ESTIMATION OF SPATIAL SPECTRA IN A NON-STATIONARY ENVIRONMENT

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ABSTRACT

The problem of computing the time-bearing representation for data from a sensor array is considered. A novel approach to dealing with situations where the sources are mobile is presented. The methodology adopted is based on ideas borrowed from the field of time-frequency analysis. Through simulations it is demonstrated that the new approach produces time-bearing plots with greater resolution than conventional methods. Further it is demonstrated that this approach yields bearing estimates with smaller variances than can be achieved using conventional techniques.

1. INTRODUCTION

In a sonar system data is often presented in the form of a time-bearing representation (display) [1]. These representations exploit the data from an array of L hydrophones to construct an estimate of the energy arriving at the array from a particular bearing during a time interval. Here a narrow-band time-bearing representation is considered; broad-band representations can be considered as summing the results from a series of narrow-band representations.

Most work related to these representations considers the problem of estimating the spatial spectrum for a particular time, this corresponds to effectively computing a single line of a time-bearing plot. The problem of computing a spatial spectrum from signals measured via an array is closely related to the time domain problem of estimating a conventional spectrum from a time series. Indeed methods derived for one purpose can almost always be applied to the other [2].

It is one purpose of this paper to draw a parallel between the problems of computing a time-bearing representation from sensor array data and computing a time-frequency representation [3] from a single time-series. As shall be shown, whilst these problems do have similarities they are not directly equivalent and some modification is required before time-frequency methods can be applied to time-bearing representations.

This paper begins with a recap of conventional methods for spatial spectrum estimation and how these are applied to time-bearing representations. The salient points from time-frequency analysis will then be presented. Finally the application of some of the philosophies underlying time-frequency analysis is considered to the problem of constructing a time-bearing display.

2. SPATIAL ENERGY SPECTRA

The conventional problem of spatial spectral estimation can be summarized as: Given the set of vector ($L\times1$) measurements, $\underline{x}_n$, $n=0,1,2,\ldots N-1$, can one construct an estimate of the energy spectrum $S(\theta)$? The underlying data model is [4,5]:

$$\underline{x}_n = H(\underline{\theta})\underline{a}_n + \underline{w}_n$$

where there are assumed to be K, distinct, point sources, the ($K\times1$) vector, $\underline{a}_n$, contains the K source amplitudes at time n, with $H(\underline{\theta})$ being the ($L\times K$) steering matrix (each column of which is the steering vector associated with one of the discrete sources), $\underline{\theta}$ is the ($K\times1$) vector containing the source bearings and $\underline{w}_n$ is a ($L\times1$) vector representing the spatially disperse noise. A random model for the source amplitudes $\underline{a}_n$ is assumed. Much of the following is independent of array geometry but to aid discussions a Uniform Linear Array (ULA) geometry is assumed.

Conventional approaches to estimation of a spatial spectrum are based on analysis of the data correlation matrix, namely by consideration of:

$$R = \frac{1}{N}\sum_{n=0}^{N-1} \underline{x}_n \underline{x}_n^{\,h}$$

where ^{h} denotes conjugate transposition and the scaling factor $1/N$ may be omitted without penalty (it is included here so that the R can be truly considered as an approximation to the spatial correlation matrix).

Given the spatial correlation matrix one can compute various estimates of the spatial spectrum, three commonly considered examples of which are: the conventional beamformer estimate, $S_{CBF}(\theta)$ [5], the Capon estimate, $S_{Capon}(\theta)$ [5] and the MUSIC spectrum $S_{MUSIC}(\theta)$ [4,5], these are defined as:

$$S_{CBF}(\theta) = \underline{h}(\theta)^h R \underline{h}(\theta)\,,\ S_{Capon}(\theta) = \left\{\underline{h}(\theta)^h R^{-1} \underline{h}(\theta)\right\}^{-1},$$

$$S_{MUSIC}(\theta) = \left\{ \sum_{k=K+1}^{L} \left| \underline{h}(\theta)^h \underline{u}_k \right|^2 \right\}^{-1}$$

where $\underline{u}_k$ are the eigenvectors of R, in order of increasing eigenvalues. For each of these methods to be successful one needs to assume that the sources are stationary over the N snapshots used to compute R.

If one, or more, of the sources move during the N snapshots then it is apparent that a degradation in performance will be incurred (the construction of R is based on a stationarity assumption that is no longer valid). Specifically the spectral peak associated with the moving source will be spread, leading to a loss of resolution. The degree of spreading will depend upon the rate of change of bearing and the length of the observation. The significance of this spreading effect depends upon several factors. For bearings close to end-fire, where bearing resolution is relatively poor, a greater degree of spreading can be tolerated than for sources close to broadside, where the bearing resolution is initially best. Similarly for large arrays, with good bearing resolution, only modest source motions lead to significant degradations in performance.

The conventional approach to constructing a time-bearing plot is to divide the incoming array data into blocks of N snapshots and make a quasi-stationarity assumption (*i.e.* to assume that N is sufficiently small to make the stationary assumption justifiable). This allows one to apply any of the conventional spatial spectral estimators to each block. In general one aims to select N as large as possible without violating the quasi-stationarity assumption.

3. TIME-FREQUENCY ANALYSIS

Time-frequency analysis aims to compute a representation that reflects the time-varying spectrum for a time-series, $x(t)$. The oldest approach to time-frequency analysis is the spectrogram; which is constructed by windowing the data stream and invoking a quasi-stationarity assumption to allow one to perform a Fourier based spectral estimation. The resolution of the spectrogram is controlled by choice of the duration of windowing function, which controls both the temporal and frequency resolution.

A more general class of bilinear time-frequency representations (Cohen's class) is obtained through smoothing the Wigner-Ville Distribution (WVD). The WVD, $W(t,f)$, is defined as [3]:

$$W(t,f) = \int_{-\infty}^{\infty} x(t-\tau/2)^* x(t+\tau/2) e^{-2\pi i f\tau} d\tau = \int_{-\infty}^{\infty} r(t,\tau) e^{-2\pi i f\tau} d\tau$$

$$r(t,\tau) = x(t-\tau/2)^* x(t+\tau/2)$$

where * denotes complex conjugation. Evidently the WVD is the Fourier transform (w.r.t. τ) of the instantaneous correlation function $r(t,\tau)$.

The WVD is particularly well suited to analyzing signals that are linear chirps. This can be seen by considering a linear chirp signal of the form:

$$x(t) = Ae^{2\pi i(\alpha t^2 + \beta t)}$$

in which case one can show that:

$$r(t,\tau) = |A|^2 e^{2\pi i(2\alpha t + \beta)\tau}$$

To compute the WVD this is Fourier transformed yielding a delta function located at $f=2\alpha t+\beta$ [6], which can be regarded as the ideal representation for a linear chirp.

Note that the WVD has succeeded in converting the linear chirp into a constant frequency signal that is perfectly suited for Fourier analysis. One can view this as applying a non-linear transformation (computation of the instantaneous correlation function) to convert a non-stationary signal into a stationary signal.

The parallels between the spectrogram and a conventional time-bearing display should be evident, both rely upon segmenting the signal into short time intervals over which a quasi-stationarity assumption is appropriate. This paper seeks to illustrate how a principle, inspired by the WVD, can be applied to the time-bearing problem. The goal is to identify a non-linear transformation that will render array data more stationary.

4. THE WVD PRINCIPLE APPLIED TO ARRAYS

4.1 Principles

The non-stationary situation considered here is that of a signal model of the form:

$$\underline{x}_n = H(\underline{\theta}_n)\underline{a}_n + \underline{w}_n$$

The columns of the steering matrix remain constant but the bearing varies as a function of sample number. The manner of this variation is assumed to be such that the individual steering vectors can be written as:

$$\underline{h}(\theta_n) = \left[1 \quad e^{i(\alpha n+\beta)} \quad e^{2i(\alpha n+\beta)} \quad \cdots \quad e^{(L-1)i(\alpha n+\beta)}\right]^t$$

Hence the assumption is that the electric angle (rather than bearing) varies as a linear function of time. Such variations are not unreasonable, since a source undergoing constant velocity linear motion will induce a linearly varying electric angle on a ULA. For a ULA the electric angle, ϕ, is related to bearing through:

$$\phi = 2\pi d \sin(\theta)/\lambda$$

where d is the inter-sensor spacing within the ULA and λ is the wavelength of the source signal.

The non-linear transformation used in this case is:

$$\underline{v}_n = \underline{x}_n \otimes \underline{x}_{N-n-1} \quad n = 0,1,...,(N/2-1) \qquad (1)$$

where for ease it has been assumed that N is even (the extension to the case for odd N is trivial) and $\otimes$ represents a Hadamard (element-wise) product. It is also obvious that $\underline{v}_{N-k-1}=\underline{v}_k$. The effectiveness of this transformation can be seen if the case of a single non-stationary source is considered in the absence of noise, so that:

$$\underline{x}_n = a_n \underline{h}(\theta_n)$$

where the bearing is assumed to conform to the model of a linearly varying electric angle, $\phi_n=\alpha n+\beta$. In which case the vector $\underline{v}_n$ is given by:

$$\underline{v}_n = a_n a_{N-n-1}\left[1 \quad e^{i(\alpha(N-1)+2\beta)} \quad \cdots \quad e^{(L-1)i(\alpha(N-1)+2\beta)}\right]^t$$

$$= a_n a_{N-n-1}\underline{h}(2\theta_m) = b_n \underline{h}(2\theta_m) \qquad b_n = a_n a_{N-n-1}$$

where θ_m is the mean angle given by $\alpha(N\text{-}1)/2+\beta$. The critical observation is that the vector $\underline{v}_n$ is stationary in the sense that its steering vector is invariant with n.

The methodology proposed here to compute the correlation matrix for the vectors $\underline{v}_n$:

$$C = \frac{2}{N}\sum_{n=0}^{N/2-1} \underline{v}_n \underline{v}_n^h$$

The summation only extends to $N/2-1$ on account of the symmetry in the vectors $\underline{v}_n$. The matrix C is a quartic function of the data, so contains information about 4th order statistics.

The matrix C can be employed in place of the matrix R in the standard array processing algorithms, *e.g.* CBF, Capon and MUSIC. Once this substitution is made one also needs to employ the steering vector $\underline{h}(2\theta)$. The effect of these substitutions is that the algorithms then become insensitive to motions that induce linear variations in the electric angle, leading to an increase in resolution of the time-bearing plot. This can be compared with the manner in which the WVD has increased resolution in comparison to the spectrogram (although the two mechanisms by which the resolution increases are achieved are subtly different). To reflect this similarity the technique is referred to as the Wigner Beamformer (only application of these techniques to a CBF is presented here).

It is interesting to note that the amplitudes b_n are formed as the product of two of the original amplitudes. Hence if one assumes Gaussianity for a_n, then the b_n will not have a Gaussian distribution.

4.2 Limitations

The above methodology does introduce a variety of issues that need to be addressed. This section aims to outline how these limitations can be overcome. Details of the approaches will be given elsewhere.

The 2 major problems that need to be overcome are considered in the next two subsections.

4.2.1. Introduction of Aliasing

The multiplication used to form $\underline{v}_n$ doubles the spatial resolution needed to accurately represent the data. If this is not appropriately dealt with then aliasing will be introduced. This problem is identical to that encountered when computing the WVD of a real signal [7]. In the case of the WVD the most widely employed solution is to convert the signal into its analytic form prior to computing the WVD. In its analytic form the signal only occupies half the frequency band and can be multiplied by itself without introducing aliasing. Such a solution can not be adopted in this application because the array data is already in complex form and so one can not generate an analytic representation.

The solution adopted here is to implement (1) through convolution in the frequency domain, where the bandwidth can be readily extended by appending zeros to the Fourier representation. Alternatively this can be interpreted as interpolating the data. The implementation of this scheme is made simple by the choice of a ULA structure.

4.2.2. Interference Terms

The major drawback of the WVD is the fact that it exhibits interference terms: if a signal contains 2 distinct components then the resulting WVD contains 3 elements, the additional element arising from the interaction of the 2 “real” elements. The generation of these terms arises from the quadratic nature of the WVD. In time-frequency analysis the interference terms are characterized by their oscillatory nature in the time-frequency plane. This oscillatory nature means that the interference terms can be attenuated through application of a 2-D smoothing operation.

The application of the preceding methodology generates interference terms through a mechanism similar to that encountered in the WVD. However the interference terms generated through (1) have the undesirable property of being non-oscillatory. Similar non-oscillating interference terms are also associated with the 4th order Wigner distribution [8,9]. The methodology applied in [8,9] can be used to reduce interference terms in this application.

5. RESULTS

Figure 1 shows time-bearing plots computed for simulated data that mimics a source moving parallel to a 32 element ULA (sensor spacing 0.5λ). During this maneuver 1000 snapshots are generated with the bearing varying between

-80° and 80°. The measurements are corrupted by spatially uncorrelated Gaussian noise at 0 dB.

The first three frames in Figure 1 are computed using different block sizes (*N*). The blocks are overlapped such that 10 new snapshots are introduced for each new line. In these plots time is shown down the page, bearing is plotted horizontally (0º corresponding to broadside) and dark shades represent high energies on a linear scale. One can see that short block sizes result in little spreading of the source track. However the use of short blocks implies that the bearing estimates are less reliable (they will have greater variance). Larger data block sizes result in more spreading of the track, directly leading to greater uncertainty in the source bearing estimates. The final frame shows the result of applying the Wigner Beamformer. The computed track is much better resolved. Note also that the full range of azimuth (allowing for left-right ambiguity in the array) has been exploited, highlighting that no aliasing problems appear.

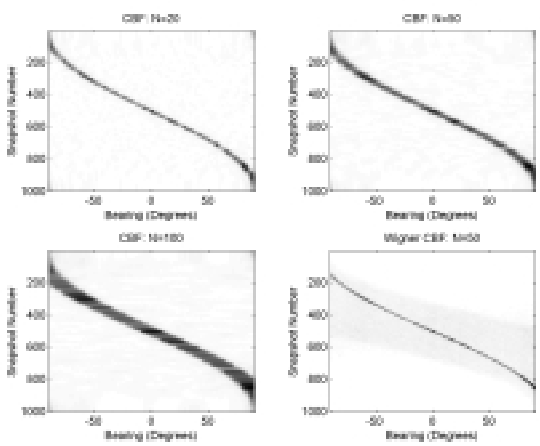

Figure 1: Time-Bearing Plots for a Source Undergoing a "Drive Passed" Maneuver

Figure 2 illustrates the results of a series of simulations exploring the accuracy of the Wigner Beamformer. For each trial 100 different realizations of a single block of 100 snapshots is computed (the SNR is 0dB). The spatial spectrum is computed for each block and the peak of that spectrum is taken as the estimate of the source bearing. The variance of these estimates is computed for CBF and the Wigner beamformer. For each trial a different maneuver is considered, starting at -θ and ending at θ. Figure 2 plots the variances (on a dB scale) as a function of the angle traversed (2θ).

Figure 2 illustrates several properties of the Wigner Beamformer. When there is no source motion CBF has a lower variance than the Wigner Beamformer. This is to be expected since the CBF is known to be optimal for a single stationary source in spatial white Gaussian noise. However the effect of relatively small motions <1º is to make the Wigner Beamformer yield a better estimate of mean bearing. As the motion is increased, so that very large motions are experienced during a block, the performance of the Wigner Beamformer remains stable, whilst the CBF's performance degrades dramatically.

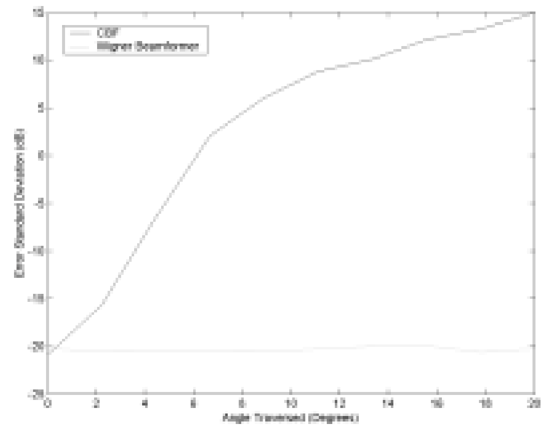

Figure 2: Performance of Wigner Beamformer for Different Levels of Source Motion

6. CONCLUSIONS

This paper has described a novel methodology for obtaining time-bearing representations. The technique relies upon employing non-linear operations on the data to partly mitigate some of the effects of source motion. It is shown that this technique yields significantly better behavior for mobile targets than that which can be achieved by the CBF.

7. REFERENCES

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