MULTI-SPLIT EQUALIZERS FOR HDSL CHANNELS

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ABSTRACT

The application of multi-split equalizers is proposed as a means of diminishing the training sequence length. Reductions in the order of four to six times are presented for practical HDSL channels. The new structure is suitable for numerous applications, for instance, as the front end of any joint equalization and decoding scheme that makes use of a DFE in an iterative way.

I. INTRODUCTION

Digital communications through band-limited linear filter channels are affected by both additive noise and channel distortion [1]. Channel distortion leads to intersymbol interference (ISI), which can be compensated by means of an equalizer. When channel distortion is not known a priori, the equalizer must be of an adaptive nature. If adaptive, then the equalizer relies on a training sequence or on a self-learning process. In this paper we are concerned with equalizers adapted via a training sequence in a noiseless environment. Throughout this paper, the equalizer will be considered part of the digital communication system depicted in Figure 1.

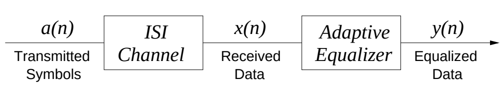

Fig. 1. Communication system with ISI

Our objective is to present a new equalizer structure that is able to achieve the same performance as the traditional one, but with a shorter training sequence. In other words, we intend to improve the overall system throughput. To do so, we make use of a novel filtering technique, the multi-split adaptive filtering [2]. The multi-split operation can decrease the eigenvalue spread of the autocorrelation matrix of the channel output. This eigenvalue spread is a limiting factor in the performance of the least mean square (LMS) algorithm [3], which is used to update the equalizer. Reducing such a spread improves the convergence rate of the equalizer parameters, and thus a shorter training sequence can be used.

The digital communication channels to be considered here are from high-bit-rate digital subscriber line (HDSL) systems. With the development of rate-adaptive lines, HDSL now sees a broad new market, as campus LAN's using a single copper pair [4]. In the U.S., for instance, 70% of all loops contain nonloaded twisted pairs up to 18,000 feet, thus qualifying to take advantage of such rate-adaptive lines [5]. It is also worth to say that there are about 700 million copper pairs in telephony around the world [4]. And it is much less expensive to optimize the use of such a wire network than to provide every home with a fiber optic connection. The results presented here are directly applicable to the development of more efficient HDSL protocols.

This paper is organized as follows. Section II presents the derivation of the multi-split equalizers, based on [2] and [6]. In Section III we present the results of the application of multi-split equalizers to HDSL practical channels. Finally, in Section IV we draw our conclusions.

II. MULTI-SPLIT EQUALIZER

Let us consider the classical scheme of an adaptive transversal filter as shown in Figure 2, where $W(n)$ is a vector of N–by–1 coefficients, and $N = 2^M$. If we split up this filter into its symmetric and anti-symmetric parts then:

$$W(n) = W_s(n) + W_a(n) \tag{1}$$

where $W_s(n) = \frac{1}{2}[W(n) + JW(n)]$, $W_a(n) = \frac{1}{2}[W(n) - JW(n)]$, and J is the reflection matrix.

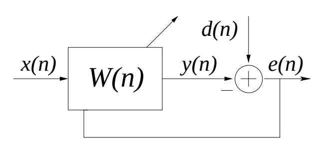

Fig. 2. Traditional adaptive transversal filter.

The symmetry and anti-symmetry conditions of $W_s(n)$ and $W_a(n)$ can be easily introduced through a linearly constrained approach [6]. It consists of making:

$$C_s = \begin{bmatrix} I_K \\ -J_K \end{bmatrix}, \quad C_a = \begin{bmatrix} I_K \\ J_K \end{bmatrix}, \tag{2}$$

$$F_s = F_a = 0_K$$

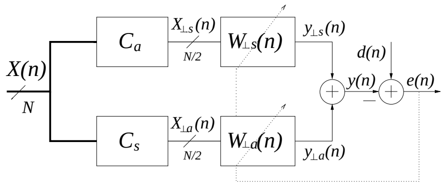

Fig. 3. GSC implementation of the split filter.

for N even and $K = \frac{N}{2}$, and of imposing:

$$C_s^t W_s(n) = F_s \quad \text{and} \quad C_a^t W_a(n) = F_a \tag{3}$$

in a constrained optimization process of the mean-square value of the estimation error $e(n)$, which is defined as the difference between the desired response $d(n)$ and the filter output $y(n)$.

Now, using the Generalized Sidelobe Canceller (GSC) structure with the symmetry and anti-symmetry constraints, the split filtering scheme can be represented in the form of Figure 3 [6].

As far as the adaptation process is concerned, the LMS algorithm can be applied independently in each branch in a normalized fashion. Thus, the algorithm for the symmetric filter can be defined as:

$$W_{\perp s}(n) = W_{\perp s}(n-1) + \frac{\mu}{r_s(n)} X_{\perp s}(n) e(n) \tag{4}$$

and for the anti-symmetric filter as:

$$W_{\perp a}(n) = W_{\perp a}(n-1) + \frac{\mu}{r_a(n)} X_{\perp a}(n) e(n) \tag{5}$$

where:

$$r_i(n) = \gamma r_i(n-1) + \frac{1}{n}(|X_{\perp i}(n)|^2 - \gamma r_i(n-1)), \tag{6}$$

for $i = (a, s)$, γ is the forgetting factor and μ is the adaptation step-size.

Now, if each branch in Figure 3 is considered separately, the transversal filters $W_{\perp s}$ and $W_{\perp a}$ can also be split into their symmetric and antisymmetric parts. By proceeding continuously with this process and also splitting the resulting filters, we arrive, after M steps with 2^{m-1} splitting operations $(m = 1, 2, \ldots, M)$, at the multi-split scheme shown in Figure 4. C_{sm} and C_{am} are 2^{M-m+1}–by–2^{M-m} matrices such as in (3) and $w_{\perp i}$ for $i = 0, 1, \ldots, N-1$, represent the single parameters of the resulting zero-order filters.

The above multi-split scheme can be viewed as a linear transformation of $X(n)$ denoted by

$$X_{\perp}(n) = T^t X(n) \tag{7}$$

where

$$T = \begin{bmatrix} C_{aM}^t & C_{aM-1}^t & \cdots & C_{a1}^t \\ C_{sM}^t & C_{aM-1}^t & \cdots & C_{a1}^t \\ & \vdots & & \\ C_{sM}^t & C_{sM-1}^t & \cdots & C_{s1}^t \end{bmatrix}_{N \times N}^t . \tag{8}$$

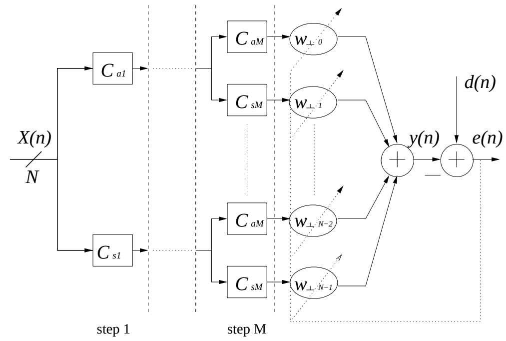

Fig. 4. Multi-split adaptive filtering.

It can be verified by direct substitution that T is a matrix of $+1$'s and -1's, in which the inner product of any two distinct columns is zero. The columns of T can be permuted in order to re-arrange the single parameters of Figure 4 in different sequences. Then, there are $N!$ possible permutations. One of them turns T into the N-order Hadamard matrix H_N. Another very interesting linear transform is obtained making:

$$C_{sm} = \begin{bmatrix} J_2^{M-m} \\ -I_2^{M-m} \end{bmatrix} \text{and} \quad C_{am} = \begin{bmatrix} I_2^{M-m} \\ J_2^{M-m} \end{bmatrix} \tag{9}$$

for $m = 1, 2, \ldots, M$. Applying (9) to (8), we obtain a linear transformation of $X(n)$ with the butterfly structure depicted in Figure 5, for $N = 8$. The value of any butterfly state is defined as the sum of the values of each branch entering that state. If a branch is dashed it has the negative value of its starting state. Otherwise it has the same value as its starting state. Note that only 24 addition operations are necessary to perform the multi-split operation for a eight-taps filter. Most important than that, no multiplication is required.

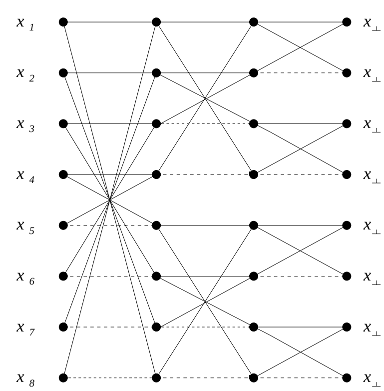

Fig. 5. Butterfly structure for the multi-split operation. The dashed (solid) lines represent the subtraction (addition) operation.

Regarding the adaptation process, the LMS algorithm can be applied independently to each single parameter as:

$$w_{\perp i}(n) = w_{\perp i}(n-1) + \frac{\mu}{r_i(n)} x_{\perp i}(n) e(n) \tag{10}$$

where $e(n) = d(n) - y(n)$,

$$y(n) = \sum_{i=0}^{N-1} x_{\perp i}(n) w_{\perp i}(n-1), \quad (11)$$

and:

$$r_i(n) = \gamma r_i(n-1) + \frac{1}{n}(|x_{\perp i}(n)|^2 - \gamma r_i(n-1)), \quad (12)$$

for $i = 0, 1, \ldots, N-1$.

Finally, when applying the multi-split filtering in channel equalization, $X(n)$ represents the channel output and $W_\perp(n) = [w_{\perp 0}(n), w_{\perp 1}(n), \ldots, w_{\perp N-1}(n)]^t$ the linear equalizer. The desired response $d(n)$ is the so-called training sequence.

III. EQUALIZING THE HDSL CHANNEL

As stated early, the split operation can reduce the eigenvalues spread of the autocorrelation matrix of the channel output, in order to improve the LMS algorithm performance. One channel having considerable eigenvalues spread is the 2kft-AWG26 channel [7], frequently encountered in HDSL applications. The frequency response and the zeros diagram of this channel can be visualized in Figures 6 and 7 respectively.

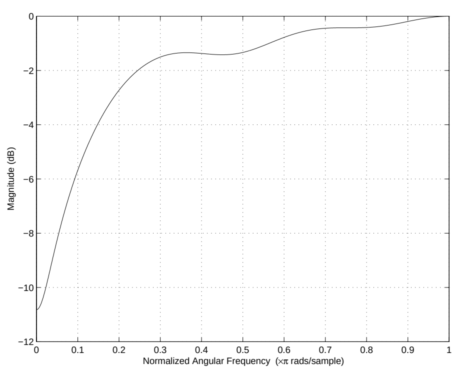

Fig. 6. Frequency response of the 2kft-AWG26 channel

Considering a 4-PAM input and an equalizer with 32 taps following the channel, one can estimate the eigenvalue spread of such a channel as being $\chi = 90.8$. When applying the multi-split operation at the channel output, the spread is reduced to $\chi = 11.9$. Thus, we can expect the multi-split equalizer to have a much better performance than the traditional equalizer. Figure 8 shows the ISI evolution for both equalizers. The solid line in the plot grid represents the amount of ISI that can be considered as the edge of the open-eye condition. The open-eye condition requires that the training error and the direct-decision error are the same.

The ISI parameter is defined as in [8]:

$$ISI = \frac{\sum_i t_i^2 - max_i t_i^2}{max_i t_i^2} \quad (13)$$

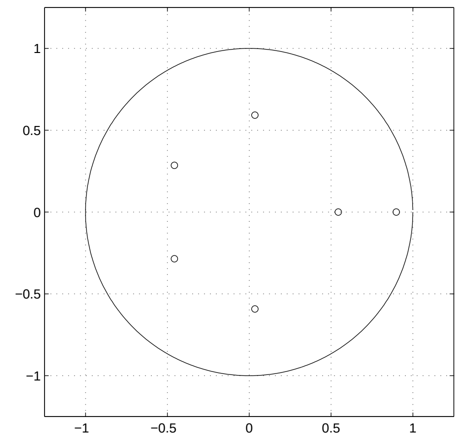

Fig. 7. Zeros diagram of the 7-taps long 2kft-AWG26 channel

where

$$t_i = h_i * w_i \quad (14)$$

and h_i and w_i are the channel and the equalizer impulse responses coefficients, respectively.

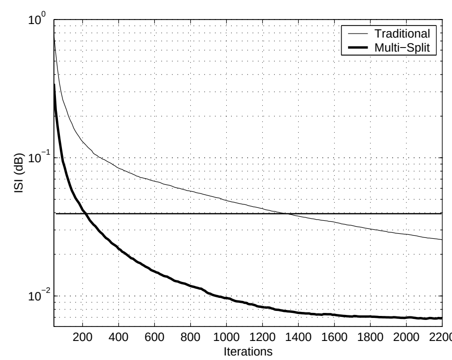

Fig. 8. ISI evolution for the 2kft-AWG26 channel

Figure 8 shows that the multi-split equalizer requires a training sequence of just 200 symbols to reach the ISI level for the open-eye condition, while the traditional equalizer requires a sequence more than 1200 symbols long. This means a reduction of six times in the length of the training period.

Following our investigation, let us analyze the case of a more interfering channel. Consider the ANSI CSA #4 channel [7]. This loop is known to be one of the most difficult test channels to equalize. Figures 9 and 10 present the frequency response and the zeros diagram for this channel. The zeros on the unit-circle reflect in the spectral null present in the frequency response.

The eigenvalue spread of this channel, when considering a 4 - PAM input and an equalizer with 64 taps, can be estimated to be $\chi = 291.78$. When applying the split operation such a spread is reduced to $\chi = 51.25$. Again, we can expect that the multi-split equalizer will converge much faster than the traditional one. Figure 11 shows the ISI evolution for both the traditional and the multi-split equalizers. The solid

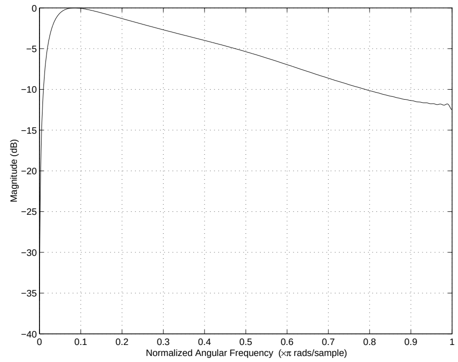

Fig. 9. Frequency response of the ANSI CSA #4 loop channel

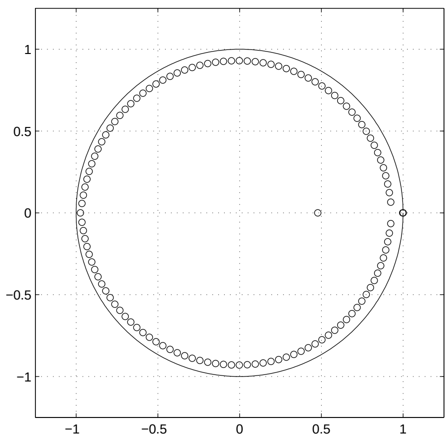

Fig. 10. Zeros diagram of the 121-taps long ANSI CSA #4 loop channel.

line in the plot grid represents the amount of ISI that can be considered as the edge of the open-eye condition.

For this second channel, the multi-split equalizer requires a training sequence almost 3000 symbols long, while the traditional equalizer needs about 12000 symbols to reach the open-eye condition. Thus, the training sequence can be reduced by a factor of four when using multi-split equalizers.

In both simulations we use the normalized LMS algorithm, either for the traditional equalizer or the multi-split equalizer. The adaptation step-size was always set to $\mu = \frac{1}{4\times \mathrm{N}}$.

IV. CONCLUSION

The results presented in this paper indicate that the multi-split equalizer can diminish considerably the length of the training sequence required by an adaptive equalizer. Channels as the ANSI CSA #4 loop channel, having zeros close to the unit-circle, need more than a feed-forward filter to be perfectly equalized. A feedback filter is also necessary. Thus, the multi-split equalizer could be used as the feed-forward filter, quickly opening the eye and allowing the feedback filter to start operation much sooner. This would increase the overall throughput of the system.

Recently, structures combining equalization and decoding in an iterative way, and using a DFE as the front end, were

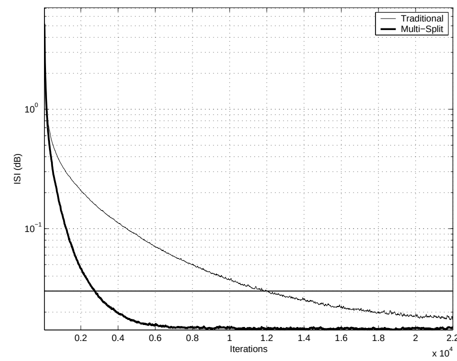

Fig. 11. ISI evolution for the ANSI CSA #4 loop channel

introduced [9], [10]. Such structures require the forward filter to open the eye before the feedback filter and the decoder start to operate. If we consider the same training sequence length for both the traditional and the multi-split equalizers, the later would present a smaller ISI level at the end of the learning process. Thus, coming back to the case of joint equalization and decoding, the use of a multi-split equalizer would yield a better first iteration performance. As stated in [9], the performance of the whole structure is highly dependent on the performance of the first iteration.

Finally, as shown in Section II, the split operation does not require any additional multiplication, just a set of addition operations. Moreover its butterfly structure is very suitable for VLSI implementation.

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