Regularized Restoration of Color-Quantized Images

Yuk-Hee Chan and Yik-Hing Fung *

Center for Multimedia Signal Processing
Department of Electronic and Information Engineering
The Hong Kong Polytechnic University
Hung Hom, Kowloon, Hong Kong

ABSTRACT

This paper studies the restoration of color-quantized images. Restoration of color-quantized images is rarely addressed in the literature, and direct applications of existing restoration techniques are generally inadequate to deal with this problem. We propose a restoration algorithm specific to color-quantized images, which makes a good use of the available color palette to derive useful *a priori* information for restoration. The proposed restoration algorithm is shown to be able to improve the quality of a color-quantized image to a certain extent.

1 Introduction

Color quantization is the process of reducing the number of colors in a digital image by replacing them with a representative color selected from a palette [1]. It is widely used nowadays as it lessens the burden of massive image data on storage and transmission bandwidth that are bottlenecks in many multimedia applications.

A color-quantized image can be considered as a degraded version of the original full-color image. Accordingly, there should be some image restoration techniques for recovering the original image from its color-quantized version whenever it is necessary. Although there is quite an amount of work in the literature on digital restoration of noisy and blurred color images [2]-[10], by far little effort has been seen in the literature to address the restoration of color-quantized images. Obviously, the degradation models of the two cases are completely different and hence a direct adoption of conventional restoration algorithms does not work effectively. This paper is devoted to formulating the problem of color-quantized image restoration and developing a restoration algorithm specific to restoration of color-quantized images.

The organization of this paper is as follows. First, we describe the basic concept of color quantization and how quality degradation is introduced. Second, we formulate the *a priori* information we can have about the degradation process of color quantization and about the original image. We then present the derivation of a restoration algorithm specific to color-quantized images. In Section 4, simulation results and comparative study are provided to evaluate the performance of the proposed algorithm. Finally, conclusions are given in Section 5.

* This work was supported by POLYU Grant A412.

2 Image Degradation in color quantization

A pixel of a 24-bit full color image generally consists of three color components. The intensity values of these three components form a vector in a 3D space. In color quantization, each pixel vector is compared with a set of representative color vectors, $\hat{\mathbf{v}}_i$, $i = 1, 2, \cdots, N_c$, which are stored in a previously generated *color palette*. The best-matching color is selected based on the minimum Euclidean distance criterion, where the Euclidean distance measure between vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ is defined as $d(\mathbf{v}_1, \mathbf{v}_2) \equiv \|\mathbf{v}_1 - \mathbf{v}_2\|$. In other words, a pixel vector $\mathbf{v}$ is represented by color $\hat{\mathbf{v}}_k$ if and only if $d(\mathbf{v}, \hat{\mathbf{v}}_k) \leq d(\mathbf{v}, \hat{\mathbf{v}}_j)$ for all $j = 1, 2, \cdots, N_c$. Once the best-matching colors for all pixel vectors of the source image have been selected from the color palette, the indices of the selected colors are transmitted to the receiver with the color palette. At the receiver, with the same color palette, the color quantized image can be reconstructed based on the received indices.

3 Restoration of color quantized images

3.1 Formulation of A Priori Information

An image consisting of N pixels is represented by an $N \times 3$ matrix $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3)$, where $\mathbf{x}_i$ is the lexicographically ordered i^{th} color components of the pixels. In formulation, we have $\mathbf{x}_i = (x_{1,i}, x_{2,i}, \cdots, x_{N,i})^t$, where $x_{k,i}$ is the i^{th} color component of the k^{th} pixel of the image. Consider the case that image $\mathbf{x}$ is encoded as $\mathbf{y}$ by color quantization with a color palette C containing N_c colors:

$$C = \left\{ \hat{\mathbf{v}}_i \;\middle|\; i = 1, 2, \cdots, N_c \right\}. \quad (1)$$

According to color quantization theory, the N_c colors in the palette partition the whole color vector space, say V, into N_c non-overlapped Voronoi regions. For a particular Voronoi region $R_k = \{\mathbf{v} | \mathbf{v} \in V$ and $d(\mathbf{v}, \hat{\mathbf{v}}_k) \leq d(\mathbf{v}, \hat{\mathbf{v}}_j)$

for $j = 1, 2, \cdots, N_c\}$, the associated variance vector is given as

$$\vec{\sigma}_k = (\sigma_{k,1}^2, \sigma_{k,2}^2, \sigma_{k,3}^2) \tag{2}$$

where

$$\sigma_{k,m}^2 = \frac{1}{N_{o,k}} \sum_{\mathbf{v} \in R_k} \left(v_m - \hat{v}_{k,m} \right)^2. \tag{3}$$

Here, $N_{o,k}$ is the total number of vectors in R_k, v_m is the m^{th} element of $\mathbf{v}$, and $\hat{v}_{k,m}$ is the m^{th} element of $\hat{\mathbf{v}}_k$. In practice, in order to reduce the realization complexity, one may select some typical images to form a training set Ω_t and then approximate $\vec{\sigma}_k$ with vectors in $\Omega_t \cap R_k$ instead of R_k. Note that all $\vec{\sigma}_k$'s are solely determined by the color palette which is attached with the image or known by default and hence no extra information is required for their estimation.

Let $\mathbf{x}_b$ and $\mathbf{y}_b$ be the 3-dimensional vectors representing the b^{th} pixels of the original image $\mathbf{x}$ and the encoded image $\mathbf{y}$, respectively. If $\hat{\mathbf{v}}_k \in C$ is the color used to represent $\mathbf{x}_b$, then we will have $\mathbf{y}_b = \hat{\mathbf{v}}_k$, and the distance between $\mathbf{y}_b$ and $\mathbf{x}_b$ will be bounded by the boundary of the Voronoi region R_k. For any particular color component $y_{b,i}$ in $\mathbf{y}_b$, its deviation from the corresponding color component in $\mathbf{x}_b$, $x_{b,i}$, is also bounded. In formulation, we have

$$(y_{b,i} - x_{b,i})^2 \leq \varepsilon_{b,i} \tag{4}$$

where $\varepsilon_{b,i}$ is the bound for the uncertainty of pixel $y_{b,i}$ in encoded image $\mathbf{y}$. Since we have $\mathbf{y}_b = \hat{\mathbf{v}}_k$, $y_{b,i} = \hat{v}_{k,i}$, where $\hat{v}_{k,i}$ is the i^{th} element of $\hat{\mathbf{v}}_k$, holds and the bound $\varepsilon_{b,i}$ can be estimated to be a function of $\sigma_{k,i}^2$. By assuming that $\varepsilon_{b,i}$ is proportional to $\sigma_{k,i}^2$, we have $\varepsilon_{b,i} = \mathcal{K}\sigma_{k,i}^2$, where $\mathcal{K}$ is a constant. Note that this assumption can easily be satisfied as long as $\mathcal{K}$ is sufficiently large. In practice, by assuming a normal distribution, $\mathcal{K} \approx 10$ is a reasonable estimate as the range of $[\ \hat{v}_{k,i} - 3\sigma_{k,i}, \hat{v}_{k,i} + 3\sigma_{k,i}\ ]$ covers more than 99% of the possible values of $x_{b,i}$.

The information carried by $\sigma_{k,i}^2$ provides us an important constraint to seek the original image $\mathbf{x}$. To formulate this constraint, we first rewrite equation (4) as

$$\omega_{b,i}^2 (y_{b,i} - x_{b,i})^2 \leq \mathcal{K}, \tag{5}$$

where $\omega_{b,i}^2 = 1/\sigma_{k,i}^2$. The overall distortion in $\mathbf{y}$ can then be described as

$$\sum_{b=1}^{N} \sum_{i=1}^{3} \omega_{b,i}^2 (y_{b,i} - x_{b,i})^2 \leq 3N\mathcal{K} \equiv \varepsilon_d, \tag{6}$$

where N is the total number of pixels of the original image. In matrix-vector formulation, we have

$$J_d(\hat{\mathbf{x}}) \equiv \sum_{i=1}^{3} \| \mathbf{A}_i (\mathbf{y}_i - \mathbf{x}_i) \|^2 \leq \varepsilon_d, \tag{7}$$

where $\mathbf{y}_i = (y_{1,i}, y_{2,i}, \cdots, y_{N,i})^t$, $\mathbf{x}_i = (x_{1,i}, x_{2,i}, \cdots, x_{N,i})^t$, and $\mathbf{A}_i = \begin{bmatrix} \omega_{1,i} & & & 0 \\ & \omega_{2,i} & & \\ & & \ddots & \\ 0 & & & \omega_{N,i} \end{bmatrix}$.

Typical images would generally have weak high frequency components as the intensity of neighboring pixels is highly correlated. This feature can be exploited as an additional *a priori* information in the restoration of color-quantized images. To incorporate this information into restoration, we assume that $\| \mathbf{H}\mathbf{x}_i \|^2$ for $i \in \{1, 2, 3\}$ is bounded by ε_{s_i}, where $\mathbf{H}$ is a linear highpass operator, which is generally a spatial 2D Laplacian filter, and ε_{s_i} is the upper energy bound of the high frequency components of $\mathbf{x}_i$. The smoothness constraint for the restored image can then be described by

$$J_s(\hat{\mathbf{x}}) \equiv \sum_{i=1}^{3} \| \mathbf{H}\mathbf{x}_i \|^2 / \varepsilon_{s_i} \leq 3 \tag{8}$$

3.2 Formulation of Restoration Algorithm

Based on the *a priori* information concerning the color quantization process and the original image, two constraint sets have been defined for the restoration of color-quantized images. By assuming that constraints (7) and (8) are equally important, the restored image can then be obtained by minimizing the objective function

$$J(\hat{\mathbf{x}}) = \sum_{i=1}^{3} \| \mathbf{A}_i (\mathbf{y}_i - \hat{\mathbf{x}}_i) \|^2 + \sum_{i=1}^{3} \alpha_i \| \mathbf{H}\hat{\mathbf{x}}_i \|^2 \tag{9}$$

where $\hat{\mathbf{x}}_i$ is the estimate of $\mathbf{x}_i$ and $\alpha_i = \varepsilon_d / 3\varepsilon_{s_i}$. The minimization of $J(\hat{\mathbf{x}})$ with respect to $\hat{\mathbf{x}}_i$ leads to the normal equation

$$\left(\mathbf{A}_i^t \mathbf{A}_i + \alpha_i \mathbf{H}^t \mathbf{H} \right) \hat{\mathbf{x}}_i = \mathbf{A}_i^t \mathbf{A}_i \mathbf{y}_i \tag{10}$$

In practice, an iterative technique is applied to successively approximate the solution. This leads to the following iteration for the restoration solution:

$$\hat{\mathbf{x}}_{i,(k+1)} = \hat{\mathbf{x}}_{i,(k)} + \beta \left\{ \mathbf{A}_i^t \mathbf{A}_i (\mathbf{y}_i - \hat{\mathbf{x}}_{i,(k)}) - \alpha_i \mathbf{H}^t \mathbf{H} \hat{\mathbf{x}}_{i,(k)} \right\}, \tag{11}$$

where k is the iteration index ($k = 0, 1, 2, \ldots$) and β is the relaxation parameter. A sufficient condition for the convergence of this iteration is $0 < \beta < 2/\lambda_{\max}$, where $\lambda_{\max}$ is the largest eigenvalue of matrix $(\mathbf{A}_i^t \mathbf{A}_i + \alpha_i \mathbf{H}^t \mathbf{H})$ [11]. It is common to set the initial estimate $\hat{\mathbf{x}}_{(0)}$ to be $\mathbf{y}$, although theoretically no restriction are imposed on the initial estimate.

Parameters α_i should be determined prior to performing the iteration. Recall that parameter α_i is defined as $(N\mathcal{K})/\varepsilon_{s_i}$, where ε_{s_i} is the bound of the smoothness constraint set defined in (8), N is the image size, and $\mathcal{K}$ is a constant to make (4) hold. In practice, since $\mathbf{x}$ is unavailable, we set ε_{s_i} to be $\mathcal{K}' \|\mathbf{H}\mathbf{y}_i\|^2$, where $\mathcal{K}'$ is a tuning parameter for image smoothness. As for $\mathcal{K}$, as we have mentioned previously, $\mathcal{K}$=10 is a reasonable estimate.

4 Simulation and Comparative Study

Simulation has been carried out to evaluate the performance of the proposed algorithm on a set of color-quantized images. In our simulation, a number of 24-bit full color test images were first color-quantized with a color palette of 256 colors. In our study, two color palettes were generated with median cut algorithm [12] and octree algorithm [13] respectively and used to investigate if the algorithm works with different color palettes. No halftoning is performed during the quantization. The test images applied are a set of *de facto* standard 24-bit full color images of size 512×512 each.

The proposed restoration algorithm was then applied to restore the quantized images. A set of 10000 uniformly distributed 3D random vectors were generated and used as the training set Ω_t to estimate $\vec{\sigma}_k$, and, $\mathcal{K}'$ was assigned to be 0.5 in the realization of the proposed algorithm. By considering that the extreme case happens when the whole color space is uniformly partitioned into slices of equal width and that colors are uniformly distributed in the color space, the lower bound of $\sigma^2_{k,i}$ is set to be $\frac{1}{12}(\frac{\nu_i}{N_c})^2$, where ν_i is the size of the i^{th} dimension of the color space and N_c is the total number of colors in the palette.

Some other restoration algorithms which were originally proposed for restoring noisy and blurred color images were also evaluated for comparison. They were simulated here for comparative study as few schemes had been proposed for restoring color-quantized images and they are typical examples of the type ([2]-[10]). In particular, Galatsanos's algorithm [6] is based on the constrained least square approach, which is the same as ours, and Hunt's algorithm [7] is based on Wiener filtering.

Table 1 shows the SNR improvement (SNRI) achieved by different algorithms. Mathematically, the SNRI is defined as

$$\text{SNRI} = 10\log\frac{\sum_{i=1}^{3}||\mathbf{x}_i-\mathbf{y}_i||^2}{\sum_{i=1}^{3}||\mathbf{x}_i-\hat{\mathbf{x}}_i||^2} \tag{12}$$

where $\mathbf{x}_i$, $\mathbf{y}_i$ and $\hat{\mathbf{x}}_i$ are the i^{th} color components of the pixels in the original, the color-quantized and the restored images, respectively.

In realizing Galatsanos's algorithm [6], the noise power of each channel was estimated with the original full-color image. In realizing Hunt's algorithm [7], three separate Wiener filters were used in three different channels and, during the design of the filters, the noise spectrum of each channel was estimated with the original full-color image. Note that, in practical situation, these parameters must be estimated from the degraded image and hence the restoration results would not be as good as those shown in the table.

From Table 1, one can see that the proposed restoration algorithm outperforms the other algorithms. This can be expected as the noise introduced by color quantization is basically signal dependent and is not white, which violates the assumptions adopted in most of the current multichannel restoration algorithms [2]-[10]. One can also see that the performance of the proposed algorithm is consistent even though the input is color-quantized with different color palettes. On average, by applying the proposed algorithm to restore the color-quantized images, an SNRI of 1.3 dB in image quality was achieved.

For visual evaluation of the restoration results, some color-quantized outputs of image “fruits” and the restoration results obtained with various restoration algorithms are provided in Figure 1 for comparison.

5 Conclusions

By far very little research has been carried out to address the restoration of color-quantized images. Besides, direct applications of existing restoration techniques are generally inadequate to deal with the problem. This paper has proposed a restoration algorithm specific to color-quantized images. This algorithm makes good use of the color palette to derive useful *a priori* information for restoration. It has been demonstrated by simulation results and comparative study that the proposed restoration algorithm is capable of improving the quality of a color-quantized image, as compared with other existing restoration approaches such as [6] and [7].

The proposed restoration algorithm requires no extra information other than the color palette to carry out the restoration. Besides, it is not tailor-made for a particular color quantization scheme and hence no *a priori* knowledge about the construction of the color palette is required during the restoration. This makes it be always able to provide a reasonable restoration performance whatever color quantization scheme with which it works.

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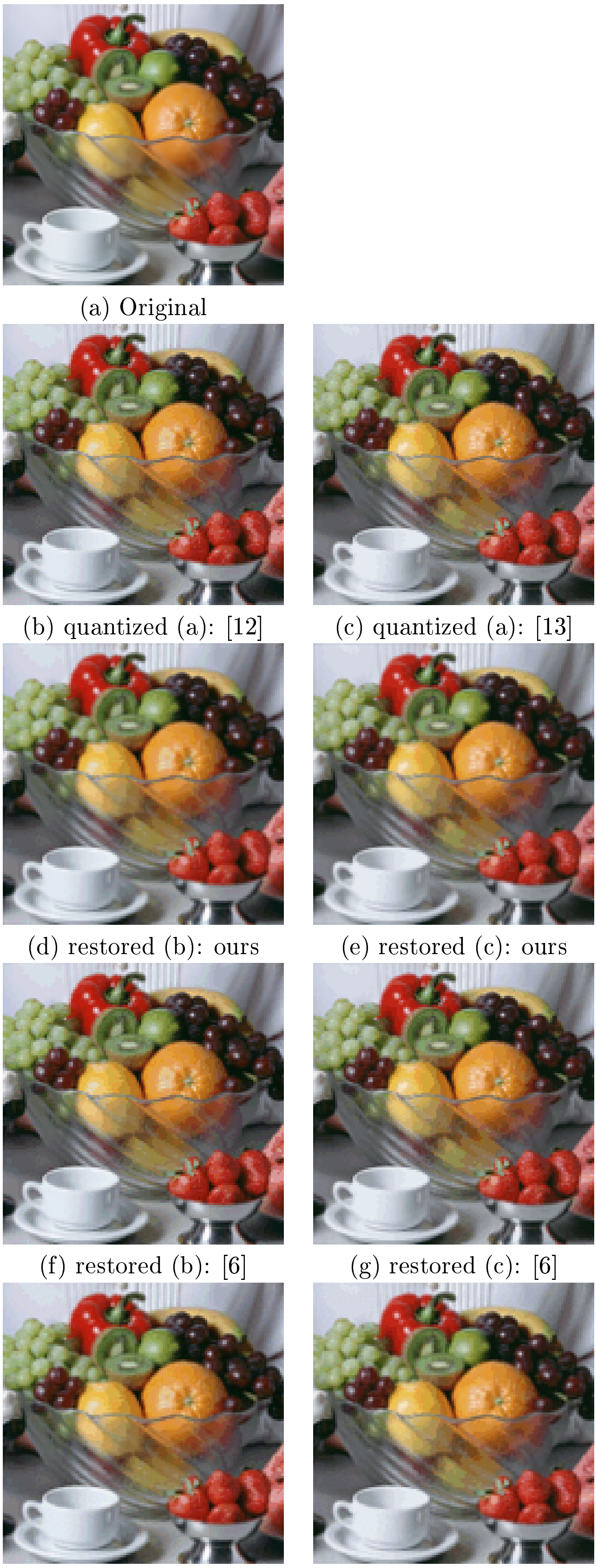

(a) Original

(b) quantized (a): [12]

(c) quantized (a): [13]

(d) restored (b): ours

(e) restored (c): ours

(f) restored (b): [6]

(g) restored (c): [6]

(h) restored (b): [7]

(i) restored (c): [7]

Figure 1: Comparison of the restoration performance of various approaches in restoring color-quantized "fruits"

	SNR Improvement (dB)		
	Proposed	[6]	[7]
Lenna	1.3529	0.3881	1.4077
Baboon	0.8630	0.2082	0.2381
Boat	1.3127	0.1985	0.3990
Airplane	0.6026	0.0345	0.9266
Peppers	2.2394	0.1881	1.5510
Average	1.2741	0.2035	0.9045

(a)

	SNR Improvement (dB)		
	Proposed	[6]	[7]
Lenna	1.3078	0.3493	0.8224
Baboon	1.0725	0.1592	0.6033
Boat	1.3326	0.2956	0.1443
Airplane	0.7949	0.1487	0.4570
Peppers	2.5221	0.3457	1.4042
Average	1.4060	0.2597	0.6862

(b)

Table 1: SNR Improvements of various algorithms in restoring images color-quantized with (a) median cut algorithm [12] and (b) octree algorithm [13]

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