ADAPTIVE KALMAN FILTERING FOR SPEECH SIGNAL RECOVERY IN COLORED NOISE

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ABSTRACT

This paper deals with the problem of speech enhancement when a corrupted speech signal with an additive colored noise is the only information available for processing. Kalman filtering is known as an effective speech enhancement technique, in which speech signal is usually modeled as autoregressive (AR) process and represented in the state-space domain. In the above context, all the Kalman filter-based approaches proposed in the past, operate in two steps: first, the noise and the signal parameters are estimated, and second, the speech signal is estimated by using Kalman filtering. In this paper a new sequential estimators are developed for sub-optimal adaptive estimation of the unknown a priori driving processes statistics simultaneously with the system state and a recursively least-squares lattice (RLSL) algorithm is used for adaptive estimation of the speech and noise AR parameters. The algorithm provides improved speech estimate at little computational expense.

1 INTRODUCTION

Speech enhancement using a single microphone system has become an active research area for audio signal enhancement. The aim is to minimize the effect of noise and to improve the performance in voice communication systems when input signals are corrupted by background noise.

Kalman filtering is known as an effective speech enhancement technique, in which speech signal is usually modeled as autoregressive (AR) process and represented in the state-space domain. Many approaches using Kalman filtering have been referenced in the literature. They usually operate in two steps: first, the noise and the signal parameters are estimated, and second, the speech signal is estimated by using Kalman filtering. These approaches differ essentially one from the other by the choice of the algorithm used to estimate the parameters of such model, the models adopted for the speech signal and the additive noise. In [1], [2] and [3] the noise under a simplified assumption is considered as an white Gaussian process, but in [4], [5] and [6] the noise is considered colored. Paliwal and Basu [1] have used estimates of the speech signal parameters from clean speech, before being contaminated by white noise. They then used a delayed version of Kalman filter in order to estimate the speech signal. In [2], Oppenheim et al. have used a time-adaptive algorithm to adaptively estimate the speech model parameters and the noise variance. Gannot et al. [6] have proposed the use of EM (Espectation-Maximisation) algorithm to iteratively estimate the spectral parameters of speech and noise parameters. The enhanced speech signal was obtained as a byproduct of the parameter estimation algorithm. In [5], the coefficients of the AR processes and the AR driving processes variances are estimated based on EM algorithm. Gabrea and O'Shaughnessy [3] have proposed estimating the noise and driving process variances using the property of the innovation sequence, obtained after a preliminary Kalman filtering with an initial gain.

In this paper a new adaptive Kalman filter based method is proposed to recover the speech signal from a sequence of the speech signal corrupted by an additive colored noise. The speech signal and the additive noise are modeled as the AR processes.

The RLSL [7] algorithm is proposed for adaptive estimation of the speech and noise AR parameters because it has a rate of convergence typically an order of magnitude faster than the least mean squares (LMS) algorithm used in [2].

The sequential estimators are derived for sub-optimal adaptive estimation of the unknown a priori driving processes statistics simultaneously with the system state by reformulating and adapting the classical approach used for control applications.

A limited memory algorithm is developed for adaptive correction of the a priori statistics, which are intended to compensate for time varying model errors. The algorithm involves using the state corrections to estimate the driving processes variances and provides improved state estimates at little computational expense.

The paper is organized as follows. In Section II we present the speech enhancement approach based on the Kalman filter algorithm. Section III is concerned with the estimation of AR parameters and driving processes statistics. Simulation results are the subject of Section IV.

2 NOISY SPEECH MODEL AND KALMAN FILTERING

The speech signal $s(n)$ and the additive noise $v(n)$ are modeled as the pth-order order and qth-order AR processes:

$$\begin{aligned} s(n) &= \sum_{i=1}^{p} a_i s(n-i) + u(n) && (1) \\ v(n) &= \sum_{j=1}^{q} b_j v(n-j) + w(n) && (2) \\ y(n) &= s(n) + v(n) && (3) \end{aligned}$$

where $s(n)$ is the nth sample of the speech signal, $v(n)$ is the nth sample of the additive noise, $y(n)$ is the nth sample of the observation, a_i is the ith AR speech model parameter and b_j is the jth AR noise model parameter.

This system can be represented by the following state-space model:

$$\begin{aligned} \mathbf{x}(n) &= \mathbf{F}\mathbf{x}(n-1) + \mathbf{d}(n) && (4) \\ y(n) &= \mathbf{H}\mathbf{x}(n) && (5) \end{aligned}$$

where:

1. $\mathbf{x}(n)$ is the $(p+q) \times 1$ state vector
$$\mathbf{x}(n) = [s(n-p+1), \cdots s(n), v(n-q+1), \cdots, v(n)]^T \qquad (6)$$
2. $\mathbf{d}(n)$ is the $(p+q) \times 1$ vector
$$\mathbf{d}(n) = [0, \cdots, 0, u(n), 0, \cdots, 0, w(n)]^T \qquad (7)$$
3. the sequences $u(n)$ and $w(n)$ are uncorrelated Gaussian white noise sequences with means $\bar{u}(n)$ and $\bar{w}(n)$ and the variances $\sigma_u^2(n)$ and $\sigma_w^2(n)$
4. $\mathbf{F}$ is the $(p+q) \times (p+q)$ transition matrix
$$\mathbf{F} = \begin{bmatrix} \mathbf{F_s} & 0 \\ 0 & \mathbf{F_v} \end{bmatrix} \qquad (8)$$
$$\mathbf{F_s} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ a_p & a_{p-1} & a_{p-2} & \cdots & a_1 \end{bmatrix} \qquad (9)$$
$$\mathbf{F_v} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ b_q & b_{q-1} & b_{q-2} & \cdots & b_1 \end{bmatrix} \qquad (10)$$
5. $\mathbf{H}$ is the $1 \times (p+q)$ observation row vector
$$\mathbf{H} = [0, \cdots, 0, 1, 0, \cdots, 0, 1] \qquad (11)$$

The standard Kalman filter [8] [9] provides the updating state-vector estimator equations:

$$\begin{aligned} e(n) &= y(n) - \mathbf{H}\hat{\mathbf{x}}(n/n-1) && (12) \\ \mathbf{K}(n) &= \mathbf{P}(n/n-1)\mathbf{H} \times \\ &\times \ (\mathbf{H}\mathbf{P}(n/n-1)\mathbf{H}^T)^{-1} && (13) \\ \hat{\mathbf{x}}(n/n) &= \hat{\mathbf{x}}(n/n-1) + \mathbf{K}(n)e(n) && (14) \\ \mathbf{P}(n/n) &= (\mathbf{I} - \mathbf{K}(n)\mathbf{H})\mathbf{P}(n/n-1) && (15) \\ \hat{\mathbf{x}}(n+1/n) &= \mathbf{F}\hat{\mathbf{x}}(n/n) + \bar{\mathbf{d}}(n) && (16) \\ \mathbf{P}(n+1/n) &= \mathbf{F}\mathbf{P}(n/n)\mathbf{F}^T + \mathbf{Q}(n) && (17) \end{aligned}$$

where:

1. $\hat{\mathbf{x}}(n/n-1)$ is the minimum mean-square estimate of the state vector $\mathbf{x}(n)$ given the past observations $y(1)$, ..., $y(n-1)$
2. $\tilde{\mathbf{x}}(n/n-1) = \mathbf{x}(n) - \hat{\mathbf{x}}(n/n-1)$ is the predicted state-error vector
3. $\mathbf{P}(n/n-1) = E[\tilde{\mathbf{x}}(n/n-1)\tilde{\mathbf{x}}^T(n/n-1)]$ is the predicted state-error correlation matrix
4. $\hat{\mathbf{x}}(n/n)$ is the filtered estimate of the state vector $\mathbf{x}(n)$
5. $\tilde{\mathbf{x}}(n/n) = \mathbf{x}(n) - \hat{\mathbf{x}}(n/n)$ is the filtered state-error vector
6. $\mathbf{P}(n/n) = E[\tilde{\mathbf{x}}(n/n)\tilde{\mathbf{x}}^T(n/n)]$ is the filtered state-error correlation matrix
7. $\mathbf{Q}(n) = E[(\mathbf{d}(n) - \bar{\mathbf{d}}(n))(\mathbf{d}(n) - \bar{\mathbf{d}}(n))^T]$ is the driving processes correlation matrix
8. $e(n)$ is the innovation sequence
9. $\mathbf{K}(n)$ is the Kalman gain

The estimated speech signal can be retrieved as the pth component of the state-vector estimator $\hat{\mathbf{x}}(n/n)$. However, the transition matrix and the driving processes statistics are unknowns and hence must be estimated.

3 PARAMETERS ESTIMATION

The estimation of the transition matrix, which contains the AR models parameters, was made using two adaptive filters based on RLSL algorithm, to provide the best prediction in the sense of least-squarer error of the present value of the speech and the noise. The RLSL algorithm is in fact rewriting of the QR-decomposition-based least-squares lattice algorithm (QRD-LSL), which represent the most fundamental form of an order-recursive adaptive filter. This algorithm enjoys many of the properties of the QRD-LSL algorithm, namely, fast convergence, modularity, and an integral set of useful parameters and variables for signal processing applications. For a such purpose we estimate the transition matrix in two steps: first, we estimate the noise AR parameters during the silence period and second, the speech AR parameters using the pth component of the state-vector estimator $\hat{\mathbf{x}}(n/n)$.

The estimation of driving process statistics needed to compute the vector $\bar{\mathbf{d}}(n)$ and the matrix $\mathbf{Q}(n)$ is derived under the assumption of the constant mean and variance over N samples $u(n), u(n-1), \cdots, u(n-N+1)$ and $w(n), w(n-1), \cdots, w(n-N+1)$, respectively by reformulating and adapting the approach proposed in control by Myers and Tapley [10]. Using the state propagation equation (4) the samples of the driving process $u(n)$ are given by the equation:

$$u(n) \quad = \quad \mathbf{H}_1[\mathbf{x}(n) - \mathbf{F}\mathbf{x}(n-1)] \qquad (18)$$

where $\mathbf{H}_1 = [0, \cdots, 0, 1, 0, \cdots, 0, 0]$. The true state vectors $\mathbf{x}(n)$ and $\mathbf{x}(n-1)$ are unknown, so $u(n)$ cannot be determined, but the approximation:

$$\alpha(n) \quad = \quad \mathbf{H}_1[\hat{\mathbf{x}}(n/n) - \hat{\mathbf{x}}(n/n-1)] \qquad (19)$$

can be used [10]. The samples $\alpha(n)$ are assumed to be representative of $u(n)$ and can be considered independent and identically distributed. Based on the last N measurements the mean $\bar{\alpha}(n)$ and the variance $\sigma^2_\alpha(n)$ are estimated [11]. An unbiased esimator for $\bar{\alpha}(n)$ is taken as the sample mean:

$$\hat{\bar{\alpha}}(n) \quad = \quad \frac{1}{N}\sum_{i=0}^{N-1} \alpha(n-i) \qquad (20)$$

and an unbiased estimator for $\hat{\sigma}^2_\alpha(n)$ is obtained by:

$$\hat{\sigma}^2_\beta(n) = \frac{1}{N-1}\sum_{i=0}^{N-1}[\alpha(n-i) - \hat{\bar{\alpha}}(n)]^2 \qquad (21)$$

The estimation of the driving process mean is:

$$\hat{\bar{u}}(n) \quad = \quad \hat{\bar{\alpha}}(n) \qquad (22)$$

If the samples $\alpha(n)$ are considered independent and identically distributed the expected value of $\hat{\sigma}^2_\alpha(n)$ is

$$E\{\hat{\sigma}^2_\alpha(n)\} = \frac{1}{N}\sum_{i=0}^{N-1} E\{[\alpha(n-i)]^2\} \qquad (23)$$

The analysis reduces to expanding $E\{[\alpha(n-i)]^2\}$ in term of $\sigma^2_u(n)$. We write $\alpha(n)$ in term of the filtered state-error vectors:

$$\alpha(n) = -\mathbf{H}_1\tilde{\mathbf{x}}(n/n) + \mathbf{H}_1\mathbf{F}\tilde{\mathbf{x}}(n-1/n-1) + u(n) - \bar{u}(n) \qquad (24)$$

Since the filtered state-error vectors errors are not independent, the correlation are avoited by writing:

$$\alpha(n) + \mathbf{H}_1\tilde{\mathbf{x}}(n/n) = \mathbf{H}_1\mathbf{F}\tilde{\mathbf{x}}(n-1/n-1) + u(n) - \bar{u}(n) \qquad (25)$$

The variance of this equation is:

$$E\{[\alpha(n)+\mathbf{H}_1\tilde{\mathbf{x}}(n/n)]^2\} = \mathbf{H}_1\mathbf{F}\mathbf{P}(n-1/n-1)\mathbf{F}^T\mathbf{H}_1^T + \sigma^2_u(n) \qquad (26)$$

Now we develop $E\{[\alpha(n) - \mathbf{H}_1\tilde{\mathbf{x}}(n/n)]^2\}$ in term of $E\{[\alpha(n)]^2\}$ and of other computed terms in the Kalman filter:

$$\begin{aligned} E\{[\alpha(n) + \mathbf{H}_1\tilde{\mathbf{x}}(n/n)]^2\} \quad &= \quad E\{[\alpha(n)]^2\} \\ &+ \quad 2E\{\alpha(n)\tilde{\mathbf{x}}^T(n/n)\mathbf{H}_1^T\} \\ &+ \quad \mathbf{H}_1\mathbf{P}(n/n)\mathbf{H}_1^T \qquad (27) \end{aligned}$$

Using the Kalman filter equations the filtered state-error vector can be rewriting as

$$\tilde{\mathbf{x}}(n/n) = [\mathbf{I} - \mathbf{K}(n)\mathbf{H}_1]\tilde{\mathbf{x}}(n/n-1) - \mathbf{K}(n)[v(n) - \bar{v}] \quad (28)$$

and the second term in (26) is

$$\begin{aligned} E\{\alpha(n)\tilde{\mathbf{x}}^T(n/n)\mathbf{H}_1^T\} \quad &= \quad -\mathbf{H}_1\mathbf{P}(n/n)\mathbf{H}_1^T \\ &+ \quad \mathbf{H}_1\mathbf{P}(n/n-1) \times \\ &\times \quad [\mathbf{I} - \mathbf{K}(n)\mathbf{H}_1]^T\mathbf{H}_1^T \quad (29) \end{aligned}$$

By combining (26)(27) and (29) the resulting expression for is $E\{[\alpha(n)]^2\}$ is

$$\begin{aligned} E\{[\alpha(n)]^2\} \quad &= \quad \mathbf{H}_1\mathbf{F}\mathbf{P}(n-1/n-1)\mathbf{F}^T\mathbf{H}_1^T \\ &+ \quad \mathbf{H}_1\mathbf{P}(n/n)\mathbf{H}_1^T \\ &- \quad 2\mathbf{H}_1\mathbf{P}(n/n-1)[\mathbf{I} - \mathbf{K}(n)\mathbf{H}_1]^T\mathbf{H}_1^T \\ &+ \quad \sigma^2_u(n) \qquad (30) \end{aligned}$$

Using (21) (23) and (26) an unbiased estimator of $\sigma^2_u(n)$

is given by:

$$
\begin{aligned}
\hat{\sigma}_u^2(n) &= \frac{1}{N-1}\{\sum_{i=0}^{N-1}[\alpha(n-i)-\hat{\bar{\alpha}}(n)]^2 \\
&- \frac{N-1}{N}\mathbf{H}_1\mathbf{F}\mathbf{P}(n-i-1/n-i-1)\mathbf{F}^T\mathbf{H}_1^T \\
&- \frac{N-1}{N}\mathbf{H}_1\mathbf{P}(n-i/n-i)\mathbf{H}_1^T \\
&+ 2\frac{N-1}{N}\mathbf{H}_1\mathbf{P}(n-i/n-i-1)\times \\
&\times [\mathbf{I}-\mathbf{K}(n-i)\mathbf{H}_1]^T\mathbf{H}_1^T\} \qquad (31)
\end{aligned}
$$

The samples of the driving process $w(n)$ can be approximated by:

$$\beta(n) = \mathbf{H}_2[\hat{\mathbf{x}}(n/n)-\hat{\mathbf{x}}(n/n-1)] \qquad (32)$$

where $\mathbf{H}_2 = [0,\cdots,0,0,0,\cdots,0,1]$. Based on the last N measurements an unbiased estimator for the mean of the driving process $w(n)$ is obtained by:

$$\hat{\bar{w}}(n) = \hat{\bar{\beta}}(n) \qquad (33)$$

and an unbiased estimator of $\sigma_w^2(n)$ is obtained by:

$$
\begin{aligned}
\hat{\sigma}_w^2(n) &= \frac{1}{N-1}\{\sum_{i=0}^{N-1}[\beta(n-i)-\hat{\bar{\beta}}(n)]^2 \\
&- \frac{N-1}{N}\mathbf{H}_2\mathbf{F}\mathbf{P}(n-i-1/n-i-1)\mathbf{F}^T\mathbf{H}_2^T \\
&- \frac{N-1}{N}\mathbf{H}_2\mathbf{P}(n-i/n-i)\mathbf{H}_2^T \\
&+ 2\frac{N-1}{N}\mathbf{H}_2\mathbf{P}(n-i/n-i-1)\times \\
&\times [\mathbf{I}-\mathbf{K}(n-i)\mathbf{H}_2]^T\mathbf{H}_2^T\} \qquad (34)
\end{aligned}
$$

4 SIMULATION RESULTS

The approach was tested using a speech signal and an additive noise. The speech signals are sentences from the TIMIT database and the noise signals are the samples from the NOISEX database. Table 1 offers a comparison with others approaches, by showing averaged SNR gain based on 10 speech signals and 10 noise simulations for each speech signal.

Compared to the method similar in structure previously proposed by the author in [5] and to the Gibson's algorithm [4], the proposed method provides increases in SNR, as well as improved speech quality and intelligibility for input SNR between -5 and 15 dB. Gibson's algorithm needs two or three iterations to get the highest SNR gain and lead to computational requirements higher than those corresponding to the proposed approach.

	Output SNR		
Input SNR (dB)	[4] (dB)	[5] (dB)	proposed (dB)
-5.00	1.24	3.14	3.81
0.00	4.16	4.78	5.11
5.00	7.35	7.89	8.29
10.00	11.21	11.56	12.14
15.00	15.62	15.93	16.07

Table 1: Output SNR for an Input Speech Signal plus Colored Noise

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