Propagation Delay Estimation in Asynchronous DS/CDMA Using Multiple Antennas

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ABSTRACT

This paper addresses the problem of channel and propagation delay estimation in asynchronous DS/CDMA systems. We consider the uplink connection in DS/CDMA with long spreading codes. The MIMO stochastic gradient algorithm proposed in [6] is estimating a linear combination of the channel impulse responses and the propagation delays. This estimate suffices for the equalization purposes. The propagation delays are estimated with a simple matching scheme.

1 Introduction

In uplink DS/CDMA, the receivers have to cope with multiple active users as well as with their asynchronous transmissions. Thus, receiver schemes that incorporate both multi-channel estimation and propagation delays estimation are necessary.

In [7] a propagation delay estimator based on subspace decomposition (MUSIC) was proposed. The algorithm was further improved in [3]. Another method based on subspace decomposition (ESPRIT) was proposed in [5]. In [4] a subspace based algorithm for time varying channel conditions is proposed. Several subspace tracking techniques are tested there. A maximum likelihood (ML) delay estimator was proposed in [1], where the multiple access interference (MAI) was modeled as Gaussian distributed. The proposed schemes listed above are near-far resistant and achieve good performance with high resolution. However, these algorithms require a static channel (with the exception of [4]), assume no multi-path propagation as well as the number of antennas in the receiver is required to be larger than the number of active users. In practice these conditions are impossible to fulfill.

The algorithm proposed in this paper uses a different system model which covers both the multi-path propagation and time varying channels. There are no restrictions in the maximum accepted propagation delays and the number of receive antennas can be smaller than the number of active users. In our previous work, [6], we proposed an adaptive channel estimation and tracking algorithm for uplink DS/CDMA. A perfect knowledge of the propagation delay for different users was assumed. In this paper we show that for equalization purposes there is no need for explicit estimation of the propagation delays. They are included in the estimated MIMO channel matrix. However, a simple delay estimation method is presented and its performance is studied.

The paper is organized as follows: the system model is presented in Section 2, the delay estimation algorithm is described in Section 3, some experimental results are provided in Section 4 and the conclusions are drawn in Section 5.

2 System model

We assume that K mobile stations (MS) are transmitting their signals to a single base station equipped with Q receive antennas.



Figure 1: The uplink in DS/CDMA

The k-th user information sequence $s_k(n)$ is spread with the corresponding spreading code $c_k(n)$ with spreading factor N_c . We assume that the spreading code period is much longer than the symbol period, so called long code. The channel from user k to the q-th antenna is assumed to be time-varying Rayleigh fading channel with the sampled impulse response $h_{kq}(i, n)$. The channel taps are chip spaced. We consider that the channels are time invariant over the symbol period. The receiver antennas are assumed to be significantly more than half

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of the wavelength apart so the subchannels can be considered independent. At each receive antenna the signal is corrupted by additive white Gaussian noise $w_q(n)$. The received sequence sampled at the chip rate at the *q*-th receive antenna can be written:

$$x_q(n) = \sum_{k=1}^{K} \sum_{i=0}^{L_h - 1} h_{kq}(i, n) s_{c,k}(n - i - \tau_k) + w_q(n) \quad (1)$$

where L_h is the channel length considered to be the same for all subchannels, $s_{c,k}(n)$ is the transmitted chip sequence of the k-th user and τ_k is the propagation delay of the k-th user. Without loss of generality we assume that the delay corresponding to a user is the same for all receive antennas. Our goal is to somehow transform the asynchronous system in described by (1) into a synchronous one. The delay of the user k can be rewritten as:

$$\frac{\tau_k}{T_c} = p_k + d_k \tag{2}$$

where p_k is the integer part, d_k is the fractional part of the delay and T_c is the chip period. Following the same rationale as in [6], we can define a new sequence which is synchronous with the received sequence:

$$r_{c,k}(n) = (1 - d_k)s_{c,k}(n - p_k) + d_k s_{c,k}(n - p_k - 1)$$
(3)

Consequently, the asynchronous model is transformed into a synchronous one. The received sampled sequence at the *q*-th antenna may be then rewritten as:

$$x_q(n) = \sum_{k=1}^{K} \sum_{i=0}^{L_h - 1} h_{kq}(i, n) r_{c,k}(n - i) + w_q(n) \qquad (4)$$

By stacking N_c received samples into a vector $\mathbf{x}_q[n] = [x_q(n), ..., x_q(n - N_c + 1)]^T$, we can write the following matrix form equation:

$$\mathbf{x}_{q}[n] = \sum_{k=1}^{K} \mathcal{H}_{q}^{(k)}[n] \mathbf{r}_{c,k}[n] + \mathbf{w}_{q}[n]$$
(5)

where $\mathbf{r}_{c,k}[n] = [r_{c,k}(n), \dots, r_{c,k}(n-T+1)]$ with $T = N_c + L_h - 1$, $\mathcal{H}_q^{(k)}[n]$ is the channel convolution matrix from the user k to antenna q of dimension $N_c \times T$ and $\mathbf{w}_q[n] = [w_q(n), \dots, w_q(n-N_c+1)]^T$.

By stacking the signal vectors from all the receive antennas column-wise we obtain the following expression:

$$\mathbf{x}[n] = \sum_{k=1}^{K} \mathcal{H}^{(k)} \mathbf{r}_{c,k}[n] + \mathbf{w}[n] = \mathcal{H}[n] \mathbf{r}_{c}[n] + \mathbf{w}[n] \quad (6)$$

where

$$\mathcal{H}^{(k)}[n] = [\mathcal{H}_1^{(k)T}[n], \dots \mathcal{H}_Q^{(k)T}[n]]^T$$

$$\mathcal{H}[n] = [\mathcal{H}^{(1)}[n], \dots \mathcal{H}^{(K)}[n]]$$

$$\mathbf{r}_c[n] = [\mathbf{r}_{c,1}^T[n], \dots, \mathbf{r}_{c,K}^T[n]]^T$$

$$(7)$$

Let us assume that the maximum delay propagation is M chip periods. Then we can write in matrix form for the user k:

$$\mathbf{s}_{c,k}[n-p_k] = B_{p_k}\mathbf{s}_k[n] \tag{8}$$

where $\mathbf{s}_k[n] = [s_{c,k}(n), \dots, s_{c,k}(n-T-M+1)]$ and the matrix B_{p_k} is defined as:

$$B_{p_k} = \begin{bmatrix} \mathbf{0}_{T \times p_k} & \mathbf{I}_T & \mathbf{0}_{T \times (M-p_k)} \end{bmatrix}$$
(9)

where $\mathbf{0}_{m \times p}$ is the zero matrix of dimension $m \times p$ and \mathbf{I}_m is the identity matrix of dimension $m \times m$.

Substituting (8) in (6) we obtain:

$$\mathbf{x}[n] = \sum_{k=1}^{K} \mathcal{H}^{(k)} \left((1 - d_k) B_{p_k} + d_k B_{p_k+1} \right) \mathbf{s}_k[n] + \mathbf{w}[n]$$

$$\mathbf{x}[n] = \sum_{k=1}^{n} \mathcal{H}^{(k)} B_k \mathbf{s}_k[n] + \mathbf{w}[n]$$
(10)
$$= \mathcal{H}[n] B\mathbf{s}[n] + \mathbf{w}[n] = C[n] \mathbf{s}[n] + \mathbf{w}[n]$$

The matrix B is the block diagonal matrix composed from the matrices B_k .

3 Algorithm

The channel estimation algorithm proposed in [6] can be applied on the system model described by (10), the only difference being that we estimate the product $\mathcal{H}[n]B$ instead of $\mathcal{H}[n]$ only. Thus we have a scheme of joint channel and delay estimation. Actually, the MMSE equalizer proposed in [6] can be applied directly on the matrix $\hat{C}[n]$ and then we don't have to estimate the propagation delays as separate parameters. However, for other purposes than equalization, such as mobile location estimation and for the performance study of the receiver, the propagation delay estimation is necessary.

The matrix $C[n] = \mathcal{H}[n]B$ is formed from blocks of size $N_c \times (T + M)$ corresponding to the channel from user k to antenna q. For the k-th user the corresponding delay matrix B_k is given by:

$$B_{k} = \begin{bmatrix} 0 & \dots & 1 - d_{k} & d_{k} & 0 & 0 & \dots & 0 \\ 0 & \dots & 0 & 1 - d_{k} & d_{k} & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & 0 & 1 - d_{k} & d_{k} & \dots & 0 \end{bmatrix}$$
(11)

In the matrix $\mathcal{H}_q^{(k)}[n]B_k$ the first p_k columns are zero and the non-zero elements on each line are a linear combination of the channel taps and propagation delay corresponding to the user k. For example in the first row the non-zero elements are:

$$(1 - d_k)h_{kq}(0, n)$$

$$d_k h_{kq}(0, n) + (1 - d_k)h_{kq}(1, n)$$

$$\dots$$

$$d_k h_{kq}(Lh - 2, n) + (1 - d_k)h_{kq}(L_h - 1, n)$$
(12)

If we try to solve the set of equations corresponding to these linear combinations, we end up with an equation in d_k of order L_h for each antenna. Thus we would have L_h solutions for the propagation delay. The true delay is the common solution for all the Q equations corresponding to a user.

 $d_k h_{ka}(Lh-1,n)$

Unfortunately this holds only in the ideal case of perfect channel estimation and noise free transmission. In practice, the noise level as well as the estimation error make impossible to find a common solution among all the Q equations.

At each time instance n we have an estimate $\hat{C}[n]$ and we know the structure of matrix B. Thus the estimate of the channel matrix $\mathcal{H}[n]$ is given by:

$$\hat{\mathcal{H}}[n] = \hat{C}[n]B^{\#} \tag{13}$$

where $(.)^{\#}$ denotes the pseudo-inverse. By taking into account the equations (3), (9) and (10) the propagation delay can be computed as:

$$\hat{d}_{k} = argmin_{d_{k}} \{J(d_{k})\}$$
(14)
$$J(d_{k}) = \sum_{n=N_{st}+1}^{N_{st}+S} \left(\mathbf{e}^{(k)}[n]^{H}\mathbf{e}^{(k)}[n]\right)$$

$$\mathbf{e}^{(k)}[n] = \mathbf{x}[n] - \hat{C}^{(k)}[n]B_{k}^{\#}\mathbf{r}_{c,k}[n]$$

In the previous equation, N_{st} is the steady-state index and S is the lag for which we compute the cost function. The cost function $J(\delta_k)$ is evaluated for all the possible delays. The normalized cost function is depicted in Figure 2 for the first user in the ideal case when the channel is known.

4 Results

We considered the case where 3 users are transmitting their information symbols to a base station equipped with two antennas. The users are moving in a typical urban environment with the speeds of 3 km/h, 30 km/h and 50 km/h. We used the wide sense stationary uncorrelated scattering (WSSUS) channel model to describe the time varying channels [2]. The spreading codes are Gold codes with the period much larger than the symbol period (long codes) and the spreading factor of 7. They



Figure 2: The cost function $J(d_1)$

are pulse shaped with a raised-cosine filter with excess bandwidth of 35%.

The algorithm proposed in [6] is used to estimate the composed delay and channel matrix C[n] and then the algorithm for delay estimation is applied after the convergence of the channel estimator. All the results are averaged over 100 independent runs. The quantization step for estimating the propagation delays is $0.1T_c$.

We consider that the maximum propagation delay is one symbol period even if it can be set at higher values.

The error in delay estimation as a function of the SNR per user is presented in Figure 3. The cost function was computed over 1000 chip periods. It can be observed



Figure 3: The error in delay estimation vs the SNR

that if the first two users delays are fairly estimated even at low SNR's, the third user require a higher SNR for a correct delay estimation.

In the delay estimation scheme the cost functions $J(d_k)$ are computed over a interval of S chip periods. We are interested to see how fast can be achieved a good delay estimation. Thus, in Figure 4 the estimation error as a function of the lag parameter S is depicted. It can observed that for the first two users we obtain a good performance if the cost function is computed over 200-300 chips while the third user requires a longer time.



Figure 4: The error in delay estimation vs lag parameter $(SNR_{user}=5dB)$

An important factor in the performance degradation in CDMA networks is the near-far effect. In the previous simulation results we considered perfect power control. In Figure 5 the degradation in algorithm performance as a function of the near-far ratio (NFR) is presented for the first user.

5 Conclusions

A propagation delay estimation technique have been presented. A linear combination of the channel impulse responses and the delays is estimated by using a MIMO stochastic gradient algorithm. Thus for pure equalization purposes there is not need for explicit delay estimation. A simple matching technique for delay estimation is presented also and its performance is studied through simulations. This scheme achieves good performance for lower to medium mobile user speeds. It is also near-far resistant. Since these estimates does not influence the equalization process performance, the interval on which the delay estimation is done can be selected to be rather long in order to have reliable estimates for all the active users in the system.

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Figure 5: The algorithm performance in near-far effect scenario

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