# A CLASS OF PERFECT-RECONSTRUCTION NONUNIFORM COSINE-MODULATED FILTER-BANKS WITH DYNAMIC RECOMBINATION

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## ABSTRACT

This paper presents a class of perfect-reconstruction (PR) nonuniform cosine-modulated filter-banks (CMFBs) with dynamic recombination, which based on a two-stage structured nonuniform filter-bank (FB). By dynamically merging consecutive subbands in an original uniform FB with pre-designed transmultiplexers (TMUXs) having different numbers of channels, PR nonuniform FBs with adjustable time-frequency resolution can be obtained. In particular, we emphasize the issue of the PR recombination nonuniform FB when the coprime condition on the numbers of channels in the original FB and the recombination TMUX does not hold. A new optimization criterion is proposed to handle the case. Finally, detailed design procedure and example are shown to illustrate this method.

## I. INTRODUCTION

The theory and design of *M*-channel uniform filter-bank (FB) has been extensively studied [1]. In some applications, such as audio analysis/coding and broadband array signal processing, nonuniform FBs with adjustable time-frequency resolution may lead to better performance and reduced arithmetic complexity. The theory and design of nonuniform FB have been studied in previous literatures [2-11]. For a recent review and the open research problems in nonuniform FBs, see [8] and the references therein. In this paper, we consider a class of nonuniform FB that was first proposed by Cox [3]. Fig. 1 shows the general structure of this FB, where successive channels of an *L*-channel uniform FB are recombined by the synthesis FB of another FB with smaller number of channels. In Fig. 1, one such nonuniform subband obtained by merging $m_s$ outputs from the uniform FB is illustrated. For convenience, we call this method the indirect or recombination (merging) method. In Cox's original work, the FBs were derived from the pseudo-quadrature mirror filters, and it is not perfect-reconstruction (PR). In addition, the possibility of using biorthogonal FBs to reduce the system delay was not studied. In [4], Kovacevic and Vetterli presented a detailed theory and classification of nonuniform FBs. They compared the indirect method with their proposed direct structure and pointed out that the indirect method does not have an equivalent be linear time-invariant (LTI) filters ($H_0,...,H_{N-1}$) representation as the direct structure, which makes it difficult to perform the optimization. Because of this reason, the two stages of analysis bank in the indirect method are usually designed separately. On the other hand, the direct method allows one to have complete control over the desired frequency characteristics of the analysis filters ($H_0,...,H_{N-1}$). Recently, the authors have found that the recombination nonuniform FB can achieve PR if the original FB and the recombination transmultiplexer (TMUX) are both PR [5,6]. Furthermore, if the number of channels in the original FB and the recombination TMUX are coprime to each other, the analysis and synthesis filters of the merged channel can be shown to LTI. In other words, we can minimize the stopband attenuation of this LTI filter, which considerably simplifies the design of the nonuniform FB. It should be noted, however, that the direct structure in [4] for the coprime situation is more general than the two-stage recombination FBs because the analysis filters of the latter have an interpolation filter-like structure. This has the advantage of higher stopband attenuation but at the expense of longer system delay. In contrast, the direct structure does not have this limitation and is thus very useful in designing FBs with short filter support and low system delay, where all the freedom can be fully utilized under the given constraint of filter length and system delay.

The design of the indirect recombination FBs can further be simplified by using the cosine modulated filter-banks (CMFBs) as the original and merging FBs. It was shown in [5, 6] that the two CMFBs can be designed separately, except that certain simple transition band matching condition has to be satisfied. PR Nonuniform FBs with very high stopband attenuation can be obtained with very low design and implementation complexities. Moreover, by using biorthogonal CMFB, the system delay of the recombination nonuniform FB can be significantly reduced [6]. Another interesting application of this result is that FBs with different time-frequency resolutions can be achieved by dynamic merging consecutive subbands of the original FBs with pre-designed TMUX having different number of channels, since the original and merging FBs can be designed separately after appropriate matching of the transition band. These dynamically recombination nonuniform FBs are most useful in situations where the input sampling rate is very high. The uniform FB, which usually has a large number of channels, helps to reduce the sampling rate through downsampling to a much lower value for further processing, such as adaptive filtering, of individual subband signals. However, due to the downsampling, the time resolution will be lower and significant aliasing components will be generated for adaptation, which is sometimes undesirable. Through recombination, wider signal bandwidth is obtained with improved time-resolution as well as aliasing cancellation. Towards this goal, we need more general design procedure for nonuniform recombination FBs when the numbers of channels in the two FBs are not coprime to each other. In this case, the analysis and synthesis filters will be linear periodic time varying. A new design method is therefore proposed to minimize the undesirable components in the bi-frequency responses of the analysis filters. Because of the various desirable properties of the CMFBs, they are also employed in this study. A detailed design example is given to demonstrate the principle and design procedure of the proposed nonuniform FBs.

The design results also confirm that the proposed dynamic recombination nonuniform FB is useful in improving the time resolution of certain channels in the original uniform FB by merging adaptively its sub-channels. Design examples show that low implementation and design complexity nonuniform FB with good frequency characteristics and capable of recombining dynamically can be obtained by the proposed method. The paper is organized as follows: Section II describes the principle of the proposed PR recombination nonuniform FB. The design procedure and example are shown in Section III. Finally, we summarize the conclusions in Section IV.

## II. THE PRINCIPLE OF THR PROPOSED PR RECOMBINATION NONUNIFORM FBS

The referred recombination nonuniform FBs is shown in Fig. 1. The *s*-th recombination section inside the box in dotted line constitutes an $m_s$ -channel TMUX. It can be seen that if the *L*-channel original uniform FB and all TMUXs are PR, then the whole system will be PR, provided that the certain delay of the TMUXs are compensated in other channels. Here, the filters $G_{s,i}$ and $G'_{s,i}$, $i=0...m_s$, are respectively the synthesis and analysis filters of the $m_s$ -channel TMUX. While the filters $H_l$ and $F_l$, $l=0...L$, are respectively the analysis and synthesis filters of the *L*-channel original FB. Because of this structural advantage, it is relatively simple for the whole system to obtain PR.

As mentioned in Section I, the recombination nonuniform FBs do not generally have an equivalent LTI filter representation like the direct structure in [4]. The analysis filters are linear time periodic varying.

In the case where *L* and $m_s$ are coprime, each subband of the nonuniform FB will have an equivalent LTI filter representation. The frequency response of the nonuniform FB can be evaluated directly to minimize some performance measures such as the stopband attenuation and passband ripples. In [5], the authors have given a detailed discussion on this case. The theory and design of the biorthogonal nonuniform CMFBs [6] have been presented, where the system delay can be greatly reduced.

In the case where *L* and $m_s$ are not coprime, the analysis and synthesis filters are linear periodic time varying. A new design method has to be proposed. To gain more insight into the problem at hand, let's consider the discrete-time Fourier transform (DTFT) of the merged output in the FB. Assume that the first channel to be merged by the transmultiplexer under consideration is the $r_s$-th subband of the *L*-band uniform FB, Fig. 1. Also, assume that the output of the merged channel is the *s*-th channel, (*s=0,…S*-1), of the nonuniform FB. Its DTFT can be written as:

$$Y_s(e^{j\omega}) = \tfrac{1}{L}\sum_{i=0}^{m_s-1} G_{s,i}(e^{j\omega})\sum_{v=0}^{L-1} H_{r_s+i}(e^{j\frac{m_s\omega-2\pi v}{L}})X(e^{j\frac{m_s\omega-2\pi v}{L}})$$

$$= \tfrac{1}{L}\sum_{i=0}^{m_s-1} G_{s,i}(e^{j\omega})H_{r_s+i}(e^{j\frac{m_s\omega}{L}})X(e^{j\frac{m_s\omega}{L}})$$

$$+\tfrac{1}{L}\sum_{i=0}^{m_s-1} G_{s,i}(e^{j\omega})\sum_{v=1}^{L-1} H_{r_s+i}(e^{j\frac{m_s\omega-2\pi v}{L}})X(e^{j\frac{m_s\omega-2\pi v}{L}}), \qquad (1)$$

where $X(e^{j\omega})$ and $Y_s(e^{j\omega})$ are respectively the DTFT of the input and the *s*-th channel output. The first part of (1) is the desired signal component that will be generated by an ordinary LTI analysis filter. The second part is the additional aliasing components generated by the linear periodic time varying analysis filters. For ideal analysis filters with infinitely sharp cutoff, all the aliasing terms will be zero. For practical filters with finite length, we would expect that the aliasing terms would be small when the analysis and recombination filters have sufficiently large stopband attenuation and sharp transition band. This would require filters with very long filter length. Though paraunitary FBs can be completely characterized by the lossless lattice structure, the design and implementation complexities is rather high due to the nonlinear dependence of the frequency response on the lattice parameters. PR FB based on CMFB on the other hand is relatively simple to design and the filter characteristics and stopband attenuation are very good. Therefore, they are considered in this study. The new optimization is to minimize the additional aliasing terms in (1). As an illustration, the frequency response of a LTI filter with cutoff frequency at $\pi/4$ and its bi-frequency response are shown in Fig. 2(a) and (b), respectively. The bi-frequency response plots the amplitudes and phases of all the frequency components being generated for input sinusoidal at certain input frequency with unit amplitude and zero phase. The response corresponding to the line at 135 degree is associated with the frequency response at negative frequencies. The response on the diamond-shaped ring in Fig. 2(d) corresponds to the aliasing terms of the LTPV analysis filter. To visualize more clearly the magnitude of the various responses, only the one at 45 degree is plotted in Fig. 2(c). The dotted line is the projection of its corresponding aliasing term. Our goal is to minimize these aliasing components on the diamond ring and those resulting from decimation. It can be seen that there are considerable aliasing components around $\pi/8$. Next, we shall consider method for suppressing these aliasing components.

**Optimization criterion**

For simplicity, let's rewrite (1)

$$Y_s(e^{j\omega}) = \frac{1}{L}\sum_{v=0}^{L-1} A_v(e^{j\omega})X(e^{j\frac{m_s\omega-2\pi v}{L}}), \qquad (2)$$

where

$$A_v(e^{j\omega}) = \sum_{i=0}^{m_s-1} G_{s,i}(e^{j\omega})H_{r_s+i}(e^{j\frac{m_s\omega-2\pi v}{L}}),\ 0\le v\le L-1. \qquad (3)$$

The terms with $v\neq 0$ represent those additional aliasing terms. In what follows, method for minimizing $A_v(e^{j\omega})$, $1\le v\le L-1$, is derived. To simplify the notation, define $H'_{r_s+i}(e^{j\omega}) = H_{r_s+i}(e^{j\frac{m_s}{L}\omega})$. Thus, (3) can be rewritten as

$$A_v(z) = \sum_{i=0}^{m_s-1} G_{s,i}(z)H'_{r_s+i}(zW^v), \quad 0\le A_v\le L-1, \qquad (4)$$

where $W = W_{m_s} = e^{-j2\pi/m_s}$.

By the CMFB in the form of DTFT, $G_{s,i}(z)$ and $H'_{r_s+i}(zW^v)$ can be written as

$$G_{s,i}(e^{j\omega}) = G(e^{j(\omega-\frac{\pi(i+0.5)}{m_s})})e^{-j((2i+1)\theta_0+\frac{(-1)^i\pi}{4})}$$
$$+G(e^{j(\omega+\frac{\pi(i+0.5)}{m_s})})e^{j((2i+1)\theta_0+\frac{(-1)^i\pi}{4})},$$
$$H'_{r_s+i}(e^{j(\omega-\frac{2\pi v}{m_s})}) = H'(e^{j(\omega-\frac{2\pi v}{m_s}-\frac{\pi(i+0.5)}{m_s})})e^{-j((2i+1)\theta_1-\frac{(-1)^i\pi}{4})}$$
$$+H'(e^{j(\omega-\frac{2\pi v}{m_s}+\frac{\pi(i+0.5)}{m_s})})e^{j((2i+1)\theta_1-\frac{(-1)^i\pi}{4})}, \qquad (5)$$

where

$$H'(e^{j\omega}) = H(e^{j\frac{m_s}{L}\omega}); \qquad (6)$$

$G(e^{j\omega})$ and $H(e^{j\omega})$ are the prototype filters of $m_s$ - and *L*-channel uniform CMFBs, respectively;

$$\theta_0 = \frac{\pi}{2m_s}\times\frac{N_{m_s}}{2},\ \theta_1 = \frac{\pi}{2L}\times\frac{N_L}{2}. \qquad (7)$$

$N_{m_s}$ and $N_L$ are the filter lengths of $m_s$ - and *L*-channel uniform CMFBs, respectively.

For simplicity, let's consider an example with sampling rates 2/8, which can be obtained by merging the first two channels in an 8-channel PR filter bank by a 2-channel PR filter bank. It is possible to generate more general PR non-uniform FB, such as one with sampling rates $(2/8, 3/8, 3/8)$. The latter can be obtained by merging last six channels in two groups of three in an 8-channel PR FB. This frequency partition cannot be derived from conventional tree-structure FB.

For the 2/8 case, there are seven additional aliasing terms, namely

$$A_v(z) = \sum_{i=0}^{m_s-1} G_{s,i}(z)H'_{r_s+i}(zW^v), \quad 1\le A_v\le L-1, \qquad (8)$$

Careful examination reviews that if

$$G(e^{j\omega}) = H'(e^{j\omega}) \text{ and } \theta_0 = \theta_1, \qquad (9)$$

then $A_v(z)$, $1\le A_v\le L-1$ is at its minimum value.

Substituting (6) and (7) to (9), we have

$$G(e^{j\omega}) = H(e^{j\frac{m_s}{L}\omega}),\ N_{m_s}/m_s = N_L/L. \qquad (10)$$

In general, if the prototype filters of $m_s$- and $L$-channel uniform CMFBs satisfy the condition in (10), then the additional aliasing components of the nonuniform CMFB will be minimized. This is also the condition for suppressing the spurious responses in the coprime case [5, 6].

In the previous work [5, 6], due to the coprime condition on the numbers of channels in the original and recombination FBs, only the nonuniform FBs with sampling rate such as 4/7, 3/7 can be obtained. This limits the application of this class of PR nonuniform FBs. The new method, handling the case when coprime condition does not hold, increases the freedom of the design of the nonuniform FBs. By freely merging subbands of the original FBs with the recombination TMUX, nonuniform FB with arbitrary frequency partition can be achieved. FBs with different time-frequency resolutions are needed in some applications, this can be done by dynamically merging consecutive subbands of the original FB with pre-designed TMUX.

## III. DESIGN PROCEDURE AND EXAMPLE

From the principle described in Section II, it is concluded that if the original FB and the recombination TMUX satisfy the condition in (10), the additional aliasing components in (1) is minimized. The design procedure can be summarized as:

(1) Set $N_{m_s}/m_s = N_L/L$.

(2) Design the prototype filter, $H(e^{j\omega})$, of the PR $L$-channel uniform CMFB.

$$\min_h \Phi = \int_0^{\omega_p} \left|1 - H(e^{j\omega})\right|^2 d\omega + \int_{\omega_s}^{\pi} \left|H(e^{j\omega})\right|^2 d\omega$$

subjected to PR constraint,

where $\omega_p$ and $\omega_s$ are the passband and stopband cut-off frequencies. $\boldsymbol{h}$ is the vector containing the impulse response of $H(e^{j\omega})$.

(3) Design the prototype filter, $G(e^{j\omega})$, of the $m_s$-channel PR uniform CMFB by the constrained optimization problem

$$\min_g \Phi = \alpha \int_0^{\frac{L}{m_s}\omega_p} \left|1 - G(e^{j\omega})\right|^2 d\omega + (1-\alpha) \int_{\frac{L}{m_s}\omega_s}^{\pi} \left|G(e^{j\omega})\right|^2 d\omega$$

subjected to PR constraint,

where $\frac{L}{m_s}\omega_p$ and $\frac{L}{m_s}\omega_s$ are the passband and the stopband cutoff frequencies of $G(e^{j\omega})$, respectively. $\boldsymbol{g}$ is the vector containing the impulse response of $G(e^{j\omega})$. Because the relative transition bandwidth of $G(e^{j\omega})$ and $H(e^{j\omega})$ are identical and the same least square objective function is used in the optimization, $G(e^{j\omega})$ so obtained will be similar to $H(e^{j\frac{m_s}{L}\omega})$, hence approximately satisfying (10).

Taken the following as an example. Two uniform orthogonal CMFBs with 2 and 8 channels and filter lengths of 20 taps and 80 taps, respectively, are designed using the proposed method. The constrained optimizations involved are solved using the DCONF subroutine in the IMSL library. Fig. 2(f) shows the bi-frequency response of the first analysis filter at rate (2/8). The responses corresponding to the lines at 45 degree and 135 degree are associated with an ordinary LTI filter. The responses on the diamond-shaped ring correspond to the additional aliasing terms. To visualize more clearly the magnitude of the various responses, only the one at 45 degree is plotted in Fig. 2(e). The dotted line is the projection of its corresponding additional aliasing term. Obviously, the aliasing components are rather small, thanks to the condition in (10) to suppress the major aliasing terms and the high stopband attenuation of the CMFB. As another illustration, Fig. 2 (c) and (d) show a similar plot with (10) not being satisfied. Based on the above design, we perform a nonuniform CMFB with sampling rate (2/8,3/8,3/8). The length of the 3-channel orthogonal uniform CMFB is 30. The frequency response of this FB is shown in Fig. 3. Since no equivalent filter representation for the first band with sampling rate 2/8 in the nonuniform CMFB, we use dotted line for the ideal LTI filter for it.

Consecutive subbands can be combined on-line either by the present method or the one proposed in [5,6] for the coprime case to improve their time resolution, while trading the frequency resolution. Suppose an input signal, shown in the left side of the Fig. 4(a), with the frequency response in Fig. 4(b), applied to an 8-channel uniform FB. The decomposition should match the critical bands of the input signal, from $3\pi/8$ to $6\pi/8$. Therefore, to seize the important information in the critical band, we'd better merge the 4th, 5th and 6th channel in 8-channeluniform FB by a 3-channel uniform filter bank. The output of it is shown in Fig. 5(a). We can also merge the first 3 channels and the last 2 channels by a 3-channel and a 2-channel uniform FB, respectively. The outputs of them are shown in Fig. 5(b) and (c) separately. It indicates that the most component of the input signal is in the second band of the nonuniform FB. Thus, special measures could be taken to this band.

## IV. CONCLUSION

In this paper, we present a new optimization criterion to handle the case where the numbers of channels in the original FB and the recombination TMUX are non-coprime. By dynamically merging subbands in original uniform FB with TMUXs of different numbers of channels in a two-stage structured nonuniform FB, a class of PR nonuniform CMFBs with adjustable time-frequency resolution are obtained.

## REFERENCES


[1] P. P. Vaidyanathan, "Multirate digital filters, filter banks, polyphase network, and applications: A tutorial," *Proc. IEEE*, vol. 78, pp. 56-93, 1990.

[2] T. Blu, "An iterated rational filter bank for audio coding Time- Frequency and Time-Scale Analysis," *IEEE-SP* International Symposium on, pp. 81 –84, 1996.

[3] R. V. Cox, "The design of uniformly and nonuniformly spaced pseudoquadrature mirror filters," *IEEE Trans. on ASSP.*, vol. 24, pp.1090-1096, 1986.

[4] J. Kovacevic and M. Vetterli, "Perfect reconstruction filter bank with rational sampling factors," *IEEE Trans. on SP.*, vol. 41, pp. 2047-2066, 1993.

[5] S. C. Chan, X. M. Xie and T. I. Yuk, "Theory and design of a class of cosine-modulated nonuniform filter banks," *Proc. ICASSP-2000*, vol. 1, pp. 504-507.

[6] X. M. Xie, S. C. Chan and T. I. Yuk, "A class of biorthogonal nonuniform cosine-modulated filter banks with lower system delay," *ISCAS-2001*, vol. 2, pp. 25 –28.

[7] P. Q. Hoang and P. P. Vaidyanathan, "Nonuniform multirate filter banks: theory and design," *Proc. ISCAS'89.*.

[8] S. J. Akkarakaran and P. P. Vaidyanathan, "New results and open problems on nonuniform filter-banks, *Proc. ICASSP'99*.

[9] R. L. Querioz, "Uniform filter banks with nonuniform bands: post-processing design," Proc. *ICASSP'98*.

[10] K. Nayebi, T. P. Barnwell and M. J. T. Smith, "The design of perfect reconstruction nonuniform band filter banks," *ICASSP-91*, vol. 3, pp. 1781 –1784.

[11] J. Princen, "The design of nonuniform modulated filterbanks," *IEEE Trans. on SP*, vol. 43, pp. 2550 -2560, 1995.

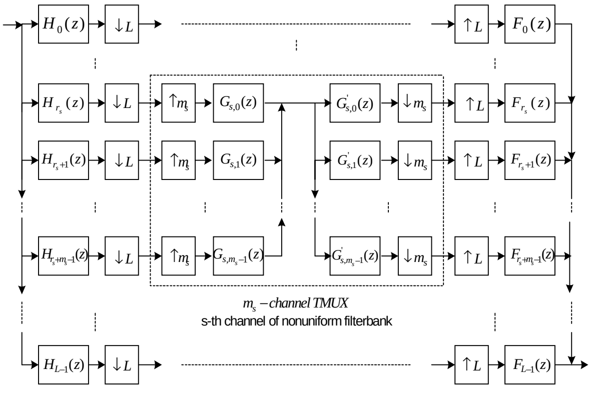



Fig. 1 Structure of recombination nonuniform PR FB

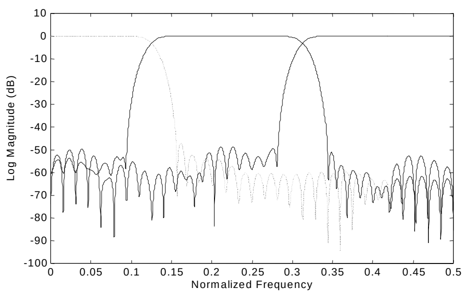
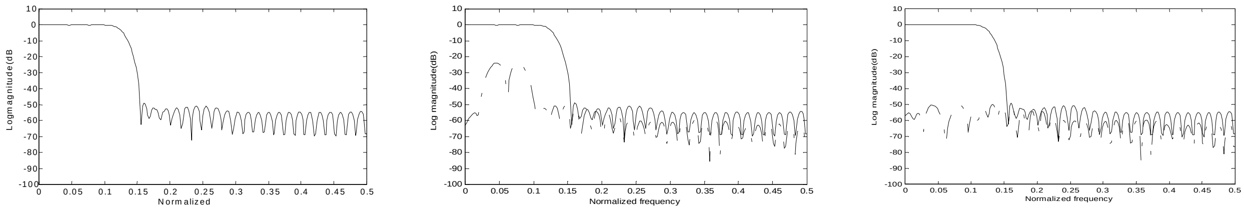
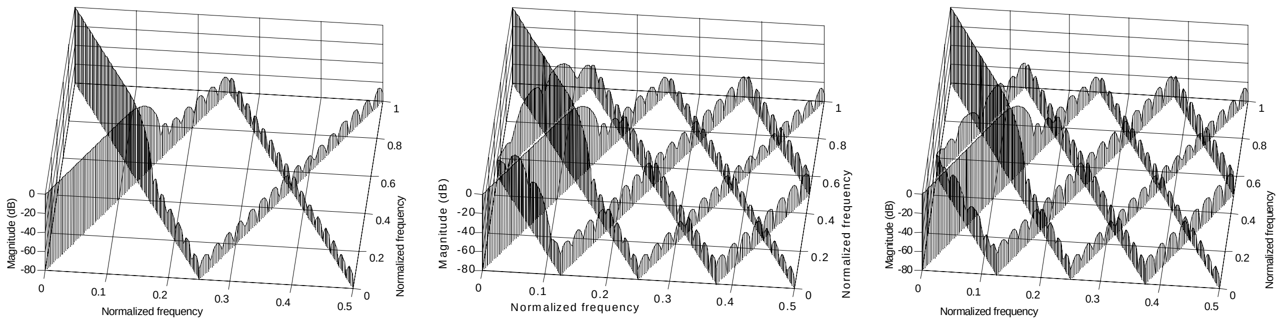



Fig. 3 Frequency response of nonuniform FB with sampling rate (2/8,3/8,3/8)

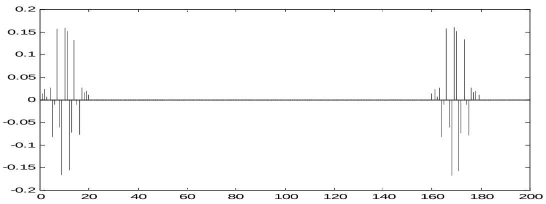



(a) (c) (e)

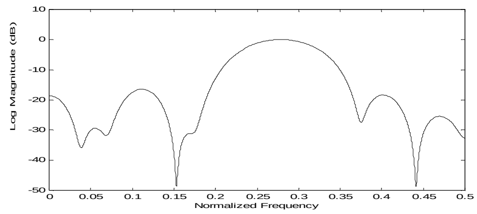



(b) (d) (f)

Fig. 2. The frequency responses of the first band of the nonuniform FB with sampling factors (2/8,…). (a) Frequency response of an LTI filter with cutoff frequency at $0.5\pi$ and (b) its bi-frequency response (frequency response at negative frequency is oriented at 135 degree); (c) Projection of the bi-frequency response at 45 degree (solid line) and its associated additional aliasing term (dotted line) – condition (10) not satisfied, (f) bi-frequency responses of the analysis filter – condition (10) not satisfied; (e) Projection of the bi-frequency response at 45 degree (solid line) and its associated additional aliasing term (dotted line) – condition (10) satisfied, (f) bi-frequency responses of the analysis filter – condition (10) satisfied.

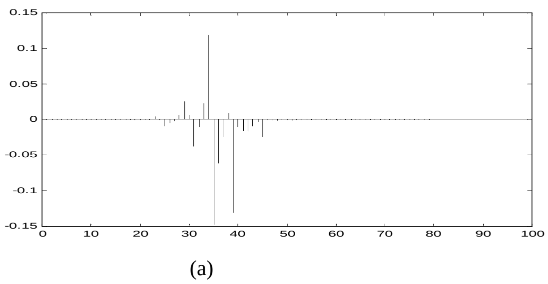


(a)




(b)

Fig. 4(a) Input (left) and output (right)of (3/8,2/8,3/8) nonuniform CMFB (b) Frequency response of the input signal



(a)

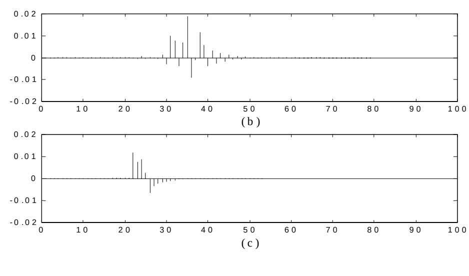



Fig. 5 Output in nonuniform CMFB (3/8,3/8,2/8) of (a) the second band (b) the first band and (c) third band.