ON-LINE SOURCE SEPARATION OF TEMPORALLY CORRELATED SIGNALS

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ABSTRACT

In this work we will show that the blind source separation problem can be addressed using a linear algebra approach. Making use of the definition of congruent pencils and matrix block operations the problem is completely characterized. We also show that it is possible to have an on-line implementation of the method.

1 Introduction

The mathematical model for the blind source separation problem is $\mathbf{y}(t) = A\mathbf{s}(t)$,where A is the mixing matrix, $\mathbf{y}(t)$ and $\mathbf{s}(t)$ are vectors of mixed and source signals at time t, respectively. Generally, it is assumed that each measured (or mixed) signal is an instantaneous mixture of the source signals. The extraction must be carried on without knowing the structure of the linear combination (the mixing matrix) and the source signals.

The problem has been addressed as a generalized eigendecomposition problem(GED). This solution comprises the simultaneous diagonalization of a matrix pencil (R_{x1}, Rx_2) computed in the mixed signals. These matrices are calculated with different strategies: Souloumiac [1]consider two segments of signals with distinct energy; Molgedey [2] and Chang [3]computes time-delayed correlation matrices; Tomé [4] and [5]consider filtered versions of the mixed signals. Using this method the separation matrix, i.e. the matrix that simultaneous diagonalizes the pencil, is the transpose of eigenvector matrix of the generalized eigendecomposition of pencil. Nevertheless, most of the solutions, for blind source separation, comprise two steps[6], [7]. In the first step, called the whitening (sphering) phase, the measured data is linearly transformed such that the correlation matrix of the output vector equals the identity matrix. This linear transformation is usually computed using the standard eigendecomposition of the mixed data correlation matrix. During this phase the dimensionality of the measured vector is also reduced to the dimension of the source vector. After that, the separation matrix, between the whitening data and the output, is an orthogonal matrix which is computed applying different strategies. In algorithms like AMUSE and EFOBI [6] a standard eigendecomposition is performed in a matrix derived from fourth-order cumulant or time-delayed correlation definitions. The global separation matrix, or an estimate of the inverse of A, is the product of the two matrices computed on the two phases of the method.

In this work, we will formulate the GED method to blind source separation using a linear algebra approach based on the definition of congruent pencils [8]. The use of congruent pencil definition and of block matrix operations constitute a very simple formulization of the GED approach to the blind source separation. We also review methods that perform the eigendecomposition of a matrix pencil based on two consecutive standard eigendecompositions. We will introduce an iterative algorithm to compute the GED of a symmetric matrix pencil. The algorithm is based on the power method and deflation techniques, to perform standard eigendecompositions, and on the use of the spectral factorization of a matrix to approximate a linear transformation. This method can be an on-line algorithm for blind source separation if the matrix pencil can be computed iteratively.

2 The Generalized Eigendecomposition Approach

The generalized eigendecomposition formulation of the blind source separation problem is based on the relation of two pencils: the source matrix pencil (R_{s1}, R_{s2}) and the mixed pencil (R_{x1}, R_{x2}). The two pencils are called congruent pencils [8] if there exists an invertible matrix A, such that

$$R_{x1} = AR_{s1}A^T \text{ and } \quad R_{x2} = AR_{s2}A^T \qquad (1)$$

In the blind source separation problem the matrix A is the instantaneous mixing matrix and can be an $m \times n$ matrix, i.e., the number of mixed signals (m) is not equal the number of source signals (n). In that case we will show that the properties which characterize congruent pencils are also applied if $m > n$. Therefore, the inverse (or the pseudo-inverse) of the mixing matrix can be estimated, using the mixed pencil, if the eigenvector

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matrix of the source pencil is diagonal. The following propositions constitute the required prove.

Proposition 1 *: Congruent pencils have the same eigenvalues.*

The eigenvalues of a pencil are the roots of the characteristic polynomial, $\chi(\lambda)$,

$$\chi(\lambda) = \det(R_{x1} - \lambda R_{x2}) = 0 \qquad (2)$$

If A is an invertible matrix, then

$$\det(R_{x1} - \lambda R_{x2}) = \det(A)\det(R_{s1} - \lambda R_{s2})\det(A^T),$$

which as the same roots as characteristic polynomial of the source matrix pencil

$$\chi(\lambda) = \det(R_{s1} - \lambda R_{s2}) = 0 \qquad (3)$$

When A is a rectangular matrix ($m > n$), if $A^T A$ is an invertible matrix, the congruent pencil ($A^T A R_{s1} A^T A$, $A^T A R_{s2} A^T A$) has also the same eigenvalues, and so the mixed pencil (with $m \times m$ matrices) has also n eigenvalues equal to the eigenvalues of the source pencil.

Proposition 2 *: The eigenvectors of the source pencil are related with the eigenvectors of the mixed pencil*

The generalized eigendecomposition statement of the mixed pencil

$$R_{x2}E = R_{x1}ED \qquad (4)$$

where E is the eigenvector matrix and it will be an unique matrix (with the columns normalized to unity length) if the diagonal matrix D has distinct eigenvalues, λ_i. Otherwise the eigenvectors which correspond to the same eigenvalue might be substituted by their linear combinations without affecting the previous equality. So, supposing that D has distinct values in its diagonal, rewriting the equation (4)

$$AR_{s2}A^T E = AR_{s1}A^T ED \qquad (5)$$

if A is an invertible matrix, we can multiply both sides of the equality by A^{-1} and changing

$$E_s = A^T E \qquad (6)$$

The new equality, $R_{s1}E_s = R_{s2}E_s D$, is the eigendecomposition of the source pencil and E_s is its eigenvector matrix. The normalized eigenvectors for a particular eigenvalue are related by $e_s = \alpha A^T e$ where α is a constant that normalizes, to unity the length, the eigenvectors

In what concerns the blind source separation problem the eigenvector matrix E will be an approximation to inverse of mixing matrix, if the E_s is the identity matrix (or a permutation). This is a fact when the matrix pencil of the source signals are both diagonal.

When the mixing matrix is a $m \times n$ ($m > n$) the equation (5) might written using block matrix notation. Considering A and E divided into two blocks: A into A_H, $n \times n$, and A_L, $(m-n) \times n$; E into $E_H, n \times m$ and $E_L, (m-n) \times m$. Therefore, performing matrix block operations the equation (5) can be written as

$$\begin{aligned} A_H R_{s1}\Phi &= A_H R_{s2}\Phi D \\ A_L R_{s1}\Phi &= A_L R_{s2}\Phi D \end{aligned} \qquad (7)$$

where $\Phi = A_H^T E_H + A_L^T E_L = A^T E$ is $n \times m$ matrix. The first equation shows that this case also resumes the relation among congruent pencils. Φ is a matrix that also represents the eigenvector matrix of the source matrix pencil having $(m-n)$ columns of zeroes paired with the eigenvalues in D that does not belong to eigenvalue decomposition of (R_{s1}, R_{s2}). Using this direct approach to solve the blind source separation, it is possible to find out the number of sources because after the separation $(m-n)$ zero amplitude signals are obtained. Nevertheless, the solution is also found using a subset of mixed signals (n signals) to compute the mixed matrix pencil.

In resume, the GED approach to Blind Source Separation is feasible if the congruent pencils have distinct eigenvalues and the eigenvector matrix of the source pencil is the identity matrix (or a permutation). Then, the source matrix pencil should be diagonal with distinct relations between its diagonal elements. In a practical situation, the diagonal constraint might be a problem because working with estimates, we may have very small values out of the main diagonal which prevent the eigenvector matrix to be an identity.

2.1 The eigendecomposition of symmetric pencils

There are several ways to compute the eigenvalues or the eigenvectors of a matrix pencil, if at least one of the matrices a symmetric positive definite pencil [8]. A very common approach is to reduce the GED statement to the standard form, i.e. , to the eigenvalue decomposition problem. Consider the problem of computing the eigenvalues and the eigenvectors of the pencil (R_{x1}, R_{x2})

$$R_{x2}E = R_{x1}ED \qquad (8)$$

The reduction of the previous equation to the standard form $\hat{C}Z = ZD$, is achieved by solving the eigendecomposition of the matrix R_{x1}. Then, if the matrix is positive definite, $R_{x1} = S\Delta S^T = S\Delta^{1/2}S^T S\Delta^{1/2}S^T = WW$, and considering $Z = WE$,we can write equation (8) as

$$W^{-1}R_{x2}W^{-1}Z = ZD \qquad (9)$$

The previous equation is an eigendecomposition statement of a real symmetric matrix $\hat{C} = W^{-1}R_{x2}W^{-1}$ if R_{x2} is also symmetric positive definite. The transformation matrix ($W^{-1} = S\Delta^{-1/2}S^T$) should be computed with the non-zero eigenvalues and the corresponding eigenvectors. With the eigendecomposition solution of $\hat{C}$, the eigenvalues of pencil (8)are also available, while the eigenvectors are computed solving the equation $E = W^{-1}Z$.

Usually, the matrix R_{x1} is decomposed using Cholesky approach [8]. A similar decomposition, $R_{x1} = S\Delta^{1/2}\Delta^{1/2}S^T$, is used in algorithms like AMUSE and EFOBI [6] to achieve the so called data whitening, but instead of performing a linear transformation on a matrix, the transformation $(\Delta^{-1/2}S^T)$ is used on the raw data. In AMUSE and EFOBI algorithms, the second step is a standard eigendecomposition of a matrix which can also be written as a product of matrices very similar to those found in equations (9) and (1). But using the proposed decomposition, the transformation matrix can be written as a spectral factorization

$$W^{-1} = \sum_i \frac{1}{\sqrt{\delta_i}} s_i s_i^T \tag{10}$$

then an iterative procedure can be implemented using power method and deflation techniques to achieve the eigendecomposition of both matrices. Therefore, as the transformation matrix can be computed iteratively, i.e., using a criterion to include a pair (s_i, δ_i) of the first eigendecomposition into the summation of equation (10) Then, after having an estimative to the transformation matrix, the eigendecomposition of $\hat{C}$ can start. The mean square error $e_i^T e_i$, where $e_i = R_{x1}s_i - \delta_i s_i$, was the criterion used[9].

2.2 Computing the matrix pencil

There are different suggestions to compute the mixed matrix pencil having the mixed signals. Let X be a $m \times N$ matrix containing a segment with N samples of each of m measured signals. The correlation matrix for X, a $m \times m$ matrix, is calculated as

$$R_{x1} = \frac{1}{N} X X^T \tag{11}$$

Let Xf be a matrix $m \times N$ having in each raw a filtered version of each raw in X. Considering that a FIR (finite impulse response) of length M $(M << N)$ was used, the convolution operation of the linear filtering is expressed as $Xf = XH^T$, where H is a $N \times N$ Toeplitz matrix with $h(n)-$the nth sample of impulse response-on the nth diagonal. The correlation matrix of Xf is $m \times m$ defined by

$$R_{x2} = \frac{1}{N} X H^T H X^T \tag{12}$$

Considering that the mixed signals are related with the source signals, i.e. , $X = AS$, the pencil (R_{x1}, Rx_2) has a congruent pencil in source domain as described in equation (1).

In the on-line implementation the correlation matrices are computed iteratively, assuming that a new sample of mixed signals and filtered signals are available on iteration i. For instance, the correlation matrix of the the mixed signals in iteration i is

$$R_{x1}(i) = (1-1/i)R_{x1}(i-i) + (1/i)X(:,i)X(:,i)^T, \tag{13}$$

and the correlation of the filtered mixed signal is computed in a similar way, substituting the vector $X(:,i)$ by the vector of filtered signals.

3 Results

The experiments reported here, involve mixtures of four sound waves, resampled at 8kHz, with 50000 samples. Different mixing matrices and different filters were used. The matrix pencil is updated for every sample and a generalized eigendecomposition is performed using power method, deflation techniques and the transformation matrix described in equation (10) as described in [9]. Nevertheless, the eigendecomposition parameters are stored with a lower rate, i.e., in intervals of 500 samples. The figure (1) shows an example of the evolution of the 100 stored eigenvalues estimated using a matrix pencil computed at the input and at the output of a FIR with coefficients. The mixed signals are separated using the corresponding 100 eigenvector matrices and the correlation coefficients, between sources and recovered signals, are calculated.

In every experiment, taking apart the initial process of convergence, the best results were achieved with the eigenvector matrix which correspond to the most accentuated eigenvalue spread for that particular simulation. The dashed box in the example of figure(1) indicates the best zone to pick up the separation parameters. During this interval it is also verified that the source signals are recovered at the same position in the vector of separated signals. Because the estimates of the eigenvalues are achieved by descending order of magnitude due to the application of power method and deflation techniques. The table (1) presents the correlation coefficients using the eigenvector matrix estimated after 35000 samples of the data. There is a maximum (approximately one) in each raw/column and all the others coefficients are approximately zero. Near the end of the segment the source signals, with exception of source 1, are not completely separated and the predominate source signal is not always at the same position in the vector of separated signals. The table (2) shows the results using the estimated parameters near the end of the data segment (with 45500 samples) where the maximum values of the correlation coefficients are lower than the ones in table (1), and there are also correlation of coefficients on the same raw/column that are significant. In this case, listening the separated signal 3, the source 4 predominates but the source 2 is also audible.

4 Conclusions

This work reports an alternative formalization for the generalized eigendecomposition approach to blind source separation. Using linear algebra concepts it was

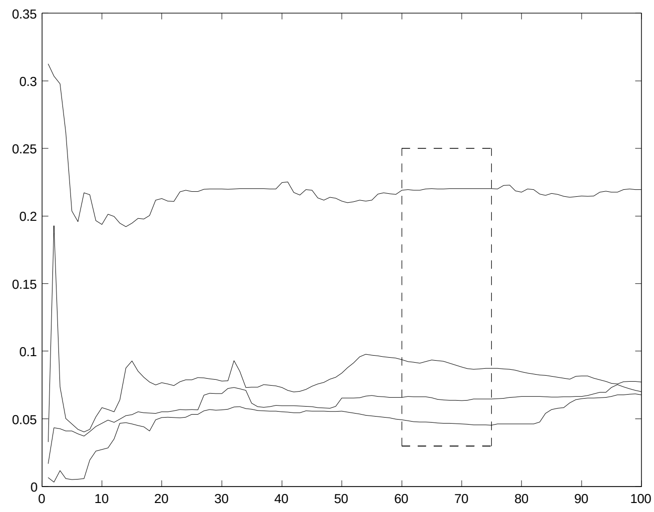

Figure 1: Evolution of the eigenvalues during the iterative matrix pencil eigendecomposition

	source 1	source 2	source 3	source 4
separated 1	**0.999**	0.003	0.007	0.005
separated 2	0.000	0.014	**0.999**	0.005
separated 3	0.006	**0.998**	0.018	0.052
separated 4	0.003	0.057	0.002	**0.998**

Table 1: Correlation coefficients between sources and separated signals using the separation matrix estimated after 35000 samples.

possible to find an on-line solution to the problem instead of using an approach based on separated steps like AMUSE and EFOBI. Furthermore, the matrix pencil is computed at the input and at the output of a simple linear filter.

The method works better when the eigenvalues are distinct and spread as it is also suggested by Souloumiac[1]. But the proposed methodology also suggests that the decision, in what concerns parameter validation, can be taken on-line. This aspect should be further exploited in order to find the best methodology to apply the criteria. Another aspect to study is the choice of the linear filter which naturally determines the numerical values of the eigenvalues, but it is not clear what should be the design constraints to achieve the best result.

	source 1	source 2	source 3	source 4
separated 1	**0.999**	0.006	0.004	0.005
separated 2	0.000	0.022	**0.999**	0.017
separated 3	0.005	0.494	0.024	**0.867**
separated 4	0.005	**0.865**	0.014	0.506

Table 2: Correlation coefficients between sources and separated signals using the separation matrix estimated after 45500 samples.

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