MODELLING AND ASSESSMENT OF SIGNAL-DEPENDENT NOISE FOR IMAGE DE-NOISING

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ABSTRACT

In this paper, a class of *signal-dependent* noise models that are encountered in image processing applications is considered. They are defined by the *gamma* exponent, which rules the dependence on the signal of the noise, and by the variance of a stationary *zero-mean* random process that generates the signal-dependent noise. An observation noise term, zero-mean, white and independent of the signal, is also considered to account for the electronics noise. A blind procedure is proposed for reliably measuring the model parameters directly from the noisy images irrespective of their texture content. Such methods are iteratively based on linear regression techniques applied to scatter-plots of local first-order statistics calculated on homogeneous areas and drawn with logarithmic scale. Adaptive LLMMSE filtering is embedded in the iteration stage to provide a rough estimate of noise-free image texture which allows to discriminate between homogeneous and textured pixels. Experiments on simulated noisy images demonstrate a high accuracy of noise assessment.

1 INTRODUCTION

In several imaging systems, the model of acquisition noise should be considered as dependent on the sensed signal. A first example is film-grain noise [1] that comes out when photographic supports are scanned and is due to the granularity of photo-sensitive crystals on the film. Another example is *speckle* noise, occurring in coherent imaging systems, like Synthetic Aperture Radar (SAR) [2] and Ultrasonic Scanner (US) [3], which is due to the random scattering of unresolvable objects inside the elementary resolution cell. The consequence is a strongly signal-dependent granular noise whose effects are degradation of the acquired image and reduced performance of post-processing algorithms as well as of visual analysis. Although incoherent averaging is usually employed to provide speckle reduction with the penalty of a loss of resolution, *adaptive* spatial filtering, i.e. driven by the noise statistics, is needed especially when advanced vision tasks are demanded [4].

A general signal-dependent noise model has been proposed to deal with several different acquisition systems [1]. The model is expressed in the following for one-dimensional signals. The two-dimensional case is straightforward. Let $u(n)$ and $w(n)$ be stationary, zero-mean random processes, with variances σ_u^2 and σ_w^2, respectively. Both are independent of the noise-less signal $f(n)$. The observed noisy image $g(n)$ can be expressed as

$$\begin{aligned} g(n) &= f(n) + f(n)^{\gamma} \cdot u(n) + w(n) \\ &= f(n) + v(n) + w(n). \end{aligned} \quad (1)$$

Since $f(n)$ is generally nonstationary, the signal-dependent noise $v(n)$ will be nonstationary as well. Furthermore, if $u(n)$ is non-white, $v(n)$ will be spatially auto-correlated. The observation noise $w(n)$ introduced by the electronic circuitry is stationary and usually modelled as white and zero-mean Gaussian.

For the acquisition processes outlined above this model has been proven to be valid for values of γ within the interval $[0, 1]$. For example, the analysis of SAR images has demonstrated that a pure multiplicative noise ($\gamma = 1$) is typical and that the signal-independent term is negligible with respect to the multiplicative term. Many de-specking filters rely on the multiplicative (or *fully developed*) speckle model [5]. The basic assumption that makes this model acceptable is that a large number of scatterers exists inside the resolution cell. In such a case, the response from single backscatters can be considered as independent of one another, with amplitudes belonging to the same statistical distribution and phases that are uniformly distributed. This is not true in general, since often the coherent radiation investigates zones of the interested area which are made up of structures comparable in dimensions to the resolution cells.

The model (1) is also suitable for *film-grain* noise, typical of photographic film scanning (*negative* film-grain). Although the γ strictly depends on the ratio between the *grain size* and the *spot size* (thickness of the illuminating beam), values of γ between $1/3$ and $1/2$ are usually assumed [6]. The case of scanned half-tone prints yields negative values of γ and is referred to as *positive* film-grain [1]. The film-grain noise is always accompanied by an electronics noise whose intensity $w(n)$ may be comparable to that of the signal dependent term $v(n)$ and must be taken into account for both estimation and filtering [7]. Its variance σ_w^2 may be easily measured starting from a *dark image*, i.e. without signal, e.g. achieved by scanning a black sheet.

For ultrasonic image generation, no characteristic values of γ have been stated, due to the great variability of scatterer size and the strong dependence of the model parameters on the pre-processing stages of the acquisition system. In the literature both the values 0.5 and 1 are accepted as typical, but experiments have revealed a larger spreading of the γ.

2 NOISE MODEL ASSESSMENT

Since the signal-dependent model (1) has a wide applicability, an iterative procedure to estimate the parameter γ and σ_u^2 is proposed. The algorithm utilizes an adaptive filter driven by the estimated noise model parameters to yield a more and more accurate estimate of the noise-free image that is exploited for a finer and finer estimation of the noise model parameters.

Since $f(n)$ is independent of $u(n)$, the mean of (1) is

$$E[g(n)] = E[f(n)]. \tag{2}$$

The variance $\sigma_g^2(n)$ of (1) is given by

$$\begin{aligned}
\sigma_g^2(n) &= E[g^2(n)] - E[g(n)]^2 \\
&= E[f^2(n)] + E[f^{2\gamma}(n)]E[u^2(n)] + E[w^2(n)] \\
&\quad +2E[f^{\gamma+1}(n)]E[u(n)] + 2E[f(n)]E[w(n)] \\
&\quad +2E[f^{\gamma}(n)]E[u(n)]E[w(n)] - E[f(n)]^2 \\
&= \sigma_f^2(n) + E[f^{2\gamma}(n)] \cdot \sigma_u^2 + \sigma_w^2
\end{aligned} \tag{3}$$

thanks to (2) and to the fact that u and w are zero-mean and independent of each other and of f.

In order to evaluate $E[f^{2\gamma}(n)]$, a Taylor's series expansion of the function $f^{2\gamma}$ is used. A 2nd order expansion of $f^{2\gamma}$ around f_0 yields

$$f^{2\gamma} \simeq f_0^{2\gamma} + \left.\frac{\partial f^{2\gamma}}{\partial f}\right|_{f_0} (f - f_0) + \left.\frac{\partial^2 f^{2\gamma}}{\partial f^2}\right|_{f_0} \frac{(f-f_0)^2}{2}. \tag{4}$$

Evaluating this expression around the mean of f, i.e. $f_0 = E[f(n)]$, and taking the expectation of the result, we have

$$\begin{aligned}
E[f^{2\gamma}] &\simeq E\left[E[f]^{2\gamma} + \left.\tfrac{\partial f^{2\gamma}}{\partial f}\right|_{E[f]} (f - E[f]) \right. \\
&\qquad \left. + \left.\tfrac{\partial^2 f^{2\gamma}}{\partial f^2}\right|_{E[f]} \tfrac{(f-E[f])^2}{2}\right] \\
&= E[f]^{2\gamma} + \gamma(2\gamma-1)E[f]^{2\gamma-2}\sigma_f^2
\end{aligned} \tag{5}$$

where we have dropped the argument n for simplicity of notation. $E\left[\left.\frac{\partial f^{2\gamma}}{\partial f}\right|_{E[f]} (f - E[f])\right] = 0$ by definition of mean.

Substituting (5) into (3) yields

$$\begin{aligned}
\tilde{\sigma}_g^2(n) &\triangleq \sigma_g^2(n) - \sigma_w^2 \\
&\simeq \sigma_f^2(n)\{1 + \gamma(2\gamma-1)E[f(n)]^{2\gamma-2} \cdot \sigma_u^2\} \\
&\quad + E[f(n)]^{2\gamma} \cdot \sigma_u^2
\end{aligned} \tag{6}$$

where $\tilde{\sigma}_g^2(n)$ can be obtained from the observed image and from the knowledge of σ_w^2.

It can be easily verified from (6) that the Taylor approximation is exact for γ equal to 0, 0.5 and 1, and approximate otherwise. The above relationship also highlights that $\tilde{\sigma}_g^2(n)$ is composed by two terms. The former is a *texture* contribution proportional to $\sigma_f^2(n)$, whereas the latter is proportional to $E[f(n)]^{2\gamma}$ and is the unique term in homogeneous areas, where $\sigma_f^2(n) = 0$. If we used only homogeneous areas to estimate the parameters of the model, $\tilde{\sigma}_g^2(n)$ could be expressed as

$$\tilde{\sigma}_g^2(n) = E[f(n)]^{2\gamma} \cdot \sigma_u^2 = E[g(n)]^{2\gamma} \cdot \sigma_u^2 \tag{7}$$

thanks to (2). By taking the logarithm of both sides of (7), an affine relationship between $E[g(n)]$ and $\tilde{\sigma}_g^2(n)$ is obtained through γ and $\log(\sigma_u)$:

$$\log[\tilde{\sigma}_g(n)] = \gamma \cdot \log\{E[g(n)]\} + \log(\sigma_u). \tag{8}$$

Eq. (8) states that the logarithms of the ensemble statistics of $g(n)$ calculated on homogeneous pixels are aligned along a straight line having γ as slope and $\log(\sigma_u)$ as intercept. In practice, for each pixel of the image, a point in the plane $(\log\{E[g(n)]\}, \log[\tilde{\sigma}_g(n)])$ may be plotted. The outcome cloud of points is called a *scatter-plot*. If only pixels belonging to homogeneous areas were used, the scatter plot should be clustered along its regression line, whose parameters would yield estimates of γ (slope) and $\log(\sigma_u)$ (intercept). Conversely, evaluation of (6) in highly textured areas results in a sparse cloud.

Therefore, the main problem becomes finding a method to discriminate between homogeneous and textured areas. This is not an immediate task especially when the image is noisy. A way to select homogeneous pixels in which (8) holds is searching for those pixels for which the term proportional to σ_f^2 in (6) is negligible with respect to the term $E[f(n)]^{2\gamma} \cdot \sigma_u^2$; in other words, finding those pixels for which the following condition holds

$$\frac{E[f(n)]^{2\gamma} \cdot \sigma_u^2}{\sigma_f^2(n)\{1 + \gamma(2\gamma-1)E[f(n)]^{2\gamma-2} \cdot \sigma_u^2(n)\}} > \theta \tag{9}$$

where θ is a constant threshold empirically chosen.

The homogeneity ratio (9) is a function of the unknown parameters γ and σ_u as well as of the variance of the noiseless image. Since an accurate estimation of $\sigma_f^2(n)$ is not required -we need to search for pixels where the denominator in (9) is negligible with respect to the numerator- the following iterative algorithm is proposed to estimate the γ and σ_u^2.

Step 1 Calculate the logarithmic scatter-plot relative to the whole set of pixels and compute an initial rough estimate of γ and σ_u ($\hat{\gamma}^{(0)}$ and $\hat{\sigma}_u^{(0)}$) from a linear best-fit of the measured data. Set iteration step $i = 0$ and a homogeneity threshold θ.

Step 2 Find an approximation of $\sigma_f^2(n)$ ($\hat{\sigma}_f^2(n)^{(i)}$) by using a parametric filter (see Sect. 3) driven by $\hat{\gamma}^{(i)}$, $\hat{\sigma}_u^{(i)}$ and σ_w^2 (assumed to be known *a priori*).

Step 3 Substitute $\hat{\sigma}_f^2(n)^{(i)}$, $\hat{\gamma}^{(i)}$ and $\hat{\sigma}_u^{(i)}$ into (9) and find homogeneous pixels.

Step 4 Set $i = i + 1$. Calculate a new log scatter-plot comprising only those pixels for which the homogeneity condition (9) holds. Compute the parameters $\hat{\gamma}^{(i)}$ and $\hat{\sigma}_u^{(i)}$ of the linear regression fitting the new set of points.

Step 5 Repeat Steps 2 to 4 until a stop criterion is met.

3 SIGNAL-DEPENDENT NOISE FILTERING

This section describes a parametric filter to be used in the iterative estimation algorithm introduced in Sect. 2. The filter should be as much insensitive as possible to inaccuracies in parameters estimation. Thus, the filtered image will converge to the optimally de-noised one while the values of the estimated parameter γ and σ_u will approach more and more closely those of the true ones.

3.1 MMSE Noise Filtering

Let $\mathbf{f}$, $\mathbf{g}$ and $\mathbf{v}$ denote the ideal noise-less sampled signal, the observed noisy signal and the additive noise, possibly signal-dependent, all arranged as 1-D vectors of size N. The MMSE estimate of $\mathbf{f}$ is its expectation conditional to the observed signal, i.e. $\hat{\mathbf{f}}_{\mathbf{MMSE}} = E[\mathbf{f}|\mathbf{g}]$, which, however, would require the knowledge of the nonstationary signal PDF's of any order. By taking a fist-order Taylor development of the MMSE solution around the unconstrained value $E[\mathbf{f}]$, the *linear* MMSE (LMMSE) estimator [8] requiring only signal and noise statistics up to the 2nd order will be given by

$$\hat{\mathbf{f}}_{\mathbf{LMMSE}} = E[\mathbf{f}] + \mathbf{C_{fg}} \mathbf{C_g^{-1}} \cdot [\mathbf{g} - E(\mathbf{g})] \qquad (10)$$

in which the $N \times N$ matrices $\mathbf{C_{fg}}$ and $\mathbf{C_g}$ are the cross-covariance between $\mathbf{f}$ and $\mathbf{g}$ and the auto-covariance of $\mathbf{g}$, respectively. Eq. (10) imposes a *global* MSE minimization over the whole image, within the constraint of a linear solution, which is optimum if the joint PDFs of $\mathbf{f}$ are multivariate Gaussian.

If we assume that $\mathbf{f}$ is spatially uncorrelated, i.e. $\mathbf{C_f} = E\{[\mathbf{f} - E(\mathbf{f})][\mathbf{f} - E(\mathbf{f})]^T\}$ is a diagonal matrix -which means that the spatial correlation of $\mathbf{f}$ is conveyed by its space-varying mean $E[\mathbf{f}]$ only- and that the noise $\mathbf{v}$ is zero-mean and uncorrelated as well, the *global* minimization (10) corresponds to a *local* minimization in a neighborhood of each sample. In fact, let $\sigma_f^2(n)$ and $\sigma_v^2(n)$ denote the ensemble variance of $\mathbf{f}$ and $\mathbf{v}$ at the nth sample position. If $E[\mathbf{v}] = \vec{0}$, the covariance matrices $\mathbf{C_{fg}}$ and $\mathbf{C_g}$ in (10) become diagonal

$$\begin{aligned} \mathbf{C_g} &= \mathrm{diag}[\sigma_g^2(1), \sigma_g^2(2), \cdots, \sigma_g^2(N)], \\ \mathbf{C_{fg}} = \mathbf{C_f} &= \mathrm{diag}[\sigma_f^2(1), \sigma_f^2(2), \cdots, \sigma_f^2(N)]. \end{aligned} \qquad (11)$$

By replacing (11) into (10), the *local* LMMSE (LLMMSE) estimate of $f(n)$ is obtained as [9]

$$\hat{f}_{\mathrm{LLMMSE}}(n) = E[f(n)] + \frac{\sigma_f^2(n)}{\sigma_g^2(n)} \cdot \{g(n) - E[g(n)]\}. \qquad (12)$$

Eqs. (2) and (6) allows to derive the variance of the noise-free image as a function of the first-order statistics of the observed image and of the noise model parameters

$$\sigma_f^2(n) = \frac{\sigma_g^2(n) - E[g(n)]^{2\gamma} \cdot \sigma_u^2 - \sigma_w^2}{1 + \gamma(2\gamma - 1) E[g(n)]^{2\gamma-2} \cdot \sigma_u^2}. \qquad (13)$$

By replacing (13) in (12), the LLMMSE estimator, namely $\hat{f}(n)$, may be specified to the noise model (1) as

$$\begin{aligned} \hat{f}(n) \quad = \quad & E[g(n)] + \{g(n) - E[g(n)]\} \\ & \times \frac{1 - \{E[g(n)]^{2\gamma} \cdot \sigma_u^2 + \sigma_w^2\}/\sigma_g^2(n)}{1 + \gamma(2\gamma - 1)E[g(n)]^{2\gamma-2} \cdot \sigma_u^2}. \end{aligned} \qquad (14)$$

The LLMMSE solution contains *first-order* ensemble statistics of the observed image, which are usually not available. The LLMMSE estimator can be reformulated by introducing local approximations of the nonstationary mean and variance of the observed image calculated as

$$E[g(n)] \cong \bar{g}(n) = \frac{1}{2W + 1} \sum_{i=-W}^{W} g(n + i) \qquad (15)$$

$$\sigma_g^2(n) \cong \bar{g^2}(n) - \bar{g}^2(n) = \frac{1}{2W} \sum_{i=-W}^{W} [g(n + i) - \bar{g}(n)]^2 \qquad (16)$$

where $2W + 1$ is the size of the local window. To prevent the estimated value for $E[g(n)]^{2\gamma} \cdot \sigma_u^2 + \sigma_w^2$ from being larger than that of $\sigma_g^2(n)$ in (14), the difference is clipped above zero after substitutions of (15) and (16) in (14). An assumption of local ergodicity is also introduced to allow estimation of local mean and variance from a neighbourhood.

4 EXPERIMENTAL RESULTS

The performance of the proposed method has been assessed by using images degraded by synthetic signal-dependent noise. The test images *Shapes*, *Peppers* and *Lenna* shown in Fig. 1 have been corrupted with additive signal-dependent noise according (1), with different values of the parameter γ and values of σ_u adjusted to yield $SNR = \bar{\sigma}_f^2/\bar{\sigma}_v^2 = 3\ dB$. Zero-mean white Gaussian random processes have been used for both u and the electronics noise w, which is assumed to be known *a priori* ($\sigma_w = 11.689$, i.e σ_w^2 is $10\ dB$ lower than

(a) (b) (c)

Figure 1: Test set of grey-scale images: (a) synthetic *ray-tracing* noise-free image *Shapes*; (b) *Lenna*; (c) *Peppers*.

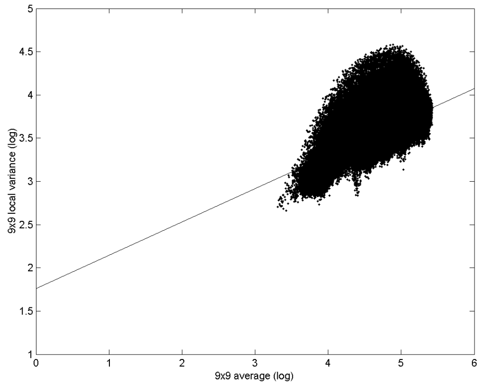
Table 1: True and estimated noise parameters for different values of γ, at fixed $SNR = \bar{\sigma}_f^2/\bar{\sigma}_v^2 = 3\ dB$.

γ	0.0	0.25	0.5	0.75	1.0
$\hat{\gamma}$ (*Shapes*)	0.0017	0.2390	0.4933	0.7296	0.9641
σ_u (*Shapes*)	36.964	13.232	4.3824	1.3782	0.4189
$\hat{\sigma_u}$ (*Shapes*)	36.811	13.896	4.5369	1.5166	0.4989
σ_v (*Shapes*)	36.977	36.977	36.912	36.925	36.855
$\hat{\sigma_v}$ (*Shapes*)	37.147	37.087	37.052	36.886	36.943
$\hat{\gamma}$ (*Peepers*)	-0.0013	0.2410	0.4902	0.7320	0.9703
$\hat{\gamma}$ (*Lenna*)	-0.0208	0.2259	0.4702	0.7077	0.9333

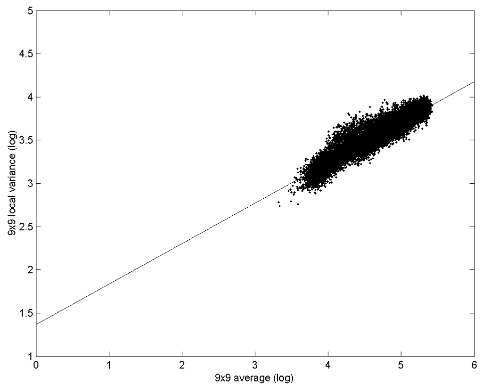
$\bar{\sigma}_v^2$). Table 1 reports true and estimated noise parameters (denoted with a tilde). Fig. 2 shows log scatter-plots of noisy *Lenna* ($\gamma = 0.5$, $SNR = 3\ dB$) calculated on all pixels and only on pixels recognized as homogeneous by the iterative procedure. As it appears, in the former case the γ (slope) is under-estimated, while $\log(\sigma_u)$ (intercept) is over-estimated because of textures.

5 CONCLUSIONS

A blind iterative procedure to assess a variety of signal-dependent noise models corrupting digital imagery has been proposed. The estimation relies on a general parametric model for the additive noise and utilizes LLMMSE filtering adjusted to the generalized noise model and embedded into the iteration stage to provide an estimate of the noise-free texture. Results show accuracy of joint estimation of the two unknown model parameters for a wide choice of values. Although only uncorrelated noise was considered in the experiments, all scatter-plot based methods, including the present one, allow to tackle correlated noise as well.



(a)



(b)

Figure 2: Log scatter-plots of $\sqrt{\bar{g^2}(n) - \bar{g}^2(n) - \sigma_w^2}$ to local average $\bar{g}(n)$ calculated on windows of length 9: (a) whole image *Lenna*; (b) homogeneous points only.

REFERENCES

[1] W. K. Pratt, Digital Image Processing, J. Wiley & Sons, New York, 1991.

[2] J. C. Curlander, R. N. McDonough, Synthetic Aperture Radar: Systems & Signal Processing, John Wiley & Sons, New York, 1991.

[3] C. B. Burckhardt, Speckle in ultrasound B-mode scans, IEEE Trans. Sonics and Ultras. SU-25 (1) (1978) 1–6.

[4] J.-S. Lee, I. Jurkevich, P. Dewaele, P. Wambacq, A. Oosterlinck, Speckle filtering of synthetic aperture radar images: a review, Remote Sensing Reviews 8 (1994) 313–340.

[5] M. Tur, K. C. Chin, J. W. Goodman, When is speckle multiplicative?, Appl. Optics 21 (7) (1982) 1157–1159.

[6] F. Naderi, A. A. Sawchuk, Estimation of images degraded by film-grain noise, Appl. Optics 17 (1978) 1228–1237.

[7] P. Campisi, J. C. K. Yan, D. Hatzinakos, Signal-dependent film grain noise generation using homomorphic adaptive filtering, IEE P-Vis. Image Sign. 147 (3) (2000) 283–287.

[8] S. M. Kay, Fundamentals of Statistical Processing, Volume I: Estimation Theory, Prentice Hall, Engl. Cliffs, NJ, 1993.

[9] D. T. Kuan, A. A. Sawchuk, T. C. Strand, P. Chavel, Adaptive noise smoothing filter for images with signal-dependent noise, IEEE Trans. Pattern Anal. Machine Intell. 7 (2) (1985) 165–177.