A BLIND PARTICLE FILTERING DETECTOR FOR JOINT CHANNEL ESTIMATION, TRACKING AND DATA DETECTION OVER FLAT FADING CHANNELS

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ABSTRACT

A new particle filtering detector is proposed for joint estimation of channel model coefficients, channel tracking, and signal detection over flat Rayleigh fading channels. The detector employs a hybrid importance function and a mixture Kalman filter which result in a highly efficient implementation. In addition, by considering the practical limitation of the system and physical interpretation of the adopted AR(2) channel model, we realize a fully blind particle filtering implementation. Simulation are provided to show the performance of the proposed detector.

1. INTRODUCTION

Detection of digital signals over flat fading distorted channels plays an important role in wireless transmissions of voice and data. The research on the topic has drawn much interest in the past decade. Especially, a class of detectors employing the maximum-likelihood sequence estimation (MLSE) techniques has been extensively studied [1]. These detectors usually implement the channel tracking and signal detection separately. It has been shown that this approach yields poorer performance than strategies based on joint implementations of tracking and detection.

Recently, novel particle filtering detectors were proposed for the problems of joint channel tracking and signal detection [2, 3]. These detectors not only achieve fully blind channel tracking but also overcome the problem of error propagation resulting from a decision feedback implementation. Also, they allow for both Gaussian and non Gaussian ambient noise as well as parallel implementations.

An important feature of particle filtering detectors is to impose a parametric structure such as AR and ARMA models on the fading channels. The parametric modeling of the channels has been shown to represent the underlying channels of many systems satisfactorily [4], which facilitates the implementation of the particle filtering detectors. However, a common assumption is that the model coefficients (AR or ARMA) are known to the detectors in advance. In [5], a hybrid algorithm is proposed to further integrate the estimation of the model parameters within the particle filtering detectors. This novel detector employs a recursive least square algorithm for the estimation objective. However, pilot symbols are required in the implementation to avoid ambiguity.

In this paper, a new detector is proposed for joint estimation of channel model coefficients, channel tracking, and signal detection. The proposed detector is constructed under a full particle filtering paradigm. In particular, a hybrid importance function is introduced which, together with the mixture Kalman filtering (MKF) [2], reduces significantly the computational complexity of a generic implementation of particle filtering. Furthermore, an AR(2) process is adopted to model the fading channels. This modeling imposes a direct link between the model coefficients and the underlying fading channel. The link enables us to resolve the ambiguity in the detection by considering the physical limitations of the system, which allows for a fully blind implementation of the particle filtering detector. Simulation results are provided that show the performance of the proposed detector.

2. PROBLEM FORMULATION

We consider detection of digital signals transmitted through flat Rayleigh fading channels. At the transmitter, the modulated M-ary PSK data sequence s_t is passed into a pulse shaping filter to form the baseband signal $s(t)$ and then transmitted through a flat Rayleigh fading channel. At the receiver, the received baseband signal $y(t)$ is first fed into a matched filter and then sampled with a symbol rate $1/T$. The resulting sampled sequence y_t can be expressed as

$$y_t = h_t s_t + e_t \qquad t = 1, 2, 3, \cdots \tag{1}$$

where h_t and e_t are the complex fading coefficients and additive ambient noise. The noise e_t is assumed to be complex Gaussian with zero mean and variance σ^2. Here, the fading channel is a Rayleigh process, and thus, the dynamic characteristics of the fading coefficient h_t depend on the maximum Doppler spread

$$f_D = v/\lambda \tag{2}$$

where v denotes the speed of the mobile and λ is the carrier wavelength. When v is constant, h_t is modeled by the Jakes'

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model as a stationary, circular complex Gaussian process with zero mean and autocorrelation function [10]

$$r_h(m) = E\{h_n h^*_{n-m}\} = P J_0(2\pi f_d T m) \quad (3)$$

where P denotes the power of the fading process and $J_0(\cdot)$ denotes the zero order Bessel function of the first kind. The direct application of the Jakes' model in our computation leads to intractable solutions. However, AR processes can often be used to approximate the Jakes' model with satisfactory accuracy. In this paper, a second order autoregressive (AR) process is adopted such that

$$h_t = -a_1 h_{t-1} - a_2 h_{t-2} + v_t \quad (4)$$

where a_1 and a_2 are the model coefficients, and $v_t \sim \mathcal{CN}(0, \sigma_v^2)$. The coefficients are closely related to the physical characteristics of the underlying fading process which will be discussed in detail in Section 3.3. Of our interest here is the detection of the transmitted symbol s_t without knowing the instantaneous value of h_t. In many schemes, a_1 and a_2 are assumed to be known for the detection, and the estimation of a_1 and a_2 is done separately. However, here we assume to have no knowledge about a_1 and a_2, and we intend to estimate these coefficients, track the channel h_t, and detect s_t at the same time. To that end, we further assume that the driven noise of the AR process σ_v^2 and the noise variance σ^2 are known to the receiver. It should be noted that the proposed algorithm can be easily extended to include them as unknowns.

3. THE PARTICLE FILTERING DETECTOR

3.1. State space modeling of the problem

We first formulate a state space representation of the system. It can be expressed as

$$\begin{cases} a_{1,t} = a_{1,t-1}, \quad a_{2,t} = a_{2,t-1} \\ \mathbf{h}_t = \mathbf{D}\mathbf{h}_{t-1} + \mathbf{g}v_t \\ y_t = \mathbf{g}^\mathsf{T}\mathbf{h}_t s_t + e_t \end{cases} \quad (5)$$

where $\mathbf{h}_t = [h_t \ h_{t-1}]^\mathsf{T}$, $\mathbf{g} = [1 \ 0]^\mathsf{T}$, and

$$\mathbf{D} = \begin{bmatrix} -a_{1,t} & -a_{2,t} \\ 1 & 0 \end{bmatrix}.$$

Define $\mathbf{a}_t = [a_{1,t}, a_{2,t}]^\mathsf{T}$. At any instant of time t, the unknowns are s_t, h_t, and $\mathbf{a}_t$, and our main objective is to detect the transmitted signal s_t sequentially without sending pilot signals.

3.2. The particle filtering solution with hybrid importance functions

Particle filtering is a sequential Monte Carlo sampling method built upon the Bayesian paradigm [6, 7]. From a Bayesian perspective, at time t, the posterior distribution $p(s_t|\mathbf{y}_{0:t})$ is the main entity of interest. Due to the nonlinearity of the model (5), the analytical expression of $p(s_t|\mathbf{y}_{0:t})$ cannot be obtained. Alternatively, particle filtering approximates $p(s_t|\mathbf{y}_{0:t})$ by using stochastic samples generated using a sequential importance sampling strategy. Due to the presence of the nuisance parameters, the objective is to sample from joint posterior distribution $p(s_t, \mathbf{a}_t, h_t|\mathbf{y}_{0:t})$. First, note that given $\mathbf{a}_t$ and s_t, (5) is linear in h_t. Therefore, the MKF can be used to marginalize out the nuisance parameter h_t. Our objective is then to generate samples from the distribution $p(s_t, \mathbf{a}_t|\mathbf{y}_{0:t})$. Second, define $\mathbf{x}_t^{(j)} = \{\mathbf{a}_t^{(j)}, \mathbf{s}_t^{(j)}\}$ and suppose that at time $t-1$, we have collected N sets of samples $\mathbf{x}_{0:t-1}^{(j)} = \{\mathbf{x}_0^{(j)}, \cdots, \mathbf{x}_{t-1}^{(j)}\}$ with weights $w_{t-1}^{(j)}$, $j = 1, \cdots, N$. In particular, the weighted samples $\{\mathbf{x}_{0:t-1}^j, w_{t-1}^{(j)}\}_{j=1}^N$ are distributed approximately according to $p(\mathbf{x}_{0:t-1}|\mathbf{y}_{0:t-1})$. When a new observation y_t arrives, the update of the sample sets from $t-1$ to t is carried out as follows:

The Particle filter

- For $j = 1, \cdots, N$
  - Sample $\mathbf{x}_t^{(j)}$ from an importance function $q(\mathbf{x}_t|\mathbf{x}_{0:t-1}^{(j)}, \mathbf{y}_{0:t})$ and set $\mathbf{x}_{0:t}^{(j)} = \{\mathbf{x}_{0:t-1}^{(j)}, \mathbf{x}_t^{(j)}\}$.
  - Calculate the weight by

$$\bar{w}_t^{(j)} = w_{t-1}^{(j)} \frac{p(\mathbf{x}_{0:t}{}^{(j)}|\mathbf{y}_{0:t})}{p(\mathbf{x}_{0:t-1}^{(j)}|\mathbf{y}_{0:t-1}) q(\mathbf{x}_t^{(\mathbf{j})}|\mathbf{x}_{0:t-1}^{(\mathbf{j})}, \mathbf{y}_{0:t})} \quad (6)$$

- For $j = 1, \cdots, N$, normalize the weights by:

$$w_t^{(j)} = \frac{\bar{w}_t^{(j)}}{\sum_{j=1}^N \bar{w}_t^{(j)}} \quad (7)$$

where $q(\mathbf{x}_t^{(j)}|\mathbf{x}_{0:t-1}^{(j)}, \mathbf{y}_{0:t})$ is an importance function which must be specified. The choice of the importance function is essential because it determines the efficiency as well as the complexity of the particle filtering algorithm. Two standard choices of the importance function are the posterior and the prior importance functions. The posterior importance function is considered optimal because it minimizes the variance of the importance weights. Here, we observe that, due to the presence of a_1 and a_2, the calculation of the posterior importance function leads to intractable weights. Hence one would usually resort to using the prior importance function. However the use of the prior importance function is often ineffective and leads to poor filtering performance. Here, we adopt a hybrid importance function [8], which is expressed as

$$\begin{aligned} & q(s_t, \mathbf{a}_t|\mathbf{s}_{0:t-1}^{(j)}, \mathbf{a}_{0:t-1}, \mathbf{y}_{0:t}) \quad (8) \\ &= p(s_t|\mathbf{a}_{0:t}^{(j)}, \mathbf{s}_{0:t-1}^{(j)}, \mathbf{y}_{0:t}) p(\mathbf{a}_t|\mathbf{a}_{t-1}^{(j)}) \\ &= p(s_t|\mathbf{a}_{0:t}^{(j)}, \mathbf{s}_{0:t-1}^{(j)}, \mathbf{y}_{0:t}) \delta(a_{1,t-1}^{(j)}) \delta(a_{2,t-1}^{(j)}) \quad (9) \end{aligned}$$

where $\mathbf{a}_{0:t}$ and $\mathbf{s}_{0:t}$ are defined in the same way as $\mathbf{x}_{0:t}$, $\mathbf{a}_t^{(j)} = \mathbf{a}_{t-1}^{(j)}$, and $\delta(\cdot)$ is the Dirac delta function. The last equality is obtained based on the state equations $a_{1,t} = a_{1,t-1}$ and $a_{2,t} = a_{2,t-1}$. The corresponding weight is computed by

$$\begin{aligned} w_t^{(j)} &= w_{t-1}^{(j)} p(y_t|\mathbf{a}_{0:t}^{(j)}, \mathbf{s}_{0:t-1}^{(j)}, \mathbf{y}_{0:t-1}) \\ &= w_{t-1}^{(j)} \sum_{s_t \in \mathcal{A}} p(y_t|s_t, \mathbf{a}_{0:t}^{(j)}, \mathbf{s}_{0:t-1}^{(j)}, \mathbf{y}_{0:t-1}) \quad (10) \end{aligned}$$

where $\mathcal{A} = \{A_1, \cdots, A_M\}$ is the alphabet of s_t. Note that the hybrid importance function (8) is a combination of the posterior and the prior importance functions. Intuitively, due to the use of observations, the hybrid importance function is more effective than the prior importance function. In addition, it is easier to implement than the posterior importance function in that the sampling from (8) and the computation of the weight in (10) can be readily carried out.

Now, we discuss the sampling of s_t and $\mathbf{a}_t$ from (8) and the calculation of the weight (10). First, we notice that no sampling for $\mathbf{a}_t$ is needed which simplifies the sampling process. However, the absence of sampling introduces lack of diversity on $\mathbf{a}_t$. To address this problem, kernel smoothing techniques can be used during the resampling procedure which will be discussed in Section 3.4. As for s_t, since it is discrete, the sampling of it from (8) only requires the evaluation of the importance function on $\mathcal{A}$. In particular, if by assuming a uniform prior on s_t, the sampling distribution becomes

$$p(s_t|\mathbf{a}_{0:t}^{(j)}, \mathbf{s}_{0:t-1}^{(j)}, \mathbf{y}_{0:t}) \propto p(y_t|s_t, \mathbf{a}_{0:t}^{(j)}, \mathbf{s}_{0:t-1}^{(j)}, \mathbf{y}_{0:t-1}) \quad (11)$$

Now, from (10) and (11), we see that both the sampling of s_t and the calculation of the weight $w_t^{(j)}$ are achieved by computing $p(y_t|s_t, \mathbf{a}_{0:t}^{(j)}, \mathbf{s}_{0:t-1}^{(j)}, \mathbf{y}_{0:t-1})$, $\forall s_t \in \mathcal{A}$. This distribution is the likelihood function after marginalizing out h_t and can be obtained from the predictive step of the Kalman filter and it is given by

$$p(y_t|s_t, \mathbf{a}_{0:t}^{(j)}, \mathbf{s}_{0:t-1}^{(j)}, \mathbf{y}_{0:t-1}) = \mathcal{N}(m_t^{(j)}, c_t^{(j)}) \quad (12)$$

where $m_t^{(j)} = \mathbf{g}^\top \mathbf{D}_{(t-1)}^{(j)} \mu_{t-1}^{(j)} s_t$ and $c_t^{(j)} = \mathbf{g}^\top \mathbf{\Sigma}_t^{(j)} \mathbf{g} + \sigma^2$ with $\mathbf{\Sigma}_t = \mathbf{D}_{t-1}^{(j)} \mathbf{P}_{t-1}^{(j)} (\mathbf{D}_{t-1}^{(j)})^\top + \sigma_v^2 \mathbf{g}\mathbf{g}^\top$, and

$$\mathbf{D}_{t-1}^{(j)} = \begin{bmatrix} -a_{1,t-1}^{(j)} & -a_{2,t-1}^{(j)} \\ 1 & 0 \end{bmatrix}.$$

Moreover, $\mu_{t-1}^{(j)}$ and $\mathbf{P}_{t-1}^{(j)}$ are computed from the update steps of the Kalman filter that are expressed, at t as $\mu_t^{(j)} = \mathbf{D}_t^{(j)} \mu_{t-1}^{(j)} + \mathbf{K}_t^{(j)} (y_t - m_t^{(j)})$ and $\mathbf{P}_t^{(j)} = (\mathbf{I} - \mathbf{K}_t^{(j)} \mathbf{g}^\top s_t^{(j)}) \mathbf{\Sigma}_t^{(j)}$ where $\mathbf{K}_t^{(j)} = \mathbf{\Sigma}_t^{(j)} \mathbf{g} c_t^{(j)-1} s_t^{(j)}$.

Now we have identified every element required in the implementation of the particle filtering algorithm. The resulting weighted samples $\{s_t^{(j)}, w_t^{(j)}\}_{j=1}^N$ approximate $p(s_t|\mathbf{y}_{0:t})$, and the minimum mean square error (MMSE) estimator of s_t can be easily calculated according to

$$\hat{s}_{tMMSE} = \sum_{j=1}^{N} s_t^{(j)} w_t^{(j)}. \quad (13)$$

3.3. Initial sampling of the AR(2) coefficients

At the beginning, initial N samples of $\mathbf{a}_0$ are drawn from a predefined prior distribution. Usually a uniform distribution defined on the whole variable space is chosen. However, for our problem, the sample space of the uniform distribution can be confined to enhance the efficiency and the performance of the proposed algorithm by considering the physical feature of the system.

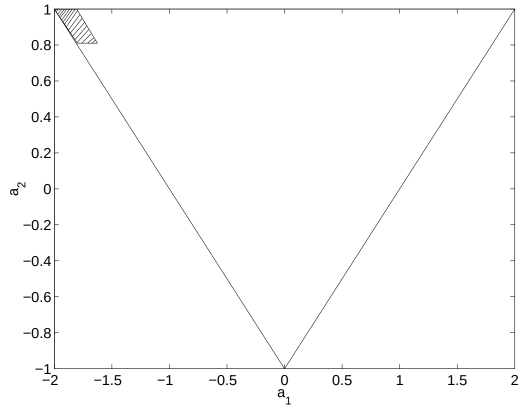

Figure 1: Plot of the sample space for the coefficients a_1 and a_2 of the AR(2) channel model. The area inside the triangle corresponds to the coefficients which induce stable and minimum phase process. The shaded area is a constrained region derived from a practical system.

First, to ensure the stability and the minimum phase of the AR(2) process, the variable space of $\mathbf{a}_0$ is defined in a triangle region depicted in Figure 1 and the sampling from the uniform distribution defined on the region can be done as in [9]. However, ambiguity in the estimation of a_1 and a_2 exists in this triangle region. The ambiguity is an inherent problem in blind detection. For instance, when the transmission is BPSK modulated, if $\bar{a}_1$ and $\bar{a}_2$ were one set of the estimates, then $-\bar{a}_1$ and $\bar{a}_2$ would also be a legitimate set of the estimates. To overcome the ambiguity, further restrictions in the sampling space need to be imposed. This can be achieved by considering the relationship between the AR coefficients and the physical parameters of the underlying fading channels. It is shown in [10] that the AR(2) coefficients are chosen by

$$a_1 = -2r_d \cos(2\pi\Omega_d/\sqrt{2}) \quad and \quad a_2 = r_d^2 \quad (14)$$

where r_d is the pole radius of the AR model and $\Omega_d = f_d T$ is the normalized maximal Doppler frequency. Since r_d determines the steepness of the power spectrum of the AR process, to closely approximate the Jakes' model, r_d is often taken between [0.9, 0.999]. Furthermore, an upper limit on Ω_d can be easily obtained from a practical viewpoint. For example, for a system with a carrier frequency of $2G$ Hz, a vehicle speed of 75 mile/hour, and symbol rates for all transmissions greater than 3600 Hz, the maximal Doppler frequency Ω_d must be less than 0.062. Then, by using the practical limits imposed on r_d and Ω_d, we can obtain from (14) a refined region for a_1 and a_2, and this region is automatically in the triangle region of a stable AR process. As a consequence, the initial samples of a_1 and a_2 can be obtained by first sampling r_d and Ω_d uniformly from the imposed regions and then compute the corresponding coefficients from (14). In Figure 1 with the shaded lines, we also plot the region corresponding to $r_d \in [0.9, 0.999]$ and $\Omega_d \in [0, 0.1]$. We see that the region is much constrained

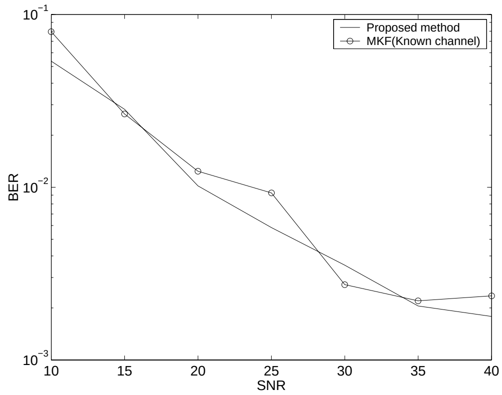

Figure 2: Plot of the BERs of the proposed particle filtering detector and the MKF with known AR coefficients

compared with the triangle region.

3.4. The resampling procedure

The resampling procedure [6] can be further incorporated to enhance the efficiency as well as the performance of the proposed particle filtering algorithm. However, since no sampling is involved for a_1 and a_2 throughout the implementation, the algorithm with simple resampling does not have the ability to rejuvenate a_1 and a_2 with the arrival of new observations. Thus, the accuracy of the final estimates of a_1 and a_2 depends greatly on the initial samples. To overcome this drawback, we adopted a scheme proposed in [11] which combines the auxiliary particle filter and a kernel smoothing technique. Modifications are made to adapt the use of the hybrid importance function. In the implementation, when resampling is needed, the proposed procedure is inserted to replace the original particle filtering step.

4. SIMULATION

The performance of the proposed particle filtering detector is studied in this section. To simulate a fading channel, the coefficients of the AR(2) model were taken as $a_1 = -1.9602$ and $a_2 = 0.9701$. They reflect a physical scenario of a normalized maximal Doppler spread of 0.0224. This AR process is normalized to have a unit power, and thus the signal-to-noise ratio (SNR) is obtained as $10\log(1/\sigma^2)$. The transmitted signal is BPSK modulated with differential coding.

In Figure 2, we provide the bit error rates (BERs) of the proposed detector under various SNRs. In the implementation of the proposed detector, 300 trajectories were maintained at every time instant. In particular, initial samples of a_1 and a_2 were drawn from the shaded region in Figure 1. To compute the BER at a given SNR, a symbol stream was transmitted continuously until 200 errors were collected. In the same figure, we also plotted the performance of the MKF with known AR coefficients. There were 200 trajectories and similarly 200 errors were collected to obtain each BER estimate. Apparently, the proposed detector has the similar performance as the MKF.

5. CONCLUSIONS

A particle filtering detector has been proposed for fully blind estimation of the parametric channel coefficients, channel tracking, and signal detection. A novel hybrid importance function has been introduced which leads to efficient implementation of the detector. The physical interpretation of the AR(2) channel model and the underlying fading channel has been used to further enhance the efficiency of the detector and to avoid ambiguity in the detections. The simulation results demonstrate good performance of the proposed detector.

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