PERFORMANCE COMPARISON OF THE FXLMS, NONLINEAR FXLMS AND LEAKY FXLMS ALGORITHMS IN NONLINEAR ACTIVE CONTROL APPLICATIONS

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ABSTRACT

This paper investigates the performance of the filtered-X LMS (FXLMS) and leaky filtered-X LMS (LFXLMS) algorithms in a nonlinear system. In addition, we derive the nonlinear filtered-X LMS algorithm (NLFXLMS), which is obtained by considering the nonlinear system function for the determination of the gradient of the cost function. We show that the fact of not using the exact gradient implies in a performance reduction of the adaptive algorithm. The adaptive algorithms are examined in the context of active noise control applications, which possess nonlinear components. Through numerical simulations we discuss the capabilities of the examined adaptive algorithms.

1. PROBLEM STATEMENT

The area of active control of sound or vibrations has received considerable attention in recent years. It has been benefited by the continuous improvement of digital signal processors. This fact permits that more sophisticated and/or efficient control algorithms can be implemented. A widely used algorithm in active control systems is the filtered-X LMS (FXLMS) algorithm. Such popularity is due to its simple implementation and robustness. This algorithm continuously modifies the adaptive filter weights, using the stochastic gradient method, which attempts to minimize the instantaneous quadratic error ($e^2(n)$). In active noise and vibration control applications, inherent nonlinearities are present in the system components (typically due to power amplifiers, sensors and actuators) as well as other originated by finite-precision effects from the algorithm implementation. A typical diagram illustrating an active noise control application is depicted in Fig. 1, where $d(n)$ and $e(n)$ represent the primary and error signals, respectively; $\mathbf{X}(n)$ is the reference signal; $z(n)$ is a Gaussian measurement noise, zero-mean with variance σ_z^2. The secondary path is composed of a nonlinear memoryless block denoted by $f[\cdot]$, which concentrates all the system nonlinearities, and a linear block modeled by a FIR filter given by $\mathbf{S} = [s_0\ s_1 \cdots s_{M-1}]$. The set of the linear and nonlinear blocks represents the secondary path.

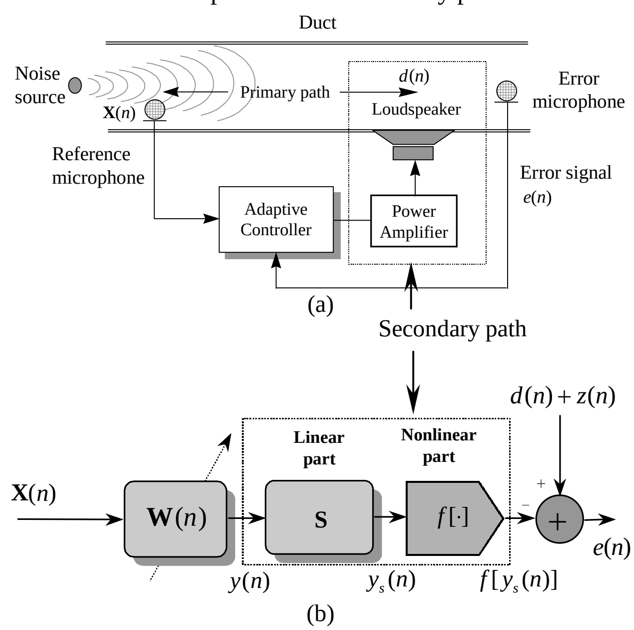


Fig. 1. Basic active noise control setup in a duct, including the system nonlinearities. (a) Physical setup, (b) block diagram.

When the LMS algorithm is implemented in this kind of system, it is common to study the effect of the nonlinearity on the adaptive algorithm performance [2,7]. The usual recursive expression to update the adaptive filter weights is the one derived for the standard FXLMS algorithm [1], given by [2]

$$\mathbf{W}(n+1) = \mathbf{W}(n) + \mu e(n)\mathbf{X}_f(n), \qquad (1)$$

with $\mathbf{X}_f(n)$ denoting the filtered reference vector, given by

$$\mathbf{X}_f(n) = \sum_{i=0}^{M-1} s_i \mathbf{X}(n-i), \qquad (2)$$

and the error signal obtained from Fig. 1(b), written as

$$e(n) = d(n) - f[y_s(n)] + z(n). \qquad (3)$$

The output of the linear block, $y_s(n)$, is determined by

$$y_s(n) = \sum_{i=0}^{M-1} s_i y(n-i) = \sum_{i=0}^{M-1} s_i \mathbf{X}^T(n-i)\mathbf{W}(n-i) . \quad (4)$$

Note that, through (4) into (3), the error signal is a nonlinear function of the adaptive weight vector. Since the LMS algorithm minimizes the instantaneous quadratic error given by $J(n) = e^2(n)$, when the gradient of $J(n)$ is computed, a different updating equation from (1) is obtained. By determining the gradient of the cost function using (3), we obtain

$$\nabla J_{\mathbf{W}}(n) = \frac{\partial e^2(n)}{\partial \mathbf{W}(n)} = 2e(n) f'[y_s(n)] \frac{\partial y_s(n)}{\partial \mathbf{W}(n)} , \quad (5)$$

where in the term $f'[y_s(n)]$, the prime represents differentiation of $f[\cdot]$ with respect to $y_s(n)$. In this way, as the adaptive filter weights are updated according to the gradient-descent algorithm, the weight updating expression reads

$$\mathbf{W}(n+1) = \mathbf{W}(n) - \frac{\mu}{2} \nabla J_{\mathbf{W}}(n) . \quad (6)$$

From (4), (3) and (5) into (6), we obtain

$$\mathbf{W}(n+1) = \mathbf{W}(n) + \mu e(n) f'[y_s(n)] \mathbf{X}_f(n) . \quad (7)$$

Note that in (1) the term $f'[y_s(n)]$ is disregarded. Consequently, the performance of the adaptive controllers will also be. The recursive expression (7) uses the exact gradient, i.e., the adaptive algorithm becomes a modified version of the traditional FXLMS algorithm, denoted as nonlinear FXLMS (NLFXLMS). Fig. 2 shows the resulting adaptive system scheme for both cases.

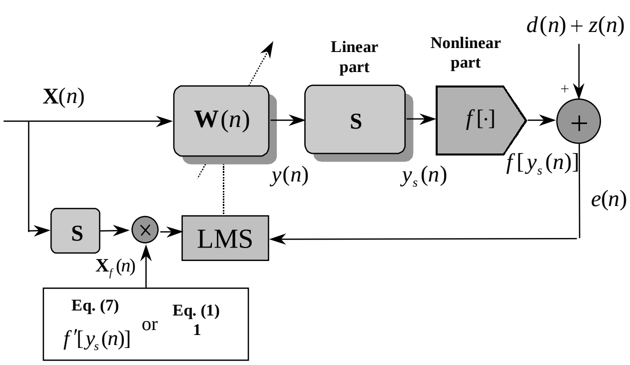


Fig. 2. Adaptive algorithm setup. Weight updating according to (1) or (7).

Another adaptive algorithm frequently used is the leaky FXLMS (LFXLMS). In this algorithm, a penalty term, proportional to the quadratic norm of the filter weights vector $\mathbf{W}(n)$, is added to the classical cost function, expressed as

$$J_L(n) = e^2(n) + \gamma \mathbf{W}^T(n)\mathbf{W}(n) , \quad (8)$$

where γ is a weighting factor denoted as leakage factor, being $\gamma \geq 0$. The introduction of a leakage factor in the LMS algorithm has been typically used for improving the algorithm performance regarding the following points: ill-conditioned input signal [3], algorithm stalling when the correction term is too small, overflow due to finite-precision arithmetic, to name a few. In active noise and vibration control applications introducing a leakage factor in the adaptive algorithm can attenuate the undesirable effects due to nonlinearities. In this case, the effect of the leakage factor is to control the level of the adaptive filter weights, hence, avoiding to drive the actuators into the nonlinear region [4]. In this work, we use the conventional weight update equation with leakage [5], given by

$$\mathbf{W}(n+1) = \nu \mathbf{W}(n) + \mu e(n) \mathbf{X}_f(n) , \quad (9)$$

where $\nu = 1 - \mu\gamma$, with $\nu < 1$. The purpose of not using the exact gradient, as derived in (5), is to show the usefulness of the LFXLMS algorithm in the presence of nonlinearities. In the section of simulation results, we will see that by comparing (7) and (9) similar results are obtained, regarding the performance of the adaptive controller. The advantages of the latter are evident, we do not need to estimate the nonlinearity of the actuator, and so the algorithm is less computationally expensive. However, the fact of not using the exact gradient makes the behavior noisier of the adaptive filter weights.

The present work is part of a project examining the effect of the system nonlinearities on the adaptive controller performance. In this paper, we present several numerical simulations, comparing the performance of the adaptive algorithms FXLMS, NLFXLMS, and LFXLMS, represented by the weight update expressions (1), (7), and (9), respectively. From this comparison, helpful implementation insights can be obtained.

2. PRACTICAL CONSIDERATIONS

2.1 Nonlinearity Model

The saturation curve is the model of the nonlinear function, which represents a large class of nonlinear systems. This kind of curve can be modeled by the hyperbolic tangent function $f(r) = \tanh(r)$. Alternatively, this class of nonlinearity can also be modeled by the error function, expressed as

$$f(r) = \int_0^r e^{-\frac{t^2}{2Fnl^2}} dt \quad (10)$$

and

$$f'(r) = e^{-\frac{r^2}{2Fnl^2}} \quad (11)$$

with Fnl being a constant factor, which controls the nonlinearity degree of $f(r)$; $f'(r)$ is the derivative of $f(r)$ with respect to *r*. The error function is a particularly attractive model for the system nonlinearity because it permits a relatively tractable mathematics. This function has been extensively studied and for further details, the reader is referred to [6]. As Fnl approaches zero the higher the nonlinearity degree of $f(r)$ is. In the limit, for $Fnl \to 0$ we obtain a hard limiter characteristic.

2.2 Implementation of Eq. (7)

The exact gradient in (7) gives rise to an increase in the computational cost of the weight update equation. However, it can be greatly reduced by implementing the computation of the $f'(r)$ term through a look-up table. In this case, there is a tradeoff between accuracy and complexity, which depends on the number of elements of the table. In considering an active noise control (ANC) application [4], the nonlinearities occur in the power amplifier and/or in the canceling loudspeaker. However, due to the state-of-the-art of electronic systems, it is very common to find power amplifiers having very low distortion rates. Thus, in ANC systems the nonlinearity can be mainly attributed to the loudspeaker.

Also, as is usual in active control applications, the transfer functions of **S** and $f[\cdot]$ are previously unknown, thus, they need to be estimated according to the methods presented, for instance, in [8].

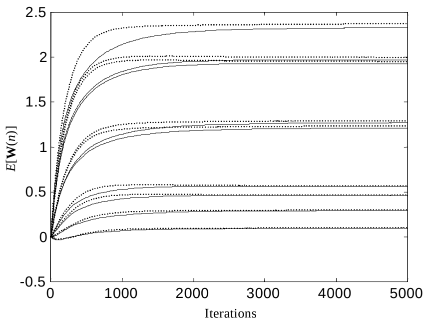
3. SIMULATION RESULTS AND DISCUSSIONS

In this section, the performance of the adaptive algorithms represented by expressions (1), (7), and (9) are compared. For this purpose, a simple active noise control system containing a primary noise source, a canceling loudspeaker and an error microphone has been considered. The primary signal $d(n)$ is obtained by passing the reference signal through a low-pass FIR filter representing the primary path, given by [0.0179, 0.1005, 0.279, 0.489, 0.586, 0.489, 0.279, 0.1005, 0.0179]. The variance of the measurement noise $z(n)$ is 0.0001. The comparison is carrying out by using several nonlinearity degrees. The mean squared error (MSE) curves are obtained by the expression $MSE = 10\log_{10}\{E[e^2(n)]/E[d^2(n)]\}$. All the Monte Carlo (MC) simulations were obtained from an average of 500 independent runs.

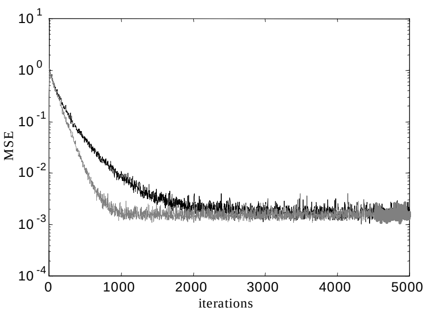
Example 1: For this example the experimental conditions are the following: the reference signal is white, zero mean and with unit variance; the step size is given by $\mu = 0.02$; the impulse response of the secondary path, **S**, is [0.7756, 0.5171,-0.362]. The value of Fnl in (10) is 4. This represents a weak degree of nonlinearity. In Fig. 3, we show the MC simulation results. In Fig. 3(a) and 3(b), we can see that an almost identical behavior of the adaptive algorithms (1) and (7) has been obtained. The only difference is in the speed of convergence. The minimum MSE values (Fig. 3(b)) are 0.0017 and 0.0016 for the FXLMS and NLFXLMS algorithms, respectively. The faster convergence is explained by examining the term $f'[y_s(n)]$ in (7). Its histogram is depicted in Fig. 4(c) (solid curve). It can be seen from that figure, that the histogram is concentrated around a value less than one, resulting in a slower speed of convergence. Due to the nonlinearity degree used in this example, the LFXLMS algorithm does not present any significant differences regarding the other two algorithms. Thus, for the sake of clarity the corresponding curves are not presented.

Example 2: In this example we use the same conditions as in the previous one, but the value of Fnl is 1, which represents a higher degree of system nonlinearity. In this numerical simulation, the limiting effect of the leakage factor on the adaptive filter weights can be observed. The MC simulation results for the adaptive algorithms are shown in Fig. 4. As can be seen from Fig. 4(a), when the standard FXLMS is used, the adaptive filter converges to inadequate values. The convergence is reached after 10000 iterations, while for the NLFXLMS and LFXLMS, it is achieved after 1000 iterations. The minimum MSE values (Fig. 4(b)) are 0.420, 0.095 and 0.114 for the FXLMS, NLFXLMS and LFXLMS algorithms, respectively. The LFXLMS algorithm, with $\gamma = 0.1$, presents nearly the same performance as the NLFXLMS algorithm. The advantages of the former over the latter, which uses the exact gradient, is that we do not need to estimate the nonlinearity model for implementing its derivative. In addition, in certain applications to obtain such an estimate is not always possible. For that reason, the LFXLMS algorithm represents an interesting alternative to implement in the adaptive controller. Regarding the LFXLMS, we also performed numerical simulations with $\gamma = 0.05$ and $\gamma = 0.2$, resulting in the following minimum MSE values, 0.152 and 0.137, respectively. This fact indicates that there exists an optimum value for γ, with which we can reach the same minimum MSE value obtained by the NLFXLMS. The fact of the LFXLMS algorithm is not using the exact gradient results in a noisier behavior for the controller weights. This can be seen when a single realization is performed. Figure 4(c) shows the histogram for the term $f'[y_s(n)]$ (dotted curve). Comparing with the previous example, we can see that due to the higher nonlinearity degree used in this case, the histogram presents a spread range of values.

Extensive numerical simulations for several primary and secondary paths and for white and colored input signals have been carried out. In all cases, similar conclusions to previous ones are verified.

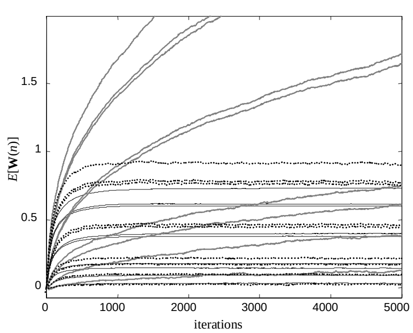


(a)

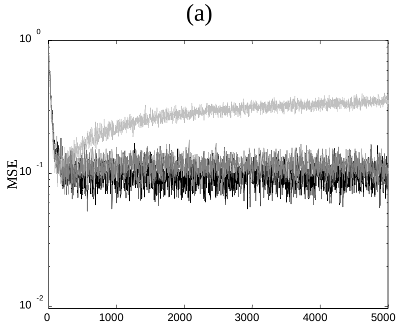


(b)

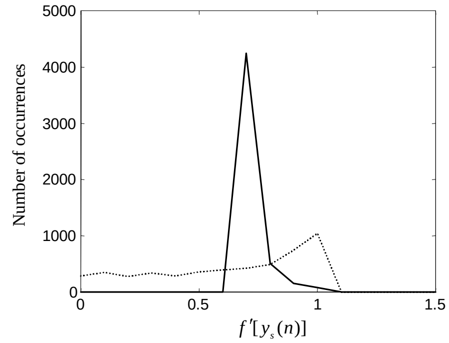
Fig. 3. MC simulation comparing the FXLMS and NLFXLMS algorithms (average of 500 runs). (a) $E[\mathbf{W}(n)]$, dotted lines: Eq. (1); solid lines: Eq. (7). (b) MSE, gray curve: FXLMS; black curve: NLFXLMS.



(a)



(b)



(c)

Fig. 4. MC simulations comparing the FXLMS, LFXLMS, and NLFXLMS algorithms (average of 500 runs). (a) $E[\mathbf{W}(n)]$, dotted lines: Eq. (1); gray lines: Eq. (9); solid lines: Eq. (7). (b) MSE, light gray curve: FXLMS; dark gray curve: LFXLMS; black curve: NLFXLMS. (c) Histogram of $f'[y_s(n)]$, solid curve: Example 1, dotted curve: Example 2.

4. CONCLUSIONS

The nonlinear filtered-X LMS (NLFXLMS) algorithm has been derived, which in a nonlinear active control (noise or vibration) application outperforms the implementation of the standard FXLMS algorithm. However, by introducing a leakage factor in the FXLMS, we can attain similar performance as with the NLFXLMS without requiring the estimation of the nonlinear function.

5. REFERENCES

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