Optimal Filtering of LORAN-C Signal via Particule Filtering Simulation Results

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Abstract: The aim of this paper is to present the application of particle filtering to LORAN-C navigation. After consideration of preprocessing treatment, one derives a complete modeling of the global problem. The model of the received wave is particularly adapted to the particle context and allows to reduce the number of particles required. Simulation results are given.

1 Introduction.

LORAN-C radionavigation system is based on the evaluation of propagation delays between terrestrial stations and receiver. Due to the propagation effects, only ground wave is reliable. For distances greater than $2000km$, the groundwave is so attenuated (about $-30dB$) that classical summation-based algorithms are not efficient as there is no way to set off the receiver movement. To this end, we present here an application of non-linear dynamic filtering using the particle technique originally introduced in [1], and patented in [2], which is practicable, thanks to the model of skywave.

2 Preprocessing and sampling.

Recall that the LORAN-C signal is defined as a repeated sequence of impulsive modulations:

$$s(t) = \left(\frac{t}{t_p}\right)^2 exp\left(\frac{-2(t-t_p)}{t_p}\right) sin(2\pi f_0 t) \qquad \textbf{(2.1)}$$

with $t_p = 65\mu s$ and $f_0 = 100kHz$.

Although main processing has to be achieved under non linear numerical form, analog linear preprocessing is required before sampling. This preprocessing must mainly satisfy the three following goals:

- Its output must produce the best signal to noise ratio.
- The reception additive white noise must remain white after filtering and sampling. This is required to apply non-linear optimal filtering theory in standard form.
- After sampling, the whole useful information must be preserved.

2.1 Matched filter.

Concerning the first point, the classical solution is to use the linear matched filter which is devoted to maximize the SNR. Recall that its impulse response is defined by $h(t) = s(-t)$. Besides the fact that such a filter is unrealizable, the main drawback of this approach is that noise is no longer white after filtering, which is in contradiction with the second point. As the spectrum of the filtered noise is the same as that of the signal, the autocorrelation of the noise is almost f_0-periodic and is defined by:

$$R(t) = \frac{1}{2} e^{\frac{-2(t-t_p)}{t_p}} cos(2\pi f_0 t) \int_R \left(\frac{t-\tau}{t_p}\right)^2 \left(\frac{\tau}{t_p}\right)^2 cos(2\pi f_0 \tau) d\tau \qquad \textbf{(2.2)}$$

It is clear that no sampling frequency can make samples to be uncorrelated.

2.2 Bandpass filter.

As the main power of the LORAN-C signal is concentrated in the band $[75kHz, 125kHz]$, an alternative solution consists in using a bandpass filter centered on $f_0 = 100kHz$ with $\Delta f = 50kHz$ as bandwidth. The loss generated by this filter, compared to the matched filter, may be quantified as follows. Let $S(f)$ be the Fourier transform of the signal $s(t)$ defined over the interval $[0, 65\mu s]$. The energies of the filtered signal (respectively of the noise), using the matched filter, may be computed as:

$$E_s^M = \int_R |S(f)|^4 df \,,\; E_b^M = \sigma^2 \int_R |S(f)|^2 df \qquad \textbf{(2.3)}$$

where σ^2 is the power spectral density of the noies.
When using a bandpass filter, these energy are:

$$E_s^B = \int_{f_0 - \frac{\Delta f}{2}}^{f_0 + \frac{\Delta f}{2}} |S(f)|^2 df \,,\; E_b^B = \sigma^2 \Delta f \qquad \textbf{(2.4)}$$

The loss involved by using the bandpass filter is then:

$$(E_s^M E_b^B)/(E_s^B E_b^M) \qquad \textbf{(2.5)}$$

In our case, this loss may be evaluated as $7dB$. The autocorrelation of the filtered white noise is defined by:

$$R(t) = 2\sigma^2 \Delta f\, cos(2\pi f_0 t) \frac{sin(\pi \Delta f t)}{\pi \Delta f t} \qquad \textbf{(2.6)}$$

The sampling period $\Delta t = 20\mu s$ makes sure that the samples are uncorrelated as these points lies on the zeros of the autocorrelation function.

2.3 Demodulation by under-sampling.

In order to achieve the third point, one usually uses a bandpass filter with a cutoff frequency f_M corresponding to the greatest frequency of the signal. The sampling frequency must then be at least equal to $2f_M$ in agreement to the Shannon theorem. Note that there is an other way to achieve this point in our case, that is to use a sampling frequency $f_e = 50kHz$. With such a procedure, the aliasing of the spectrum preserves the original signal. Note that in the band $[75kHz, 125kHz]$, the spectra coincide. One then realizes the demodulation of the signal at the frequency f_0 as it is suggested in [3]. Moreover, this sampling frequency preserves the whiteness of the sampled noise due to the term $sin(\pi \Delta f t)$ in (2.6).

2.4 Quadrature sampling.

Because the sampled signal power is not proportional to the power of the continuous signal, as it depends on the position of the samples, it is necessary to realize an other sampling in quadrature, that is delayed of the quarter of the carrier period ($2,5\mu s$). With such a procedure, ones makes sure that the sum of the power of each channel is equal to the power of the continuous signal. Note that this technique is nearly equivalent to a classical two channel demodulation.

Due to the term $cos(2\pi f_0 t)$ in (2.6), it is clear that the samples of the filtered white noise are uncorrelated as the autocorrelation is zero for all $t = k/\Delta f, \forall k \in N$ and for all $t = k/f_0 \pm 1/(4f_0), \forall k \in N$, that is for all samples separated by $(20k \pm 2{,}5)\mu s$.

3 Modelling.

3.1 Carrier dynamics.

The carrier movement is measured through the course and speed data. These measures are of course associated to an error that may be evaluated to about $10\,\%$ for classical systems. In order to derive a model which takes into account the time constant of the sensors, we adopt here as a model of the observers a first order linear system. Let δv_t and δk_t be the errors of speed and course respectively. The dynamic evolution of these errors is defined by:

$$\begin{aligned} d(\delta v_t) &= -a\delta v_t dt + \sigma_v d\beta_t \\ d(\delta k_t) &= -a\delta k_t dt + \sigma_k d\beta_t' \end{aligned} \tag{3.1}$$

where σ_v end σ_k stand for the standard deviation of the error of speed and course respectively, a is a time constant, and where β_t and β_t' are standard Wiener processes.

After sampling at the rate Δt, the evolutions of the errors may be represented as follows:

$$\begin{aligned} \delta v_{t+\Delta t} &= (1 - a\Delta t)\delta v_t + \sigma_v \sqrt{\Delta t}\; w_t \\ \delta k_{t+\Delta t} &= (1 - a\Delta t)\delta k_t + \sigma_k \sqrt{\Delta t}\; w_t' \end{aligned} \tag{3.2}$$

where w_t and w_t' are discrete normalized white noises. The evolution of the position of the carrier is then defined by:

$$\begin{aligned} x_{t+\Delta t} &= x_t + (v_t + \delta v_t) sin(k_t + \delta k_t) \\ y_{t+\Delta t} &= y_t + (v_t + \delta v_t) cos(k_t + \delta k_t) \end{aligned} \tag{3.3}$$

where v_t and k_t stand for the speed and course measured.

3.2 Observation signal.

The signal received by the antenna is the sum of the groundwave and the skywave. Theoretically, the skywave is a copy of the ground wave with a different amplitude and a time delayed. A natural way of modeling such a signal would be to introduce a new state variable for each station through the time delay of the skywave. Unfortunately, such a model would lead to an independent particle variable for each station which is not tractable in practice due to prohibiting state space dimension in such a case. Moreover, the skywave phenomenon is not really mastered and the possible deformation of the wave would not be taken in account with such a model. Also, the possible superposition of several skywaves is often considered [4]. For all these reasons, we adopt here a model of the skywave as the superposition a large number of copies of the groundwave with fixed time delays but with unknown amplitudes:

$$z_t = \sum_{k=1}^{N_s} a_t^k s^k(t - D_t^k/c) + \sum_{k=1}^{N_s} \sum_{j=1}^{M} A_t^{k,j} s^k(t - D_t^k/c - \tau_{k,j}) + v_t \tag{3.4}$$

where:

- N_s : number of stations considered.
- a_t^k : amplitude of the ground wave (unknown).
- $s^k(t)$: signal of the station k.
- D_t^k : distance between emitter and receiver (unknown).
- c : propagation speed of the wave.
- $\{A_t^{k,j}\}_{j=1}^M$: amplitude of the skywaves (unknown).
- $\tau_{k,j}$: delay of the k-th skywave (fixed).
- v_t : additive white noise.

4 Particle filtering.

4.1 Principle.

In the general case, the problem of non-linear filtering is to compute the density of the state x_t conditionally to the observation trajectory $y_0^t = \{y_0, \ldots, y_t\}$, that is $p(x_t|y_o^t)$. It is well known that the optimal filter is not computable in finite dimension in the general case, except for the gaussian-linear case which leads to the so called Kalman filter. The progress of computational power makes possible the development of particule based filtering algorithms. The aim of this technique is to construct an approximation of the probability density with a discrete measure as:

$$p(x_t|y_0^t) \cong \sum_{i=1}^{N} \rho_t^i \delta_{x_t^i}(x_t) \tag{4.1}$$

where $\delta_{x_t^i}(x_t)$ stands for the Dirac measure on the point x_t^i (called particle) and ρ_t^i is the level of this particle. Realization of the particle trajectories x_t^i is achieved via simulation of the state space model. The computation of ρ_t^i is achieved through Bayes formula from the observation equation. More precisely, let:

$$x_{t+\Delta t} = f(x_t, w_t) \tag{4.2}$$

be the state space model where w_t is white noise of known distribution (not necessary gaussian). A trajectory of x_t^i is obtain by simulation of the model, that is by an independent drawing of w_t^i according to its distribution. Then:

$$x_{t+\Delta t}^i = f(x_t^i, w_t^i) \tag{4.3}$$

This Monte Carlo procedure leads to an approximation of the a priori distribution of the state, that is:

$$p(x_t) \cong \frac{1}{N} \sum_{i=1}^{N} \delta_{x_t^i}(x_t) \tag{4.4}$$

If y_t stands for the observation process, the Bayes rule leads to:

$$p(x_0^t|y_0^t) = \frac{p(y_0^t|x_0^t)p(x_0^t)}{p(y_0^t)} \tag{4.5}$$

If one replace the approximation of the a priori density in this formula, one may write the marginal probability $p(x_t|y_0^t)$, integrating over $\{x_0, \ldots, x_{t-1}\}$:

$$p(x_t|y_0^t) \cong \frac{\frac{1}{N}\sum_{i=1}^{N} p(y_0^t|(x_0^t)^i)\delta_{x_t^i}(x_t)}{\frac{1}{N}\sum_{i=1}^{N} p(y_0^t|(x_0^t)^i)} \tag{4.6}$$

which is clearly of the form of (4.1).
The minimum variance estimator is then computed with:

$$\hat{x}_t = \sum_{i=1}^{N} \rho_t^i x_t^i \tag{4.7}$$

4.2 Conditionally gaussian case.

The number of particles may be significantly reduced by the exploitation of some linearities of the model as it will appear now.

Suppose that the state space may be separated in two sets as:

$$\begin{aligned} x_{t+\Delta t} &= f(x_t, w_t) \\ \theta_{t+\Delta t} &= F(x_t)\theta_t + G(x_t)w_t' \\ y_t &= H(x_t)\theta_t + v_t \end{aligned} \tag{4.8}$$

where w_t' and v_t are gaussian white noises. It is clear that for all x_t known, the model of θ_t is linear-gaussian. As a consequence, the optimal estimation of θ_t may be achieve using the Kalman filter. Let $\hat{\theta}^i_{t+\Delta t|t}$ be the kalman predictor of θ_t computed for the particle x^i_t. In such a case, the innovation process $\tilde{y}^i_{t+\Delta t|t} = y_t - H(x^i_t)\hat{\theta}^i_{t+\Delta t|t}$ is a gaussian white noise of known variance:

$$\Sigma^i_t = H(x^i_t)P^i_{t+\Delta t|t}H(x^i_t)^T + R \tag{4.9}$$

where R stands for the variance of the gaussian white noise v_t and where $P^i_{t+\Delta t|t}$ is the prediction variance error of the Kalman filter, that is:

$$P^i_{t+\Delta t|t} = E[(\theta_{t+\Delta t} - \hat{\theta}^i_{t+\Delta t|t})(\theta_{t+\Delta t} - \hat{\theta}^i_{t+\Delta t|t})^T] \tag{4.10}$$

The probability of the observation trajectory may now be written:

$$p(y^t_0|x^t_0) = \prod_{k=0}^{t/\Delta t} \frac{1}{\sqrt{(2\pi)|\Sigma^i_{k\Delta t}|}} e^{-\frac{1}{2}\frac{(y_{k\Delta t} - H(x^i_{k\Delta t})\hat{\theta}^i_{k\Delta t|(k-1)\Delta t})^2}{\Sigma_{k\Delta t}}} \tag{4.11}$$

The likelihood takes then the following recursive form:

$$V^i_t = V^i_{t-\Delta t} - \frac{1}{2}\left(\frac{(y_t - H(x^i_t)\hat{\theta}^i_{t|t-\Delta t})^2}{\Sigma^i_t} + \ log 2\pi(\Sigma^i_t)\right) \tag{4.12}$$

The main advantage of this approach is to reduce the dimension of the state to be estimated by a particle algorithm, and consequently to reduce the number of particle required.

4.3 Regularization.

Unfortunately, one may derive that such a procedure doesn't make sure the uniform convergence of the discrete approximation as the number N of particles would have to grow with time which is inconsistent with a real time filtering application.

However, a resampling techniques may be used to make sure the uniform convergence of the algorithm. The idea is that after some filtering time, some particles are no more relevant to represent the probability density as their level are no more significant. When such a situation occurs, it is necessary to resample the distribution representation in view to emphasize the area of great level. To achieve this goal, it suffices to realize periodically a draw of the N particles according to the probability obtained at this step. As a consequence, some particles will die as some others will birth in the high level area.

5 Application to LORAN-C signal.

5.1 Distinction between groundwave and skywave.

The main difficulty in LORAN-C filtering is to distinguish within the ground wave and skywave. This is more and more crucial when the distance between station and receiver is over $2000km$ because the ratio within skywave and groundwave may reach $20dB$. A classical error in such a situation is to catch the skywave in place of the groundwave. To avoid such a pitfall, one has to take into account that the groundwave must have an amplitude which is not null and, furthermore, limited to an interval which depends on the distance station/receiver. The values to assign to this interval may be computed from [5].

Suppose that a parameter a is assigned to lay on an interval $D = [b, c]$. Suppose that this parameter is observed trough a gaussian process of the following form:

$$y_t = h_t a + v_t \tag{5.1}$$

where v_t is a gaussian white noise of variance R. One then may derive [6] that:

$$p(a|y^t_0) = \frac{1}{\alpha_t} 1_{[x_t \in \mathcal{D}]} e^{-\frac{1}{2}(a-\bar{a}_t)^T P_t^{-1}(a-\bar{a}_t)} \tag{5.2}$$

where $\bar{a}_t$ is the kalman filter of the parameter a and where α_t is again a normalization term. P_t stands for the covariance of the kalman estimator. Furthermore, the likelihood of y^t_0 may be written as follows:

$$\begin{aligned} V(y^t_0) = &-\frac{1}{2}\sum_{\tau=0}^{t}(y_\tau - h_\tau\bar{a}_{\tau-1})^T(h_\tau P_{\tau-1}h_\tau^T + R)^{-1}(y_\tau - h_\tau\bar{a}_{\tau-1}) \\ &+ Log\left(\frac{\alpha_t}{\alpha_0}\right) - (t+1)Log(\sqrt{(2\pi)^m R}) \end{aligned} \tag{5.3}$$

where the normalization terms are defined by:

$$\alpha_t = \left(erf\left(\frac{c-\bar{a}_t}{\sqrt{|P_t|}}\right) - erf\left(\frac{b-\bar{a}_t}{\sqrt{|P_t|}}\right)\right)\sqrt{(2\pi)^n|P_t|} \tag{5.4}$$

and erf is the error function.

This property will be applied to the groundwave amplitude estimation to improve the filtering efficiency in term of rejection of the skywave.

5.2 Algorithm.

The model (3.2)-(3.3)-(3.4) is clearly of the form (4.8). The nonlinear space set is composed with the four states $\{[x_t, y_t, \delta v_t, \delta k_t]^T\}$ as the linear space set is composed of the $N_s(M+1)$ states $\{a^k_t, A^{k,j}_t, k = 1 \dots N_s, (j = 1 \dots M)\}$.

The algorithm is structured as follows:

1. Initialization.

Drawing of N particles according to the initial distribution. We adopt here to realize a uniform draw over a disk whose ray represents the initial incertitude on the position.

2. Particle evolution.

Drawing N times independently of the noises w_t and w_t'. Then use (3.2) and (3.3) to compute the evolution of the particle net.

3. Kalman filtering.

For each particles i, computation of the kalman filters conditioned in view to estimate the amplitudes of the groundwave and skywave jointly. Let:

$$H^{k,i}_t = \left[s^k\left(t - \frac{D^{k,i}_t}{c}\right) \ s^k\left(t - \frac{D^{k,i}_t}{c} - \tau_{k,1}\right) \ \dots \ s^k\left(t - \frac{D^{k,i}_t}{c} - \tau_{k,M}\right)\right] \tag{5.5}$$

the observation vector, and:

$$\xi^k_t = \left[a^k_t \ A^{k,1}_t \ \dots \ A^{k,M}_t\right] \tag{5.6}$$

the linear state space. The observation signal is then defined by:

$$z_t = \sum_{k=1}^{N_s} H_t^k \xi_t^k \tag{5.7}$$

The Kalman filter of ξ_t^k associated to the particle i and the station k is then computed according to:

$$\xi_{t|t}^{k,i} = \xi_{t-\Delta t|t-\Delta t}^{k,i} + K_t^{k,i}(z_t - H_t^{k,i}\xi_{t|t-\Delta t}^{k,i}) \tag{5.8}$$

where $K_t^{k,i}$ is the classical Kalman gain.

4. Level computation.

For each particle i, if k stands for the number of station received at time t, one computes the standard non-normalized likelihood.

$$V_t^i = V_{t-\Delta t}^i - \frac{1}{2}((z_t - H_t^{k,i}\xi_{t|t-\Delta t}^{k,i})^2(H_t^{k,i}\tilde{P}_{t|t-\Delta t}^{k,i}(H_t^{k,i})^T + R)^{-1}) \tag{5.9}$$

where $\tilde{P}_{t|t-\Delta t}^{k,i}$ is the covariance of the Kalman predictor. The levels are then computed by:

$$\rho_t^i = \frac{exp\left(-\frac{1}{2}V_t^i\right)\prod_{k=1}^{N_s}\left(erf\left(\frac{a_M^k - \bar{a}_t^{k,i}}{\sqrt{\tilde{p}_t^{k,i}}}\right) - erf\left(\frac{a_m^k - \bar{a}_t^{k,i}}{\sqrt{\tilde{p}_t^{k,i}}}\right)\right)}{\sum_{i=1}^{N}\left(exp\left(-\frac{1}{2}V_t^i\right)\prod_{k=1}^{N_s}\left(erf\left(\frac{a_M^k - \bar{a}_t^{k,i}}{\sqrt{\tilde{p}_t^{k,i}}}\right) - erf\left(\frac{a_m^k - \bar{a}_t^{k,i}}{\sqrt{\tilde{p}_t^{k,i}}}\right)\right)\right)} \tag{5.10}$$

where $[a_m^k, a_M^k]$ is the amplitude interval of the groundwave for the station k and $\bar{a}_t^{k,i}$ is the amplitude prediction of the groundwave of the station k.

5. Estimation.

The carrier position estimation is computed by:

$$\hat{x}_t = \sum_{i=1}^{N} \rho_t^i x_t^i,\ \hat{y}_t = \sum_{i=1}^{N} \rho_t^i y_t^i \tag{5.11}$$

and the standard deviations associated by:

$$\hat{\sigma}_x = \sqrt{\sum_{i=1}^{N} \rho_t^i (x_t^i - \hat{x}_t)^2},\ \hat{\sigma}_y = \sqrt{\sum_{i=1}^{N} \rho_t^i (y_t^i - \hat{y}_t)^2} \tag{5.12}$$

6. Resampling.

When the estimated distribution is degenerated, that is when the number of particle having a level under a fixed ratio of $1/N$, is greater than a treshold, the particles are resampled according to the current estimated distribution.

5.3 Simulation results.

The algorithm have been simulated for several SNR in view to evaluate the limit of detection efficiency of the particle filtering. The initial uncertainty have been fixed in a disk of ray $10km$. One has simulated 3 stations with a GRI equal to 6000. The amplitude interval was fixed to $[0.5, 1.5]$. For each stations, the amplitude of the skywave was $0dB$, $14dB$ and $20dB$ over the groundwave respectively. The delay was fixed to $65\mu s$. The interval of research of the skywave delays was fixed to $[60\mu s, 80\mu s]$ with $M = 10$ subdivisions. In all theses simulations, the clock of the receiver is supposed perfectly synchronized with those of the emitter, performing circular positioning.

On figure 1 and figure 2, one displays the longitude and latitude error associated to their standard deviations for $SNR = -30dB$, which is a typical value for distance of $2600km$ for standard stations. It appears that the algorithm converge after about $1mn30s$.

The figure 3 represent the evolution of the distribution of the longitude. It appears clearly here the multi-modes of the densities corresponding to the wavelenght of the signal, that is $3km$ corresponding to $10\mu s$.

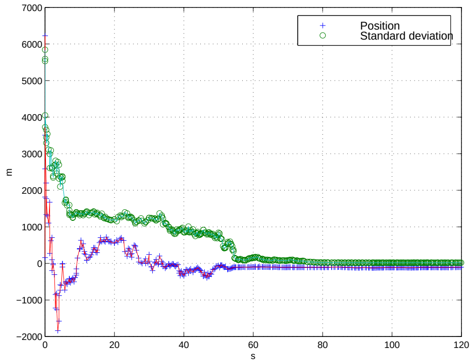

figure 1: Longitude estimation - SNR = -30 dB

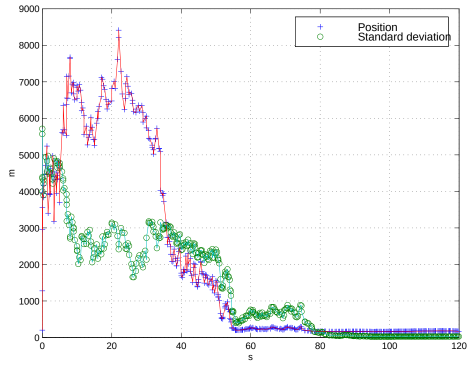

figure 2: Latitude estimation - SNR = -30 dB

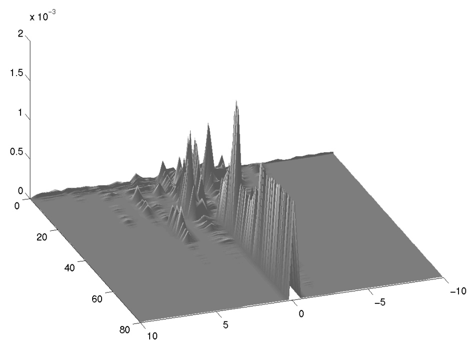

figure 3: Evolution of distribution of longitude - SNR = - 30dB

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