T_f and contains N_f time slots, each can be allocated either to the uplink or the downlink. K users share the same time slot and the the same frequency band, each user transmits a burst consisting of two data blocks, a training sequence (midamble) and a guard period. The midamble for the kth user ($k = 1, \ldots, K$) is composed of P chips $\{x_k(i)\}_{i=0}^{P-1}$, with $|x_k(i)|^2 = \sigma_x^2$. The uplink is considered, at the receiver the knowledge of the training sequences is exploited to estimate jointly the K multipath radio channels. The channel between the kth user and the receiver is modelled as a linear filter $h_k(t;\ell)$ of temporal support $t \in [0, WT]$, where $1/T$ is the chip rate (ℓ denotes the dependence on the slot as explained below). Since the channel is frequency selective ($W > 1$), the first $W-1$ symbols of the received midamble are affected from the interference of the data symbols, hence the effective training length at the receiver is $N = P - W + 1$. We assume that the K users transmit one burst in each frame, i.e., the temporal interval between two successive slots relative to the same users is the frame duration T_f. The index ℓ runs over the frames.

The discrete time model for the received data is obtained by sampling at the chip rate the output of the filter matched to the transmitted chip waveform (roll-off factor α). The N samples received during the training period can be then gathered into a $N \times 1$ vector $\mathbf{y}(\ell)$ and the channel for each user into a $W \times 1$ vector $\mathbf{h}_k(\ell)$, as

$$\mathbf{y}(\ell) = \sum_{k=1}^{K} \mathbf{X}_k \mathbf{h}_k(\ell) + \mathbf{n}(\ell) = \mathbf{X}\mathbf{h}(\ell) + \mathbf{n}(\ell), \qquad (1)$$

where the $N \times W$ Toeplitz matrix (independent on the frame) $\mathbf{X}_k = [\mathbf{x}_k(0), \cdots, \mathbf{x}_k(-W+1)]$ simplifies the convolution with the kth training sequences, $\mathbf{x}_k(i) = [x_k(i), ..., x_k(i+N-1)]^T$. The multi-user matrices $\mathbf{X} = [\mathbf{X}_1 \cdots \mathbf{X}_K]$ and $\mathbf{h}(\ell) [= \mathbf{h}_1(\ell)^T \cdots \mathbf{h}_K(\ell)^T]^T$ include the K.channels and the K training sequences. The AWGN $\mathbf{n}(\ell)$ is uncorrelated with respect to the slot: $E[\mathbf{n}(\ell)\mathbf{n}(\ell+m)] = \sigma_n^2 \mathbf{I}_N \delta_m$, the power level σ_n^2 is not necessarily known.

3 Multipath channel model

In accordance with the multipath model of propagation, the channel vector $\mathbf{h}_k(\ell)$ can be obtained as the sum of contributions relative to d_k paths, each characterized by a delay $\tau_{k,p}$ and a complex amplitude $\alpha_{k,p}(\ell)$. As discussed in Sec. 1, the set of delays $\boldsymbol{\tau}_k = [\tau_{k,1}, ..., \tau_{k,d_k}]^T$ can be considered as a constant over a window of L frames selected by taking into account the mobility of the terminals. It is

$$\mathbf{h}_k(\ell) = \sum_{p=1}^{d_k} \alpha_{k,p}(\ell)\, \mathbf{g}(\tau_{k,p}) = \mathbf{G}(\boldsymbol{\tau}_k)\boldsymbol{\alpha}_k(\ell) \quad \ell = 1, .., L. \qquad (2)$$

The slot-invariant temporal matrix $\mathbf{G}(\boldsymbol{\tau}_k) = [\mathbf{g}(\tau_{k,1}), ..., \mathbf{g}(\tau_{k,d_k})]$ collects the delayed waveforms $\mathbf{g}(\tau_{k,p}) = [g(-\tau_{k,p}), g(T - \tau_{k,p}), ..., g((W-1)T - \tau_{k,p})]^T$. The amplitudes $\boldsymbol{\alpha}_k(\ell) = [\alpha_{k,1}(\ell), ..., \alpha_{k,d_k}(\ell)]^T$ are assumed to follow the WSSUS channel model [3] and are ergodic (up to the second order) with $E[\boldsymbol{\alpha}_k(\ell+m)\, \boldsymbol{\alpha}_k(\ell)^H] = \mathrm{diag}\{A_{k,1}, ..., A_{k,d_k}\} \times \delta(m)$, $\delta(.)$ is the Kronecker delta function. Even if it is not really necessary, we assume the fading to be uncorrelated in different frames. In this case, the order of the temporal diversity depends on $\mathbf{G}(\boldsymbol{\tau}_k)$ only as $r_k = \mathrm{rank}(\mathbf{G}(\boldsymbol{\tau}_k)) \leq W$. r_k equals the number of resolvable delays (given the bandwidth of the transmitted signal) and is $r_k \leq d_k$. Let $\mathbf{U}_k$ be the orthonormal basis of the column space of $\mathbf{G}(\boldsymbol{\tau}_k)$, $\mathcal{R}(\mathbf{G}(\boldsymbol{\tau}_k))$, the channel matrix from model (2) can be equivalently parametrized as

$$\mathbf{h}_k(\ell) = \mathbf{U}_k \mathbf{b}_k(\ell) \qquad \ell = 1, .., L, \qquad (3)$$

$\mathbf{b}_k(\ell)$ is the burst-dependent $r_k \times 1$ vector. In the following $\mathcal{R}(\mathbf{U}_k)$ will be referred to as the temporal subspace.

4 Multi-slot (MS) channel estimate

The maximum likelihood estimate (MLE) of $\mathbf{h}(\ell)$ is obtained from model (1) without imposing any additional structure (i.e., a priori information) on the unknown

$$\hat{\mathbf{h}}(\ell) = \left[\hat{\mathbf{h}}_1(\ell)^T \cdots \hat{\mathbf{h}}_K(\ell)^T\right]^T = \mathbf{R}_{xx}^{-1}\mathbf{X}^H \mathbf{y}(\ell), \qquad (4)$$

here it is $\mathbf{R}_{xx} = \mathbf{X}^H\mathbf{X}$. Notice that $\hat{\mathbf{h}}(\ell)$ is the conventional least squares estimate (LSE). Let us assume that the training sequences of different users are orthogonal so that $\mathbf{R}_{xx} = \mathrm{diag}\{\mathbf{Q}_1, \cdots, \mathbf{Q}_K\}$ where $\mathbf{Q}_k = \mathbf{X}_k^H\mathbf{X}_k$. This is a close approximation of the correlation properties of the training sequences of the TDD-UTRA and TD-SCDMA standard. Imposing the parametrization (3), the negative log-likelihood function can be written, apart from uniteresting constant, as

$$\mathcal{L}(\boldsymbol{\theta}) = \frac{1}{L}\sum_{k=1}^{K}\sum_{\ell=1}^{L}\left\| \mathbf{Q}_k^{1/2}\hat{\mathbf{h}}_k(\ell) - \mathbf{Q}_k^{1/2}\mathbf{U}_k\mathbf{b}_k(\ell)\right\|^2, \qquad (5)$$

where $\boldsymbol{\theta} = [\,\mathrm{vec}(\mathbf{U}_1)^T \cdots \mathrm{vec}(\mathbf{U}_k)^T \;\; \mathbf{b}_1^T \cdots \mathbf{b}_K^T]^T$ is the vector of unknowns, $\mathbf{b}_k = [\mathbf{b}_k(1)^T \cdots \mathbf{b}_k(L)^T]^T$. The minimization of $\mathcal{L}(\boldsymbol{\theta})$ can be obtained by considering each user separately, resulting in the multi-slot channel estimator

$$\hat{\mathbf{h}}_{\mathrm{MS},k}(\ell) = \mathbf{Q}_k^{-1/2}\hat{\boldsymbol{\Pi}}_k\mathbf{Q}_k^{1/2}\hat{\mathbf{h}}_k(\ell), \qquad (6)$$

$\hat{\boldsymbol{\Pi}}_k$ is the projector onto the temporal subspace, estimated as the span of the r_k leading eigenvectors of the correlation matrix

$$\mathbf{R}_k(L) = \frac{1}{L}\mathbf{Q}_k^{1/2}\left(\sum_{\ell=1}^{L}\hat{\mathbf{h}}_k(\ell)\,\hat{\mathbf{h}}_k(\ell)^H\right)\mathbf{Q}_k^{H/2}. \qquad (7)$$

In summary, the multi-slot estimator is the projection of the LSE $\mathbf{Q}_k^{1/2}\hat{\mathbf{h}}_k(\ell)$ onto the temporal subspace spanned by the r_k leading eigenvectors of the matrix $\mathbf{R}_k(L)$ computed from the ensemble of L LSE $\{\hat{\mathbf{h}}_k(\ell)\}_{\ell=1}^{L}$.

Remark: If the training sequences are not orthogonal, it is still possible to write the negative log-likelihood function in a form similiar to (5) by introducing a correcting term that cancels the residual interference among the users. This topic is not covered in this paper.

5 Performance analysis

Since the negative log-likelihood function (5) is separable over different users, the Cramer Rao Bound (CRB) can be computed on a user-by-user basis. In the following we drop the subscript k that refers to any specific user. Gathering the channel vectors relative to the considered user into the $LW \times 1$ vector $\mathbf{h} = [\mathbf{h}(1)^T, ..., \mathbf{h}(L)^T]^T$ and the slot-depedent terms into the $L \times r$ matrix $\mathbf{B} = [\mathbf{b}(1), ..., \mathbf{b}(L)]^H$, we define the $LW \times r(W+L)$ matrix of sensitivities

$$\mathbf{D} = \left\{ \frac{\partial \mathbf{h}^H}{\partial \boldsymbol{\theta}} \right\} = \left[\begin{array}{cc} \mathbf{B}^* \otimes \mathbf{I}_W & \mathbf{I}_L \otimes \mathbf{U} \end{array} \right], \tag{8}$$

$\otimes$ denotes the Kronecker product. The $r(W+L) \times r(W+L)$ Fisher Information Matrix (FIM) of $\boldsymbol{\theta}$ is thus given by

$$\mathbf{J} = \frac{1}{\sigma_n^2} \mathbf{D}^H \breve{\mathbf{R}}_{xx} \mathbf{D}, \tag{9}$$

where $\breve{\mathbf{R}}_{xx} = \mathbf{I}_L \otimes \mathbf{R}_{xx}$. FIM (9) can be partitioned as

$$\mathbf{J} = \left[\begin{array}{cc} \mathbf{J}_{UU} & \mathbf{J}_{Ub} \\ \mathbf{J}_{Ub}^H & \mathbf{J}_{bb} \end{array} \right], \tag{10}$$

where the $Wr \times Wr$ block $\mathbf{J}_{UU}$ and the $Lr \times Lr$ block $\mathbf{J}_{bb}$ depend on the burst-independent and the burst-dependent terms, respectively. The diagonal blocks are obtained from (9):

$$\mathbf{J}_{UU} = \frac{L}{\sigma_n^2} \mathbf{R}_b(L) \otimes \mathbf{R}_{xx} \tag{11a}$$

$$\mathbf{J}_{bb} = \frac{1}{\sigma_n^2} \mathbf{I}_L \otimes (\mathbf{U}^H \mathbf{R}_{xx} \mathbf{U}), \tag{11b}$$

where $\mathbf{R}_b(L) = 1/L \sum_{\ell=1}^{L} \mathbf{b}(\ell) \mathbf{b}(\ell)^H$. Note that the matrix $(\mathbf{I}_L \otimes \mathbf{R}_{xx})$ in (9) is positive definite so that, by using the standard result on the rank of partitioned matrix [4], it can be shown that $\text{rank}(\mathbf{J}) = \text{rank}(\mathbf{D}) = r(W+L) - r^2$. As $\text{rank}(\mathbf{J}) < r(W+L)$ the FIM is singular. According to the conditions in [5], here the Cramer-Rao Bound (CRB) on the channel estimate depends on the pseudoinverse $\mathbf{J}^\dagger$:

$$\text{Cov}(\hat{\mathbf{h}}_{\text{MS}}) \geq \text{CRB}(\hat{\mathbf{h}}_{\text{MS}}) = \mathbf{D}\mathbf{J}^\dagger \mathbf{D}^H. \tag{12}$$

Next, by defining the matrices $\mathbf{D}_1 = \mathbf{B}^* \otimes \mathbf{I}_W$, $\mathbf{D}_2 = \mathbf{I}_L \otimes \mathbf{U}$, and $\mathbf{D} = [\mathbf{D}_1 \ \mathbf{D}_2]$, the CRB (12) simplifies as

$$\text{CRB}(\hat{\mathbf{h}}_{\text{MS}}) = \sigma_n^2 \breve{\mathbf{R}}_{xx}^{-1/2} \mathbf{\Pi}_{\widetilde{\mathbf{D}}} \breve{\mathbf{R}}_{xx}^{-H/2}, \tag{13}$$

$\widetilde{\mathbf{D}} = \breve{\mathbf{R}}_{xx}^{1/2} \mathbf{D}$ and $\mathbf{\Pi}_{\widetilde{\mathbf{D}}}$ is the corresponding projector. Since $\mathcal{R}\{\widetilde{\mathbf{D}}\} = \mathcal{R}\{\widetilde{\mathbf{D}}_1\} \cup \mathcal{R}\{\widetilde{\mathbf{D}}_2\}$, this can be equivalently decomposed into the orthogonal subspaces $\mathcal{R}\{\widetilde{\mathbf{D}}\} = \mathcal{R}\{\mathbf{\Pi}^\perp_{\widetilde{\mathbf{D}}_1} \widetilde{\mathbf{D}}_2\} \cup \mathcal{R}\{\widetilde{\mathbf{D}}_1\}$ such that $\mathcal{R}\{\mathbf{\Pi}^\perp_{\widetilde{\mathbf{D}}_1} \widetilde{\mathbf{D}}_2\} \cap \mathcal{R}\{\widetilde{\mathbf{D}}_1\} = \emptyset$. The projection matrix reduces to

$$\mathbf{\Pi}_{\widetilde{\mathbf{D}}} = \mathbf{\Pi}_{\mathbf{\Pi}^\perp_{\widetilde{\mathbf{D}}_1} \widetilde{\mathbf{D}}_2} + \mathbf{\Pi}_{\widetilde{\mathbf{D}}_1} \tag{14}$$

where $\mathbf{\Pi}_{\widetilde{\mathbf{D}}_1}$, $\mathbf{\Pi}^\perp_{\widetilde{\mathbf{D}}_1}$ and $\mathbf{\Pi}_{\mathbf{\Pi}^\perp_{\widetilde{\mathbf{D}}_1} \widetilde{\mathbf{D}}_2}$ denote the orthogonal projections onto, respectively, $\mathcal{R}\{\widetilde{\mathbf{D}}_1\}$, the orthogonal complement $\mathcal{R}\{\widetilde{\mathbf{D}}_1^\perp\}$ and $\mathcal{R}\{\mathbf{\Pi}^\perp_{\widetilde{\mathbf{D}}_1} \widetilde{\mathbf{D}}_2\}$. According to the model exploited here, the projectors in (14) can be easily calculated as follows: $\mathbf{\Pi}_{\tilde{\mathbf{D}}_1} = \mathbf{\Pi}_{\mathbf{B}^*} \otimes \mathbf{I}_W$, $\mathbf{\Pi}^\perp_{\tilde{\mathbf{D}}_1} \tilde{\mathbf{D}}_2 = (\mathbf{\Pi}_{\mathbf{B}^*} \otimes \mathbf{I}_W)(\mathbf{I}_L \otimes \tilde{\mathbf{U}}) = \mathbf{\Pi}^\perp_{\mathbf{B}^*} \otimes \tilde{\mathbf{U}}$, $\mathbf{\Pi}_{\mathbf{\Pi}^\perp_{\tilde{\mathbf{D}}_1} \tilde{\mathbf{D}}_2} = \mathbf{\Pi}_{\mathbf{B}^*} \otimes \mathbf{\Pi}_{\tilde{\mathbf{U}}}$. From the latter results and by combining (13) and (14), the CRB is evaluated as

$$\text{CRB}(\hat{\mathbf{h}}) = \sigma_n^2 \breve{\mathbf{R}}_{xx}^{-1/2} (\mathbf{\Pi}^\perp_{\mathbf{B}^*} \otimes \mathbf{\Pi}_{\tilde{\mathbf{U}}} + \mathbf{\Pi}_{\mathbf{B}^*} \otimes \mathbf{I}_W) \breve{\mathbf{R}}_{xx}^{-H/2} \tag{15}$$

Since $\sum_{\ell=1}^{L} \text{tr}\{\text{CRB}(\hat{\mathbf{h}}_{\text{MS}}(\ell))\} = \text{tr}\{\text{CRLB}(\hat{\mathbf{h}})\}$, the upper bound on the average performance over L slots can be obtained as

$$\begin{aligned} \text{MSE}_{\text{CRB}}(L) &= \frac{1}{L} \text{E}[\text{tr}\{\text{CRB}(\hat{\mathbf{h}})\}] = \frac{1}{L} \text{tr}\{\text{CRB}(\hat{\mathbf{h}})\} \qquad (16) \\ &= \frac{\sigma_n^2}{L} ((L-r) \text{tr}\{\mathbf{R}_{xx}^{-1/2} \mathbf{\Pi}_{\tilde{\mathbf{U}}} \mathbf{R}_{xx}^{-H/2}\} + r \text{tr}\{\mathbf{R}_{xx}^{-1}\}), \end{aligned}$$

where the operator $\text{E}[\cdot]$ denotes the expectation w.r.t. fading. For $r = W$ the result (16) equals the performance of the LSE, that is $\sigma_n^2 \text{tr}\{\mathbf{R}_{xx}^{-1}\}$. For $L \to \infty$ the lower bound reduces to:

$$\text{MSE}_{\text{CRB}}(\infty) = \frac{\sigma_n^2}{L} \text{tr}\{\mathbf{R}_{xx}^{-1/2} \mathbf{\Pi}_{\tilde{\mathbf{U}}} \mathbf{R}_{xx}^{-H/2}\}. \tag{17}$$

In the case of ideal training sequences, $\mathbf{R}_{xx}^{-1} = N\sigma_x^2$,

$$\text{MSE}_{\text{CRB}}(L) = \frac{r\sigma_n^2}{N\sigma_x^2} \frac{L+W-r}{L} = \text{MSE}_{\text{CRB}}(\infty) \frac{L+W-r}{L}. \tag{18}$$

For $L \to \infty$ the lower bound $\text{MSE}_{\text{CRB}}(\infty) = r\sigma_n^2/(N\sigma_x^2)$ depends only on the ratio between the number of parameters to be estimated on a slot by slot basis (r) and the training sequence length (N).

6 Simulation results

The performance of the multislot channel estimate is evaluated by simulating the uplink of the UTRA-TDD standard. The frame has duration $T_f = 10$ ms and is divided into $N_f = 15$ time slots. Within each time slot there are $K = 8$ active users, the carrier frequency is $f_c = 1.95$ GHz. Data blocks contain $N_s = 61$ QPSK symbols spread by a user specific code of length $Q = 16$ and chip rate $1/T = 3.84$ Mchip/sec. The training sequences are chosen according to the standard specifications and have length $P = 512$. For each user, the

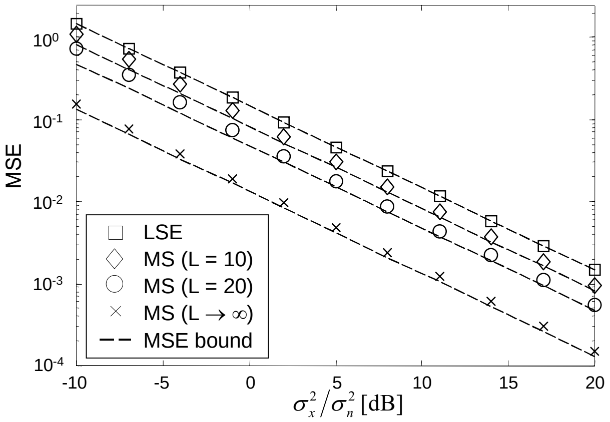

Figure 2: MSE vs. SNR for TDD-UTRA standard (TU channel model). MSE bounds (16) and (17) are in dashed-lines.

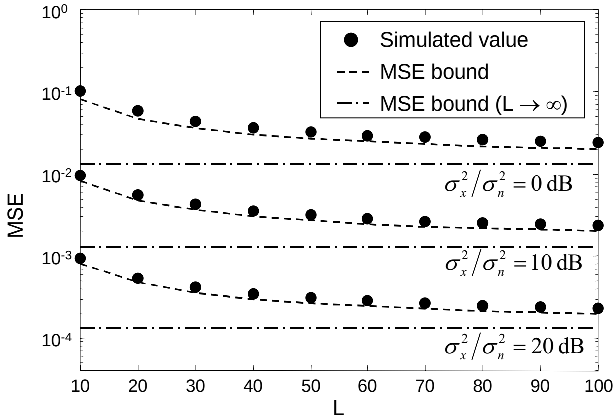

Figure 3: MSE vs. L for the multi-slot method (same parameters as Fig.2). The MSE bounds (16) and (17) are in dashed and dashed-dotted lines.

maximum number of channel unknowns is estimated, that is $W = 57$, hence $N = 456$. The multipath channel is simulated as the typical urban environment (TU, [3]). As the number of paths is $d_k = 6$ and the delays are resolvable, the temporal diversity is $r_k = 6$.

Figures 2 and 3 compare the performance of the least squares and the multi-slot estimates. The normalized MSE is evaluated for $L = 10, 20$, $L \to \infty$ and varying SNR in Fig. 2, for SNR $= \{0, 10, 20\}$ dB and varying L in Fig. 3. The signal to noise ratio is defined as SNR$= \sigma_x^2/\sigma_n^2$. The figures show both the simulated (marker) and the analitical (dashed line) bound $\mathrm{MSE}_{\mathrm{CRB}}$. Since the training sequences are almost uncorrelated, the degradation of performance with respect to the single user case is negligible (0.6 dB in SNR, see [1]), hence the analytical bounds (16)-(17) are close to the simulations. The results show that the multi-slot method outperforms the LSE and approaches the bound $\mathrm{MSE}_{\mathrm{CRB}}$ for large SNR. Asymptotically the multi-slot method provides a remarkable improvement of performance (according to (17) approx. 10 dB in SNR).

The (average) performance in terms of uncoded bit error rate for a MMSE block multi-user detector is shown in Fig. 4 for varying signal to noise ratio, defined here as SNR $= \mathrm{E_{bit}}/N_0$. The simulated BER is obtained by averaging over the fading, the noise and the data. Near-far effects are not considered. This example shows that the multi-slot estimate guarantees a gain with respect to the LSE of approximately 2 dB for $L = 20$ and up to 4 dB for a large number of slots. In this latter case, the BER performance approaches those for known channel with a negligible loss ($<$ 1dB in SNR).

7 Conclusions

The paper shows analytically and through simulations that the multi-slot method guarantees (for large L) a gain with respect to the single-slot techniques equal to W/r in terms of MSE and approximately 3 dB in terms of BER for the TDD-UTRA system.

References

[1] H. Holma, A. Toskala, *WCDMA for UMTS: Radio Access for Third Generation Mobile Communications*, John Wiley & Sons, Ltd. 2000.

[2] CWTS, “China Wireles Telecommunication Standard (CWTS) Working Group 1 (WG1): Physical channels and mapping of transport channels onto physical channels”, TS C102 v3.3.0 (2000-09).

[3] G.L. Stuber, *Principles of mobile communication*, Kluwer Academic Publisher, 1996.

[4] G.H. Golub,C.F. Van Loan, *Matrix computations*, The Johns Hopkins University Press, 1996.

[5] P. Stoica, T. L. Marzetta, “Parameter estimation problems with singular information matrices”, *IEEE Trans. Signal Processing*, vol. 49, no. 1, pp. 87-90, Jan. 2001.

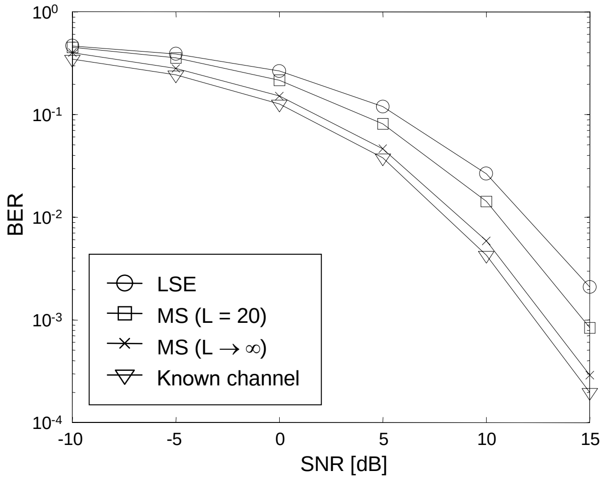

Figure 4: Average BER for MMSE block multi-user detection (TDD-UTRA standard, TU channel).