JOINT ANGLE AND DELAY ESTIMATION USING UNIFORM CIRCULAR ARRAY RECEIVER

Emmanuèle Grosicki[1], *Karim Abed-Meraim*[1], *Wrya Muhammad*[2]
[1] Dept. of Signal and Image Processing, Ecole Nationale Supérieure de Télécommunication de Paris, 46 rue Barrault, 75634 Paris Cedex 13, France
[2] Dept. of signal and image processing, Institut National des Télécommunications,
e-mail: grosicki,abed@tsi.enst.fr, wrya.muhammad@int-evry.fr

ABSTRACT

In this paper, a new method for the estimation of the direction-of-arrival (DOA) and relative propagation delay in the context of multipath rays is described. The proposed algorithm processes the received signals at a uniform circular array and uses ESPRIT-like approach for the estimation of the relevant parameters. We show, in particular that in the context of circular arrays, this algorithm increases drastically the accuracy of the estimated parameters with only a small number of sensors and limited processing complexity. These results encourage the use of such configurations in future wireless networks.

1 INTRODUCTION

In wireless communications, such as radar and sonar, emitted signals are received via multiple rays. Accurate estimation of the directions of arrival and propagation delays are therefore extremely important in order to retrieve the location of the transmitter. In particular, in mobile communications, the use of multiple antennas has been acknowledged to be an effective way to enhance the accuracy of the mobile location [1]. However, the geometrical configuration as well as the number of such antennas is still an open topic. Hence, in [2], a joint estimation algorithm of angles and delays parameters within the framework of linear antenna arrays is proposed. The latter is based on a two-dimensional (2-D) ESPRIT (Estimation of Signal Parameters via Rotational Invariance Techniques) like shift-invariance technique. In this paper, we generalize this work in the context of uniform circular array receiver. Indeed, previous studies [3] have pointed out the considerable enhancements provided by the use of circular arrays with respect to linear ones. In particular, we extend and improve the array transform technique used in [4] for 2-D angle (elevation and azimuth) estimation. The extended transform allows us to use antenna arrays with a relatively small number of sensors (contrary to [4] where a large amount of antenna arrays is needed) for recovering with precise accuracy the location parameters. The system and channel model are described in section 2 followed by a presentation of the algorithm in section 3. Section 4 is devoted to simulation results which illustrate our claims.

2 CHANNEL MODEL

In the following, upper (lower) boldface symbols will be used for matrices (column vectors) whereas lower symbols will represent scalar values, $(.)^T$ will denote transpose operator, $(.)^\star$ conjugation, $(.)^H = \left((.)^T\right)^\star$ hermitian transpose and # the Moore-Penrose pseudo inverse operator.

Consider a single source observed by a uniform circular array of N sensors. The digital sequence s_l is transmitted over a multipath channel. The received signal by the k-th sensor, in presence of additive noise, is expressed as:

$$x_k(t) = \sum_{l \in \mathbb{Z}} s_l h_k(t - lT) + w(t), \quad 1 \le k \le N \qquad (1)$$

where T is the symbol duration, $w(t)$ is an additive white Gaussian noise process and $h(t)$ is the impulse response of the channel.

For the array of N sensors, the $(N \times 1)$ impulse response model vector is given by [2]:

$$\mathbf{h}(t) = \begin{bmatrix} h_1(t) \\ \vdots \\ h_N(t) \end{bmatrix} = \sum_{i=1}^{d} \mathbf{a}(\theta_i)\beta_i g(t - \tau_i) \qquad (2)$$

where d is the number of distinct propagation paths, $g(t)$ is the known modulation pulse shape (square-root raised cosine). Each path is characterized by its delay τ_i, its complex attenuation β_i and its direction of arrival θ_i.

$\mathbf{a}(\theta_i)$ is the steering vector of the array expressing its complex response to a planar wavefront arriving from direction θ_i. The antenna considered here, is composed of sensors, assumed to be identical and omnidirectional, uniformly distributed over the circumference of a circle of radius r. The angle between sensor i and sensor 1 is noted $\gamma_i = \frac{2(i-1)\pi}{N}$. Let the array center be the phase reference point. The array response vector $\mathbf{a}(\theta_i)$ is then given by:

$$\mathbf{a}(\theta_i) = \begin{bmatrix} e^{j\xi cos(\theta_i - \gamma_1)} \\ \vdots \\ e^{j\xi cos(\theta_i - \gamma_N)} \end{bmatrix}$$

where $\xi = \frac{2\pi r}{\lambda}$ and λ is the wavelength. Denote $\mathbf{H}$ the matrix containing the impulse response samples collected at each sensor. We denote by L the channel length and P the oversampling factor. The $(N \times LP)$ dimensional matrix $\mathbf{H}$ can be written as [2]:

$$\begin{aligned} \mathbf{H} &\triangleq \begin{bmatrix} \mathbf{h}(0) & \mathbf{h}(\frac{T}{P}) & \dots & \mathbf{h}(LT - \frac{T}{P}) \end{bmatrix} \\ &= [\mathbf{a}(\theta_1) \dots \mathbf{a}(\theta_d)] \begin{bmatrix} \beta_1 & & 0 \\ & \ddots & \\ 0 & & \beta_d \end{bmatrix} \begin{bmatrix} \mathbf{g}_{\tau_1} \\ \vdots \\ \mathbf{g}_{\tau_d} \end{bmatrix} \\ &= \mathbf{A}(\theta)\mathbf{B}\mathbf{G}(\tau) \end{aligned}$$

where $\mathbf{g}_{\tau_i}$ is a $LP-$dimensional row vector containing the samples of $g(t - \tau_i)$

$$\mathbf{g}_{\tau_i} \triangleq \begin{bmatrix} g(-\tau_i) & g(\frac{T}{P} - \tau_i) & \dots & g(LT - \frac{T}{P} - \tau_i) \end{bmatrix}$$

We assume here that the channel matrix $\mathbf{H}$ has been estimated using either a training sequence or an existing blind identification method [5]. Our objective in this paper is propose a simple yet robust algorithm to extract the information of angle and delay parameters from the channel transfer function.

3 PROPOSED ALGORITHM

We introduce here our angle and delay estimation algorithm that proceeds in two steps:

- First, as in [2], a Fourier transform of the channel matrix $\mathbf{H}$ is performed and propagation delays are estimated using a standard ESPRIT algorithm.
- In the second step, we focus on the phase mode excitations of the circular array. The new procedure estimates the direction of arrival of the multipath rays with a relatively small number of sensors (in comparison with what is required in [4]).

3.1 Delay estimation

As in [2], a Fourier transform is used in order to estimate the delays since it converts them to a certain phase progression. Given the channel model in (2), the Fourier coefficient $\mathbf{h}_F(f) \triangleq TF(\mathbf{h}(t))$ can be written as:

$$\mathbf{h}_F(f) = \sum_{i=1}^{d} \mathbf{a}(\theta_i)\beta_i g_F(f) e^{-j2\pi\tau_i f}$$

where $g_F(f)$ is the Fourier transform of $g(t)$. In matrix form, this yields to

$$\mathbf{H}_F = \mathbf{A}(\theta)\mathbf{B}\mathbf{V}(\tau)\text{diag}(\mathbf{g}_F)$$

Where $\mathbf{H}_F$ is the Fourier transform channel matrix, i.e., $\mathbf{H}_F \triangleq \mathbf{HF}$ with $\mathbf{F}$ is the $(LP \times LP)$ Fourier transform matrix [2]. $\mathbf{g}_F$ represents the vector of Fourier transform coefficients of the pulse shape filter $g(t)$ and $\mathbf{V}(\tau)$ is the Vandermonde matrix given by:

$$\mathbf{V}(\tau) = \begin{bmatrix} 1 & \chi_1 & {\chi_1}^2 & \dots & {\chi_1}^{LP-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & \chi_d & {\chi_d}^2 & \dots & {\chi_d}^{LP-1} \end{bmatrix}$$

where $\chi_i = e^{\frac{-j2\pi\tau_i}{L}}, 1 \leq i \leq d$.
The structure of the matrix $\mathbf{H}_F$ has the shift-invariance properties which allows the estimation of τ_i by the ESPRIT algorithm. More precisely, delays parameters are estimated according to the following steps:

- Construct the following matrix:

$$\begin{aligned} \mathcal{H} &\triangleq \begin{bmatrix} \tilde{\mathbf{H}}_F^{(1)} \\ \tilde{\mathbf{H}}_F^{(2)} \end{bmatrix} \\ &= \mathcal{A}\mathbf{B}\mathcal{F} \end{aligned}$$

  where $\tilde{\mathbf{H}}_F = \mathbf{H}_F\text{diag}(\mathbf{g}_F)^{-1}$, $\tilde{\mathbf{H}}_F^{(i)}$ is the left shifted submatrix of $\tilde{\mathbf{H}}_F$ containing columns $i, i+1, \dots, LP+i-2$ of $\tilde{\mathbf{H}}_F$, $\mathcal{F}$ is the submatrix of $\mathbf{F}(\tau)$ containing columns $1, 2, \dots, LP-1$ and :

$$\mathcal{A} = \begin{bmatrix} \mathbf{A}(\theta) \\ \mathbf{A}(\theta)\phi(\tau) \end{bmatrix}$$

  with $\phi(\tau) \triangleq \text{diag}(e^{\frac{-j2\pi\tau_1}{L}}, \dots, e^{\frac{-j2\pi\tau_d}{L}})$

- Compute $\mathbf{U} = [\mathbf{u}_1, \dots, \mathbf{u}_d]$ the d principal left singular eigenvectors of $\mathcal{H}$.
- Estimate $\phi(\tau)$ as the diagonal matrix of the eigenvalues of $\mathbf{U}_1^{\#}\mathbf{U}_2$ where:

$$\mathbf{U} \triangleq \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{bmatrix} \begin{matrix} \}N \\ \}N \end{matrix}$$

- Estimate the delays τ_i as $\tau_i = \frac{-L\,angle(\phi_{ii})}{2\pi}$ where ϕ_{ii} is the i_{th} diagonal entry of $\phi(\tau)$ and angle(x) represents the phase of x.

3.2 DOA estimation

Once the estimation of the delays is carried out, it is possible to limit the study to matrix $\mathbf{H}_1$ obtained by right multiplying $\tilde{\mathbf{H}}_F$ by $\mathbf{V}(\tau)^{\#}$.

$$\begin{aligned} \mathbf{H}_1 &= \tilde{\mathbf{H}}_F\mathbf{V}(\tau)^{\#} \\ &= \mathbf{A}(\theta)\mathbf{B} \end{aligned} \quad (3)$$

Let us consider the excitation function $w_m(\gamma) = e^{jm\gamma}$, $\gamma \in [0, 2\pi]$ which represents the m_{th} spatial harmonic. It excites a continuous circular aperture with phase mode

m.Therefore, left multiplying $\mathbf{H}_1$ by the following normalized row vector

$$\mathbf{w}_m = \frac{1}{N}\left[\begin{array}{cccc} e^{jm\gamma_1} & e^{jm\gamma_2} & \dots & e^{jm\gamma_N} \end{array} \right] \quad (4)$$

is equivalent to excite the array sensors with the m_{th} phase mode. In [4], the expression of the resulting array excitation is derived:

$$\begin{aligned} E_m(\theta_i) &= \mathbf{w}_m \mathbf{a}(\theta_i) = \frac{1}{N}\sum_{n=1}^{N} e^{jm\gamma_n} e^{j\xi cos(\theta_i-\gamma_n)} \\ &= j^m J_m(\xi) e^{jm\theta_i} + \sum_{k=1}^{\infty}(j^g J_g(\xi) e^{-jg\theta_i} \\ &+ j^h J_h(\xi) e^{jh\theta_i}) \end{aligned} \quad (5)$$

Where $g = Nk - m$, $h = Nk + m$ and $J_m(\xi)$ represents the Bessel function of order m. It is possible to keep only the first term in (5) and to discard the other terms appearing in the sum by choosing a sufficiently large number of sensors. Indeed, the table below shows that the values of the Bessel function $J_m(\xi)$, with r the radius of the circular array has been taken equal to $\frac{\lambda}{2}$, decrease very quickly.

m	0	1	2	3	4
$J_m(\pi)$	-0.3	0.28	0.49	0.33	0.15
m	5	6	7	8	9
$J_m(\pi)$	0.05	0.01	0.003	0.0007	0.0001

One can see that for modes m higher than $M = 6$, the terms $j^m J_m(\xi) e^{jm\theta}$ can be neglected in (5). M is the maximum excitation mode considered. The condition to have the first term to be the predominant one in (5) is:

$$2M \leq N$$

This condition might be quite constraining in some applications. To relax it, we propose to take into account the two first terms appearing in the sum in (5):
– For $m \geq 0$, $E_m(\theta_i)$ is approximately given by:

$$E_m(\theta_i) \approx j^m J_m(\xi) e^{jm\theta_i} + j^{N-m} J_{N-m}(\xi) e^{-j(N-m)\theta_i} \quad (6)$$

– For $m < 0$, we have similarly:

$$E_m(\theta_i) \approx j^m J_m(\xi) e^{jm\theta_i} + j^{N+m} J_{N+m}(\xi) e^{j(N+m)\theta_i} \quad (7)$$

Therefore, only terms $J_p(\xi)$ where $p \geq N$ are neglected, and the condition on the number of sensors becomes:

$$M \leq N$$

Moreover, by using the following property $J_{-m}(\xi) = (-1)^m J_m(\xi)$, it is possible to obtain a matrix where only the first predominant term of (5) appears. Indeed, by taking an odd number of sensors N, the following result is obtained:

$$\begin{aligned} E_{2p}(\theta_i) + E^*_{-2p}(\theta_i) &= 2j^{2p} J_{2p}(\xi) e^{j(2p)\theta_i} \\ E_{2p+1}(\theta_i) - E^*_{-(2p+1)}(\theta_i) &= 2j^{2p+1} J_{2p+1}(\xi) e^{j(2p+1)\theta_i} \end{aligned}$$

In matrix form, this can be written as:

$$\bar{\mathbf{A}}(\theta) = \mathbf{J}_1 \mathbf{A}(\theta) + \mathbf{J}_2 \mathbf{A}(\theta)^* \quad (8)$$

where $\mathbf{J}_1$ and $\mathbf{J}_2$ are known matrices defined by:

$$\mathbf{J}_1 = \left[\begin{array}{c} \mathbf{w}_0 \\ \vdots \\ \mathbf{w}_M \end{array} \right], \quad \mathbf{J}_2 = \left[\begin{array}{c} (-1)^0 \mathbf{w}_0{}^* \\ \vdots \\ (-1)^M \mathbf{w}_{-M}{}^* \end{array} \right]$$

According to (6) and (7), $\bar{\mathbf{A}}(\theta)$ has the following structure:

$$\bar{\mathbf{A}}(\theta) \triangleq \mathbf{D}[\bar{\mathbf{a}}(\theta_1), \dots, \bar{\mathbf{a}}(\theta_d)]$$

with $\mathbf{D} = \mathrm{diag}(2J_0(\xi), \dots, 2j^M J_M(\xi))$
and

$$\bar{\mathbf{a}}(\theta_i) = \left[\begin{array}{cccc} 1 & e^{j\theta_i} & \dots & e^{jM\theta_i} \end{array} \right]^T$$

To apply the matrix transform in (8), we need to access directly to matrix $\mathbf{A}(\theta)$ (or, because of the complex conjugate operation to $\mathbf{A}(\theta)$ times a real-valued matrix). Up to now, we only have the estimate of $\mathbf{H}_1 = \mathbf{A}(\theta)\mathbf{B}$ where the complex attenuation matrix $\mathbf{B}$ appears. To get rid of the phases of the complex attenuations (β_i) (diagonal entries of $\mathbf{B}$), we use the fact that for mode $m = 0$. $E_0(\theta) = J_0(\xi)$ is real valued. Therefore, matrix $\mathbf{H}_1$ is transformed according to:

$$\begin{aligned} \mathbf{H}_2 &\triangleq \mathbf{J}_1 \mathbf{H}_1 \mathbf{P} + \mathbf{J}_2 (\mathbf{H}_1^* \mathbf{P}^*) \\ &= \mathbf{D} \bar{\mathbf{A}}(\theta) \bar{\mathbf{B}} \end{aligned}$$

where $\mathbf{P} = \mathrm{sign}(J_0(\xi))\mathrm{diag}(e^{-j\mathrm{phase}(\mathbf{w}_0 \mathbf{H}_1)})$ with

$$\mathrm{sign(x)} = \begin{cases} 1 & \text{if x} \geq 0 \\ -1 & \text{if x} < 0 \end{cases}$$

and $\bar{\mathbf{B}} = \mathrm{diag}(|\beta_1|, \dots, |\beta_d|)$.
Let us define $\mathbf{H}_3$ as:

$$\begin{aligned} \mathbf{H}_3 &\triangleq \mathbf{D}^{-1} \mathbf{H}_2 \\ &= \bar{\mathbf{A}}(\theta) \bar{\mathbf{B}}. \end{aligned}$$

The DOA are finally estimated according to:

$$\theta_i = \mathrm{angle}(\mathbf{h}_{3,i}^{(1)H} \mathbf{h}_{3,i}^{(2)})$$

where $\mathbf{h}_{3,i}$ denotes the i_{th} column vector of $\mathbf{H}_3$, $\mathbf{h}_{3,i}^{(1)}$ (resp. $\mathbf{h}_{3,i}^{(2)}$) is the sub-vector of $\mathbf{h}_{3,i}$ where its last entry (resp. its first entry) is removed. Note that, this estimation method provides an automatic pairing of the angle and delay parameters, i.e. the i_{th} column of $\mathbf{H}_3$ (i_{th} angle) is associated with the i_{th} row of the estimated matrix $\mathbf{V}(\tau)$ (i_{th} delay). In fact, if the rows of $\mathbf{V}(\tau)$ are permuted, then the columns of $\mathbf{H}_3$ are permuted in the same way according to equation (3).

4 SIMULATION RESULTS

In this section, we provide some simulation results to illustrate the performance of our estimation algorithm and assess the robustness of the method to channel estimation errors. A uniform circular array consisting of $N = 7$ sensors, with a circumferential spacing between two adjacent sensors of 0.45λ (the array radius r is equal to $\frac{\lambda}{2}$), is considered. N has been chosen specifically such as $J_N(\pi)$ is negligible compared to the $0th$ mode. We assume that a single source is present and each sensor receives the contribution of two rays characterized by their angle of arrival [-10, 20] degrees, their delays [0, 1/6] T and their constant fading coefficient [1, 0.8] with a randomly phase. The estimation error on the channel coefficients is modeled by a Gaussian additive noise of variance σ^2 on each sample. The SNR is defined as:

$$SNR = \frac{E(Trace(\mathbf{H}\mathbf{H}^H))}{\sigma^2(NLP)}$$

$N_r = 100$ independent Monte-Carlo runs have been performed. The performance is measured by the estimation of the Mean Square Error (MSE) defined by

$$MSE = \frac{1}{N_r} \sum_{r=1}^{N_r} \|\hat{x}_r - x\|^2$$

where $\hat{x}_r$ denotes the estimated parameter. In our case, x is either the DOA parameter θ_i, $i = 1, 2$ or the delay parameter τ_i, $i = 1, 2$. In figure 1, the normalized MSE of the estimated angles (i.e. the MSE divided by the square norm of the considered parameters) is plotted versus the SNR in dB. In figure 2, we plot the MSE of the estimated delays versus the SNR in dB (The symbol period T is normalized to one). We can observe on this example, the good performance of the proposed method of estimation for moderate and high SNR's with only 7 sensors. Further simulation examples and detailed performance analysis will be provided in future work.

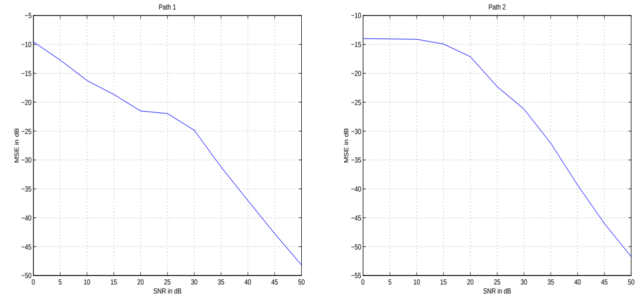

Figure 1: MSE of the estimated angles versus the SNR in dB

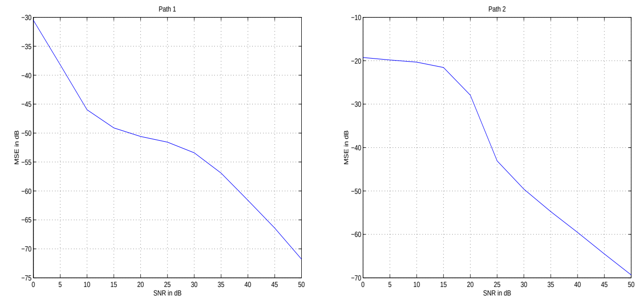

Figure 2: MSE of the estimated delays versus the SNR in dB

References

[1] J.C. Liberti,JR and T.S. Rappaport, "Smart Antennas for Wireless Communications", ch 9, ed.Prentice Hall 1999.

[2] A-J. van der Veen, M. C. Vanderveen and A. Paulraj, "Joint Angle and Delay Estimation Using Shift-Invariance Techniques," *IEEE Trans. Signal Processing* , vol. 46, no. 2, February 1998.

[3] Y. Hua, T.K. Sarkar and D.D. Weiner, "An L-Shaped Array for Estimating 2-D Directions of Wave Arrival", *IEEE Trans. Antennas and Propagation*, vol. 39, no. 2, February 1991.

[4] C. P. Mathews and M. D. Zoltowski, "Eigenstructure Techniques for 2-D Angle Estimation with Uniform Circular Arrays," *IEEE Trans. Signal Processing*, pp. 2395–2407, vol 42, no. 9, 1994.

[5] k. Abed-Meraim, W. Qiu and Y. Hua, "Blind System Identification", Proceedings of the IEEE, vol. 85, no. 8, August 1997.