GENERAL PARAMETER-BASED ADAPTIVE PREDICTORS

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ABSTRACT

A class of adaptive prediction algorithms is considered for model-based digital signal processing. Based on a single adaptive parameter, the so-called general parameter, the algorithm facilitates adaptation of the predictor properties between two boundary cases. Typically, the boundaries are set according to a quick response and good noise attenuation, respectively. The adaptation algorithm is described, two example cases are discussed, and the stability condition is derived.

1 INTRODUCTION

By predictors we in this paper mean linear Finite Impulse Response (FIR) filters which are capable of unbiased extrapolation of certain signal classes, such as polynomials and sinusoidal signals [1]. Such filters find applications, for example, in velocity measurements [2][3], control systems [4], industrial electronics [5], data smoothing [6], and hybrid nonlinear filters [7].

Adaptive filters are commonly used in signal processing applications where the characteristics of the signal or noise are time-varying, or the filtering task otherwise cannot be fully fixed in advance. Many popular general-purpose adaptation algorithms exist, such as the Least Mean Square (LMS) and Recursive Least-Squares (RLS) algorithms [8]. In the standard form, such algorithms require an arithmetic operation to be done on all of the coefficients. Coefficient updates in the LMS algorithm require $N+1$ multiplications for an N-tap transversal filter. This easily leads to a high computational burden if long filters are needed due to the length of the impulse response of the system. For specific signal models, the filter may be constrained, e.g., as an adaptive notch filter [8], or as a variable-order polynomial predictor [9]. Such constrained filters typically have only one or a few adaptive parameters [10].

An example of single parameter-based adaptive algorithms is the so-called general parameter method, originally proposed for system identification by Ashimov and Syzdykov [11]. Using this algorithm for adaptive filtering, the output is computed as

$$y(n) = \sum_{k=0}^{N-1} [h(k) + g(n)]x(n-k), \tag{1}$$

where the $h(k)$'s are the coefficients of a fixed basis filter. The general parameter $g(n)$ is updated as

$$g(n+1) = g(n) + \gamma[r(n) - y(n)] \sum_{k=0}^{N-1} x(n-k), \tag{2}$$

where $r(n)$ is the reference input against which the output is compared, and γ is a gain factor.

In this paper, we consider a general parameter-based adaptive algorithm for prediction and estimation in real-time signal processing. The algorithm is described in Section 2. Examples of adaptive prediction of polynomial and sinusoidal signals are given in Sections 3 and 4, respectively, and the stability condition is derived in Section 5. The paper is concluded in Section 6.

2 GENERAL PARAMETER-BASED ADAPTIVE PREDICTORS

We consider predictors with the transfer function of the form

$$H(z,n) = P(z) + g(n)Q(z), \tag{3}$$

where $P(z)$ and $Q(z)$ are fixed FIR transfer functions, and $g(n)$ is the time-varying general parameter, $0 \leq g(n) \leq g_{max}$. There are two typical cases where such constrained single-parameter adaptive prediction can be useful.

Case I. $P(z)$ is designed as a predictor for a low-order signal model, such as a low-order polynomial. Such low-order models support good noise attenuation properties. The general parameter is used for adapting to input signals which follow a higher-order model. $g(n)$ is raised to g_{max} when the higher-order model gives smaller prediction errors, and for low-order signals the parameter is lowered back to zero, as the output noise level will then be lower.

* This work was supported by the Academy of Finland under grant 51849.

Case II. $P(z)$ is designed as a minimum-length predictor for unbiased prediction of the specified signal type, capable of responding rapidly in transient situations. Because of the short length, its noise attenuation characteristics are poor; it may even amplify noise, and is therefore used only when the signal-to-noise ratio is high. $Q(z)$ is a longer filter, designed for good noise attenuation. When the input signal is noisy, the adaptation algorithm activates $Q(z)$ by raising the value of $g(n)$.

The output of the composite predictor is given by

$$y(n) = \sum_{k=0}^{N-1} [p(k) + g(n)q(k)]x(n-k), \tag{4}$$

where the $p(k)$'s and the $q(k)$'s are the coefficients of the FIR filters $P(z)$ and $Q(z)$, respectively. The parameter update algorithm is:

$$g(n+1) = g(n) + \gamma[r(n) - y(n)] \sum_{k=0}^{N-1} q(k)x(n-k). \tag{5}$$

Equation (5) can be considered as a generalization of the standard general parameter algorithm (2), in which all the coefficients $q(k)$ are equal to one.

3 POLYNOMIAL PREDICTION

The minimum-length FIR predictor for p-step-ahead prediction of Mth-degree polynomials has the transfer function

$$H_M(z) = \sum_{i=0}^{M} (1 - z^{-p})^i. \tag{6}$$

This is known as the Newton predictor [12]. Longer predictors can be designed, for example, by minimizing the noise gain for white noise:

$$\text{NG} = \sum_{k=0}^{N-1} |h(k)|^2. \tag{7}$$

NG increases with the polynomial order. On the other hand, for polynomial signals the higher-order differences in (6) get very small numerical values, which contribute little to the prediction, and are in practice often buried in noise. Therefore, it is advantageous to design the predictor for as small M as possible. Ultimately, the predictor for $M = 0$ is an averaging filter which is known to be the optimum estimator for constant signal levels with additive white Gaussian noise. The averager has a constant group delay and does not behave as a predictor for waveforms which have a nonzero first derivative.

Here we consider a general parameter-based adaptive scheme of the form (3), where $P(z)$ is a running average and $Q(z)$ is an estimator for the first derivative. $Q(z)$ is effectively a differentiator, optimized to have the minimum NG while producing a unity-valued response to a ramp with a slope of one [13]. The benefit of such an arrangement is that $P(z)$ has a lower NG than the ramp predictor consisting of the weighted parallel connection of $P(z)$ and $Q(z)$. The ramp prediction capability is activated only when a nonzero first derivative is detected.

As an example, consider the piecewise-polynomial signal in Fig. 1(a). Such signals are encountered, for instance, in elevator car motion control systems, where predictive filtering avoids the harmful delays of linear-phase lowpass filters [12]. The signal consists of zeroth, first, and second order polynomial segments and additive white noise. Figure 2 shows the amplitude responses of $P(z)$ and $P(z) + g_{max}Q(z)$, both of length 12, with $g_{max} = 6.5$. The NG values are 0.0833 and 0.3788, respectively. The derivative estimate produced by $Q(z)$ is shown in Fig. 1(b) and a trace of $g(n)$ is shown in Fig. 1(c). $g(n)$ rises to its maximum value for the first and second order polynomial segments, thus minimizing the prediction error, and returns to zero for constant levels, again minimizing the prediction error by producing a lower noise level.

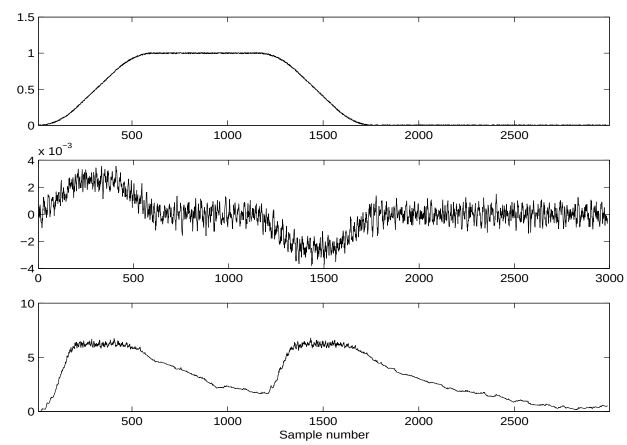

Figure 1: (a) The input signal. (b) Estimate of the first difference. (c) Trace of $g(n)$.

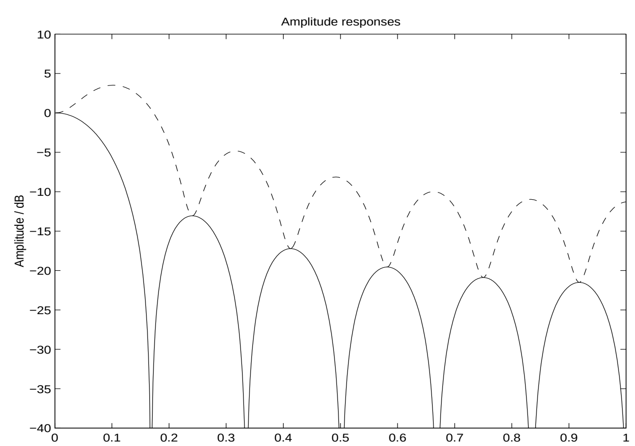

Figure 2: Amplitude responses of $P(z)$ (solid line) and $P(z) + g_{max}Q(z)$ (dashed line).

Although second-order polynomial segments are present in this example, the values of the second difference are two orders of magnitude lower than the first difference. The second or higher order differences are therefore not included in the adaptive algorithm.

A computationally efficient implementation of the proposed adaptive predictor can be constructed such that the arithmetic complexity does not depend on N. This is accomplished by using recursive running sum-based structures for both the running averager $P(z)$ and the differentiator $Q(z)$ [13].

4 SINE PREDICTORS

The coefficients of optimal p-step-ahead predictors for sinusoidal signals of frequency ω_0 are given by [1]

$$\mathbf{h}_S = \mathbf{S}(\mathbf{S}^{\mathrm{T}}\mathbf{S})^{-1}\mathbf{d}_S, \tag{8}$$

where

$$\mathbf{S} = \begin{pmatrix} \sin(0) & \cos(0) \\ \sin(1\omega_0) & \cos(1\omega_0) \\ \sin(2\omega_0) & \cos(2\omega_0) \\ \vdots & \vdots \\ \sin((N-1)\omega_0) & \cos((N-1)\omega_0) \end{pmatrix}$$

and

$$\mathbf{d}_S = \begin{pmatrix} \sin(-p\omega_0) \\ \cos(-p\omega_0) \end{pmatrix}.$$

The noise gain can be reduced by increasing N. However, a short predictor reacts more rapidly to phase changes and other transients than a long one. This observation gives rise to the use of an adaptive scheme of the form (3), where $P(z)$ is a short predictor and $Q(z)$ is an auxiliary filter which is activated for noisy signals. In transient situations, $P(z)$ is able to react more quickly, and $g(n)$ is reduced.

These properties are demonstrated by the following example. With $\omega_0 = 0.1\pi$ and $p = 1$, $P(z)$ is chosen to be the minimum-length predictor, which has the coefficient vector $\mathbf{h}_S^2 = [1.9021 - 1.0000]^{\mathrm{T}}$. $Q(z)$ is obtained by subtracting $\mathbf{h}_S^2$ from a 12-long predictor $\mathbf{h}_S^{12}$. The amplitude responses of the two predictors are shown in Fig. 3. The corresponding NG values are 4.6180 and 0.1550. The input signal is a sine wave of the nominal frequency, with superimposed white noise until sample number 2000. Then the noise is switched off and the signal goes through a series of stepwise phase changes of 0.1π. The parameter $g(n)$ and the prediction error are plotted in Fig. 4. $g(n)$ rises to unity for the noisy input signal, and is reduced by the phase changes. It is noticed that, unlike the polynomial case, prediction will be unbiased with any value of $g(n)$ since $Q(z)$ has zero gain on the nominal frequency.

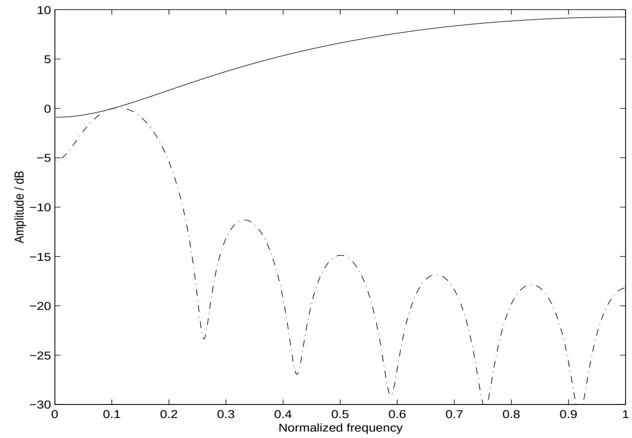

Figure 3: Amplitude responses of $P(z)$ (solid line) and $P(z) + Q(z)$ (dash-dotted line).

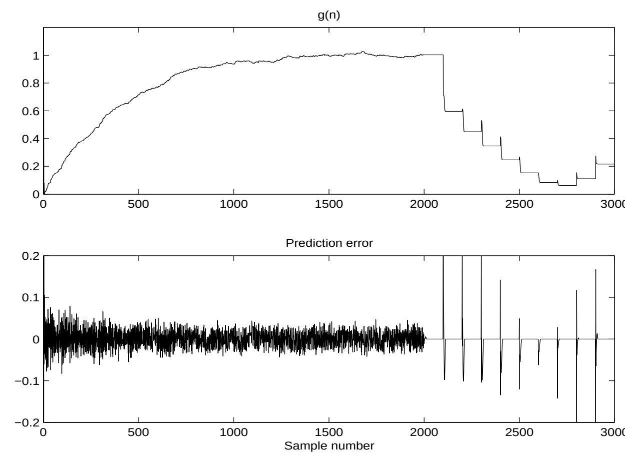

Figure 4: (a) Trace of $g(n)$. (b) The prediction error.

5 STABILITY

The stability of the single-parameter algorithm can be analyzed by following a nearly similar approach as for the standard LMS adaptive algorithm [14],[15]. Let us assume that the reference data sequence is generated by the linear time varying model

$$r(n) = \mathbf{x}(n)\boldsymbol{\Theta}_0(n) + v(n), \tag{9}$$

where $\mathbf{x}(n) = [x(n)\ x(n-1)\ \cdots\ x(n-N+1)]$. The true parameter $\boldsymbol{\Theta}_0(n)$ has a model of the form

$$\boldsymbol{\Theta}_0(n+1) = \boldsymbol{\Theta}_0(n) + \xi(n). \tag{10}$$

The variables $v(n)$ and $\xi(n)$ are considered as noise or disturbances. The adaptation error is

$$\hat{\boldsymbol{\Theta}}(n) = \boldsymbol{\Theta}_0(n) - \boldsymbol{\Theta}(n), \tag{11}$$

where

$$\boldsymbol{\Theta}(n) = \mathbf{p} + g(n)\mathbf{q} \tag{12}$$

is the composite coefficient vector, with

$$\mathbf{p} = [p(0)\ p(1)\ \cdots\ p(N-1)]^{\mathrm{T}}$$

and

$$\mathbf{q} = [q(0)\ q(1)\ \cdots\ q(N-1)]^{\mathrm{T}}.$$

For simplicity, we have assumed $\mathbf{p}$ and $\mathbf{q}$ to be of equal length. If necessary, the shorter one is padded with zeros to length N. The updated vector is

$$\mathbf{\Theta}(n+1) = \mathbf{\Theta}(n) + \gamma[r(n) - \mathbf{x}(n)\mathbf{\Theta}(n)]\mathbf{x}(n)\mathbf{qq}. \quad (13)$$

Thus,

$$\begin{aligned}&\hat{\mathbf{\Theta}}(n+1) = \\ &\mathbf{\Theta}_0(n) + \xi(n) - \mathbf{\Theta}(n) - \gamma[r(n) - \mathbf{x}(n)\mathbf{\Theta}(n)]\mathbf{x}(n)\mathbf{qq} = \\ &\mathbf{\Theta}_0(n) + \xi(n) - \mathbf{\Theta}(n) - \gamma[\mathbf{x}(n)\hat{\mathbf{\Theta}}(n) + v(n)]\mathbf{x}(n)\mathbf{qq} = \\ &\hat{\mathbf{\Theta}}(n) + \xi(n) - \gamma[\mathbf{x}(n)\hat{\mathbf{\Theta}}(n) + v(n)]\mathbf{x}(n)\mathbf{qq}. \quad (14)\end{aligned}$$

This can be written in the form

$$\hat{\mathbf{\Theta}}(n+1) =$$

$$[\mathbf{I} - \gamma\mathbf{q}\mathbf{x}(n)\mathbf{x}(n)\mathbf{q}]\hat{\mathbf{\Theta}}(n) + \xi(n) - \gamma v(n)\mathbf{x}(n)\mathbf{qq}, \quad (15)$$

where $\mathbf{I}$ is an $N \times N$ unit matrix. The stability of the algorithm therefore depends on the eigenvalues of the matrix $\mathbf{I} - \gamma\mathbf{q}\mathbf{x}(n)\mathbf{x}(n)\mathbf{q}$ which are equal to one except that given by

$$\lambda = 1 - \gamma\mathbf{x}(n)\mathbf{q}\mathbf{x}(n)\mathbf{q}. \quad (16)$$

For stability it is therefore required that the gain factor γ is limited by

$$0 < \gamma < \frac{2}{[\mathbf{x}(n)\mathbf{q}]^2}. \quad (17)$$

This condition is not suitable for dynamically adjusting the gain factor because of problems with zero-valued data. But as with LMS, one could consider a normalized algorithm, where the gain has an upper limit [15].

6 CONCLUSIONS

A single-parameter adaptive scheme was proposed for prediction and estimation. The general parameter-based adaptive method is computationally efficient and allows the predictor properties to be dynamically altered according to the noise content and other signal characteristics. Polynomial and sinusoidal prediction was demonstrated by examples. As a topic for further research, generalized predictors will be investigated, e.g., based on multiple sinusoid models.

References

[1] K. Koppinen, J. Yli-Hietanen, and J. Astola, "Optimization of generalized predictors," in *Proc. IEEE Instrumentation and Measurement Technology Conf.*, Ottawa, Canada, May 1997, pp. 54-59.

[2] J. Pasanen, O. Vainio, and S. J. Ovaska, "Predictive synchronization and restoration of corrupted velocity samples," *Measurement,* vol. 13, pp. 315-324, July 1994.

[3] O. Vainio and S. J. Ovaska, "Tachometer signal smoothing with analog discrete-time polynomial estimators," *IEEE Transactions on Industrial Electronics,* vol. 41, pp. 147-154, April 1994.

[4] S. J. Ovaska and O. Vainio, "Predictive compensation of time-varying computing delay on real-time control systems," *IEEE Transactions on Control Systems Technology,* vol. 5, no. 5, pp. 523-526, Sept. 1997.

[5] O. Vainio and S. J. Ovaska, "Digital filtering for robust 50/60 Hz zero-crossing detectors," *IEEE Transactions on Instrumentation and Measurement,* vol. 45, no. 2, pp. 426-430, Apr. 1996.

[6] P. Steffen, "On digital smoothing filters: a brief review of closed form solutions and two new approaches," *Circuits, Syst., Signal Process.,* vol. 5, pp. 187-210, 1986.

[7] P. Heinonen and Y. Neuvo, "FIR-median hybrid filters with predictive FIR substructures," *IEEE Trans. Acoust., Speech, Signal Process.,* vol. 36, pp. 892-899, June 1988.

[8] B. Farhang-Boroujeny, *Adaptive Filters Theory and Applications.* Chichester, England: John Wiley & Sons, 1998.

[9] G. X. Ming, S. J. Ovaska, and O. Vainio, "Neural network-based hybrid estimator for smoothly-varying measurement signals," in *Proc. IEEE Instrumentation and Measurement Technology Conference,* Waltham, MA, April 1995, pp. 218-223.

[10] N. G. Chernoguz, "A single-parameter adaptive comb filter," in *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing,* Salt Lake City, UT, May 2001.

[11] A. A. Ashimov and D. J. Syzdykov, "Identification of high dimensional system by the general parameter method," in *Preprints 8th Triennial World Congress of Int. Federation of Automatic Control,* Kyoto, Japan, Aug. 1981, pp. 32-37.

[12] S. Väliviita, S. J. Ovaska, and O. Vainio, "Polynomial predictive filtering in control instrumentation: a review," *IEEE Transactions on Industrial Electronics,* vol. 46, pp. 876-888, no. 5, Oct. 1999.

[13] O. Vainio, M. Renfors, and T. Saramäki, "Recursive implementation of FIR differentiators with optimum noise attenuation," *IEEE Transactions on Instrumentation and Measurement*, vol. 46, pp. 1202-1207, Oct. 1997.

[14] O. Vainio and S. J. Ovaska, "General parameter-based adaptive extension to FIR filters," in *Proc. IEEE Int. Conf. on Acoustics, Speech, and Signal Processing,* Salt Lake City, UT, May 2001.

[15] N. Kalouptsidis, *Signal Processing Systems Theory and Design.* New York, NY: John Wiley & Sons, 1997, pp. 601-604.