THIRD-ORDER CYCLIC CHARACTERIZATION OF VIBRATION SIGNALS IN ROTATING MACHINERY

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ABSTRACT

This paper introduces the application of third order cyclic statistics for vibration signals in rotating machinery. It is shown for the first time that these vibration signals display third-order cyclostationarity under some conditions. A model of signals modulated in amplitude and phase is used in order to compute the cyclic bispectrum, a third-order cyclic statistical parameter, in the frequency domain. The interpretation and the estimation techniques of this cyclic bispectrum give better understanding of the application of cyclic statistics to mechanical systems. The ambiguity concerning the use of moments or cumulants for the estimates is discussed. Moreover, the difference between them is illustrated with industrial gear signals. Application to the diagnosis of spalling in gear teeth of a U.S. Navy helicopter gearbox demonstrates the effectiveness of this new parameter for a good diagnosis.

1 INTRODUCTION

Many conventional statistical signal-processing methods treat random signals as if they were statistically stationary and ergodic. These notions are appealing because they give the possibility to estimate parameters from a single realization. However, this assumption is a mathematical idealization which, in some cases, may be valid only as an approximation of the real situation. Thus, it can exclude many real-life non stationary signals. More particularly, there is a subclass of non stationary signals called cyclostationary signals. These signals are characterized by a periodic variation of their statistical parameters. The importance of this class is that it matches the physical behavior of signals in several domains such as communication signals, and vibrating mechanical systems [1].

The theory of estimation of periodically correlated processes, i.e. second order cyclostationary, was introduced first in [2] and exploited with success in several domains especially in the diagnosis of gear faults [1] . For higher orders, the general theory of cyclic statistics has been developed in both the stochastic and fraction of time (FOT) probability frameworks. An important statistical parameter in the study of cyclostationary properties is the nth-order cyclic spectrum. Estimators for this cyclic spectrum have been proposed in [3] and [4] , respectively, for continuous time and discrete time signals. In [3], the whole study is based within a deterministic framework by using the fraction of time probability. Alternatively, in [4], estimators are built for real signals in a stochastic framework. Their estimation depends primarily on the generalization of the nth-order periodogram suggested in [5]. Applications of higher order cyclic statistical signals are limited to a few areas. In [6] and [7], an application of cyclic statistics has recently been proposed concerning the estimation of parameters of modulated signals.

This paper is concerned with the third-order cyclic characterization of vibration signals in rotating machines. The paper is organized as follows. After presenting a general model of a typical vibration signal, a study of the cyclic properties of this model, and an interpretation of the cyclic bispectrum, the third order cyclic spectrum, are proposed. Estimation techniques are reviewed and discussed in section 3. Based on the model developed, the higher-order cyclostationarity of a gear signal is demonstrated in section 4. To illustrate the use of these techniques, applications of the cyclic bispectrum, based on moments and cumulants, are also proposed in section 4. The application concerns industrial vibration signals from a helicopter gearbox of the U.S. Navy. Moreover, the possibility of the use of the cyclic bispectrum for diagnosis is discussed. Conclusions are drawn in section 5.

2 SIGNAL MODEL

A large number of vibration signals can be modeled as the sum of harmonically related exponentials modulated in amplitude and phase. They are modulated by the complex envelopes $A_n(t)$:

$$x(t) = \sum_n A_n(t)\exp(j2pf_n t) \quad \text{with} \quad f_n = nf \tag{1}$$

2.1 Analysis of the cyclostationarity of the signal

- Mathematical Expectation- First Order Cyclostationarity

$$E[x(t)] = \sum_n E[A_n(t)]\exp(j2pnft) \tag{2}$$

If the complex envelope $A_n(t) = A_n \exp(jf_n(t))$, i.e. phase modulation only, the mathematical expectation will be equal to :

$$\begin{aligned} E[x(t)] &= \sum_n A_n \exp(j2pf_n t)E[\exp(jf_n(t))] \\ &= \sum_n A_n \exp(j2pf_n t)B_{f_n}(t) \end{aligned} \tag{3}$$

with $B_{f_n}(t) = \int \exp(jf) f_{f_n}(f,t)df$ where f_{f_n} represents the probability density of $f_n(t)$. If $f_{f_n}(f,t) = f_{f_n}(f,t+kT)$ where $T = 1/f$, we obtain the result $E[x(t)] = E[x(t+kT)]$ and therefore, the signal $x(t)$ is first-order cyclostationary if the probability density of $f_n(t)$ is periodic in time (including the case of stationarity).

- Autocovariance function – 2nd order cyclostationarity

$$C_{2x}(t,t) = \sum_n \sum_m C^A_{-n,m}(t,t)\exp(j2p(m-n)ft)\exp(j2pfmt) \tag{4}$$

where $C^{A}_{-n,m}(t,t)$ is the intercovariance function of the envelopes of $A^{*}_{n}(t)$ ($A_{-n}(t)$ if $x(t)$ is real) and $A_m(t)$.
Taking into account the expression of envelope, the autocovariance function will be :

$$C_{2x}(t,t) = \sum_n \sum_m A^{*}_{n} A_m \exp(j2p(m-n)ft)\exp(j2pfmt)$$
$$\times B_{f_n f_m}(t,t+t) \quad (5)$$

where $B_{f_n f_m}(t,t+t)$ is the autocovariance function of $\exp(jf_n(t))$.
Similarly to the first-order cyclostationarity, the signal is 2nd order cyclostationary if $f_n(t)$ is wide sense 2nd order cyclostationary. This includes the cases of $f_n(t)$ being periodic and stationary.

- Third-order cumulant- Third-order Cyclostationarity

$$C_{3x}(t,t_1,t_2) = E[(x^{*}(t)-E[x^{*}(t)])(x(t+t_1)]-E[x(t+t_1)])$$
$$\times(x(t+t_2)]-E[x(t+t_2)])] = \sum_n \sum_m \sum_p C^{A}_{-n,m,p}(t,t_1,t_2)$$
$$\times\exp(j2p(m-n+p)ft)\exp(j2pfmt_1)\exp(j2pfpt_2) \quad (6)$$

where $C^{A}_{-n,m,p}(t,t_1,t_2)$ is the third-order inter-cumulant function of the envelopes $A^{*}_{n}(t), A_m(t)$ and $A_p(t)$.
After some computation, the third-order cumulant will be found equal to :

$$C_{3x}(t,t_1,t_2) = \sum_n \sum_m \sum_p A^{*}_{n} A_m A_p \exp(j2p(m-n+p)ft)$$
$$\times\exp(j2pfmt_1)\exp(j2pfpt_2) B_{f_n f_m f_p}(t,t+t_1,t+t_2) \quad (7)$$

$B_{f_n f_m f_p}(t,t+t_1,t+t_2)$ is the third-order cumulant of $\exp(jf_n(t))$.
We notice that the probability density, the conjoint probability density and $f_{f_n f_m f_p}(f_1,f_2,f_3)$ of $f(t)$ must be periodic or stationary, i.e. $f(t)$ must be third-order wide sense cyclostationary to assure the third-order cyclostationarity of the signal.

2.2 Remarks about the nth-order Cyclostationarity

We explain in this paragraph the reason for the use of cumulants to characterize the cyclostationarity of $x(t)$. There are two estimation methods based respectively on the use of moments or cumulants. The first approach deals with the periodicity of the nth-order moment. $M_{nx}(t,t_1,t_2,\ldots,t_{n-1})$. If a such case is satisfied, this moment can be decomposed into Fourier series as shown :

$$M_{nx}(t,t_1,t_2,\ldots,t_{n-1}) = \sum_a R^{a}_{nx}(t_1,t_2,\ldots,t_{n-1})\exp(j2pa\,t) \quad (8)$$

Every individual component of the sum is called an nth-order sine wave [3]. It is often the case that an nth-order moment sine wave is impure in the sense that it is made up in part or wholly of products of km_j th-order moment sine wave with $\sum_{j=1}^{k} m_j = n$ and $k \le n$, for various values of k. This property results from the properties of the nth-order moments.

To purify the nth-order impure sine waves, we must extract all the impure terms coming from lower orders. This is realized by the use of cumulants in contrast to moments and therefore represents the second approach [3]. For $n = 2$ and 3, the purification is easy, all products of first–order moment sine waves can be subtracted from the second-order moment sine waves to obtain the pure second-order sine waves. The use of the nth-order cumulants can therefore characterize specifically the nth-order properties without the influence of lower orders. In practice, the calculation and subtraction of a synchronous average, first-order moment, is sufficient to compute the second and third-order cumulants.

2.3 Interpretation of third-order cumulant spectra

In this paper, we have chosen to be limited to the third-order because orders higher than 3 involve a higher cost of calculation and give difficulties in the representation and interpretation of results.
The computation of the third-order cyclic cumulant of the modulated signal $x(t)$ gives rise to the following expression :

$$c^{k}_{3x}(t_1,t_2) = \sum_n \sum_m \sum_p C^{A,k+N(n-m-p)}_{-n,m,p}(t_1,t_2)\exp(j2pf_m t_1)\exp(j2pf_p t_2) \quad (9)$$

where $N = Pf$ with P the period of cyclostationarity and $C^{A,k+N(n-m-p)}_{-n,m,p}(t_1,t_2)$ the cyclic intercumulant of envelopes $A^{*}_{n}(t), A_m(t)$ and $A_p(t)$ for the cyclic frequency $k+N(n-m-p)$.

If we take into consideration the fact that the statistics of the envelopes have a small fluctuations, let us introduce new variables : $s = n-m, l = s\text{-}p$. The cyclic cumulant will be simplified into the following expression :

$$c^{k}_{3x}(t_1,t_2) \approx \sum_n \sum_s C^{A,r}_{-n,n-s,s-l}(t_1,t_2)\exp(j2pf_{n-s}t_1)\exp(j2pf_{s-l}t_2)$$
with $k = r + lN$ (10)

The third-order cumulant spectrum, often called the cyclic bispectrum, can be deduced from (10)

$$S_{3x}(a,n_1,n_2) \approx \sum_k \sum_n \sum_s S^{A,r}_{-n,n-s,s-l}(n_1 - f_{n-s}, n_2 - f_{s-l})d(a-k/T) \quad (11)$$

Therefore, the interpretation of the cyclic bispectrum is the following : for the cyclic frequency k/T and near the frequency pairs, $(n_1 = f_m; n_2 = f_p)$, $S_{3x}(a, f_m + l_1, f_p + l_2)$ is approximately equal to $S^{A,r}_{-n,m,p}(l_1, l_2)$, i.e. to the intercyclic bispectrum of the envelopes $A^{*}_{n}(t), A_m(t)$ and $A_p(t)$. For the cyclic frequencies $k = lN$, we obtain the interbispectrum $S^{A}_{-n,m,p}(l_1, l_2)$.

Sometimes, in the literature [8], the term bispectral correlation is used in the context of third-order cyclostationarity. Let us use the notation $B(n_1,n_2)$. It is equal to $\sum_k S_{3x}(a,n_1,n_2)d(a-k/T)$, i.e. the dimension of its support which embodies (a,n_1,n_2) is three. However, the dimension of the cyclic bispectrum is two, because it is specific for one cyclic frequency.

3 ESTIMATION TECHNIQUES FOR CYCLIC SPECTRA

Several methods exist for the estimation of higher-order cyclic spectra. Some of them are based on the computation of the Fourier transform of a windowed estimate of the higher-order cyclic cumulants. In this paper, we will not focus on these methods. Other methods issuing from [3] and [4], concern particularly the estimation of higher-order statistics for non stationary signals. For these methods, two estimators are proposed : nth-order cyclic spectra of moments and of cumulants.

3.1 Estimator of nth-order cyclic spectra of moments :

The method of estimating the cyclic spectra of moments is based on the nth-order cyclic periodogram of $x(t)$ defined by :

$$I_{nx}(a, \mathbf{f}) = \frac{1}{T} X_T(a - \sum_{i=1}^{n-1} f_i) X_T(f_1) \ldots X_T(f_{n-1}) \qquad (11)$$

with $X_T(f) = \sum_{t=0}^{T-1} x(t) e^{-2p\ jf\ t}$ its Fourier transform.

It is shown in [3], [4] that the nth-order cyclic periodogram is a sample estimator of nth-order cyclic spectra of moments for all $\mathbf{f}$ except those that lie on a submanifold D^a .This submanifold is the domain of the vector $\mathbf{f}$ such that every partial sum of its components gives a cyclic frequency of lower order than n. Such estimators are unbiased; however, they are inconsistent. A suitable smoothing in the frequency domain can make the estimators consistent and resolve the problem of estimation on the submanifold.

3.2 Estimator of nth-order cyclic spectra of cumulants :

In this paragraph, we recall that moments and cumulants share common frequency points away from the submanifold D^a . Therefore, the cyclic periodogram is a potential estimator for cumulant spectra at least away from D^a . The idea is to estimate the cyclic periodogram for all frequencies not lying on D^a . Thus, we can obtain a correct estimation of the cyclic spectra of cumulants for these harmonic frequencies. To obtain the remaining values on the submanifold D^a , two techniques are used, in the frequency domain and in the time domain.

For the frequency domain, the nth-order cyclic periodogram is constructed, masking it by a special function which is equal to one except for those frequencies that lie on the submanifold and in the end convolving with a multidimensional smoothing window. For the time domain, unlike the frequency-smoothing method, the time-averaging method needs an additional modification which is an interpolation between values near to the submanifold D^a , in order to estimate the nth-order cyclic cumulant spectra for frequencies on D^a .

Another simple way of formulating these techniques concerns the second and third order cyclostationarity. The idea is to extract the pure frequencies directly from the signal and not after computing the cyclic periodogram. As mentioned in paragraph 2, the difference between moments and cumulants for these two orders, is the first-order cyclostationary. By consequence, we can simply estimate these frequencies and extract them directly from the signal.
Recently, [9] introduced the median-based methods for the nth-order cyclic spectrum in order to avoid the problems mentioned above.

4 APPLICATIONS TO GEAR SIGNALS

Several studies using the cyclic statistics of gear signals have been developed. It was shown in [1] that vibration signals measured on gear systems display second-order cyclostationarity. In [10], signal processing methods are developed to separate first-order from second-order components of the signal. In this section, we intend to focus on the third-order cyclostationarity of gear signals. The gear signal fits the model in section 1 with $f_n = nf_e$ where f_e is the meshing frequency. By using (7), we can see that the meshing signal is third-order cyclostationary if $C^A_{-n,m,p}(t, t_1, t_2)$ is periodic. This means that the meshing harmonics (at least three between them) must be correlated and periodic. This condition is satisfied for real signals.
In order to illustrate the third-order cyclostationarity of gear signals, applications of the cyclic bispectrum are presented here for industrial systems. Signals consist of vibration data recorded from the aft main power transmission of a U.S. Navy CH-46E helicopter. The frequency-smoothed cyclic biperiodogram is used for computation. The size of FFT used is equal to 1024.

4.1 Cyclic bispectrum of moments

In this example, we have used the cyclic bispectrum based on moments, i.e. *impure* estimation. Fig.1 represents the magnitude of the cyclic bispectrum for $a = f_{r_1}$, rotating frequency of the gear wheel, of a helicopter vibration signal resulting from a spalling fault (established damage). Several local peaks inducing links appear in the magnitude of the cyclic bispectrum. However, the origin of these local peaks is undetermined because they may be the result of the cyclostationary components of order 1,2 or 3. From one point of view, these dominant peaks resulted from order 1 or 2, and therefore, can mask the pure information of the third-order cyclostationarity. From another point of view, for the same order 3, these parameters based on moments contain more information than the same parameters based on cumulants. The third-order moment contains information that we can access sequentially by using all the cumulants of order less than or equal to 3.

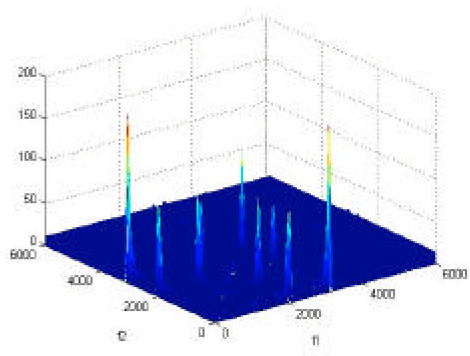
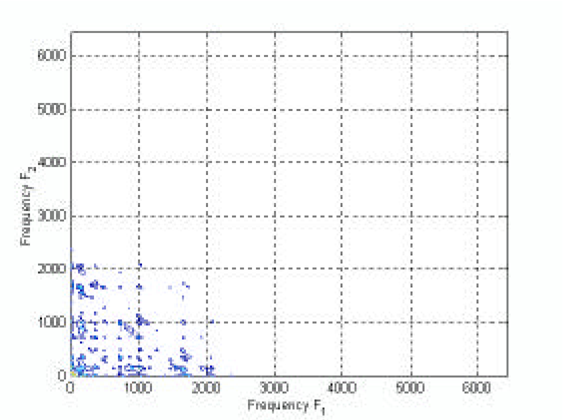


Fig.1 Magnitude of the cyclic bispectrum of moments

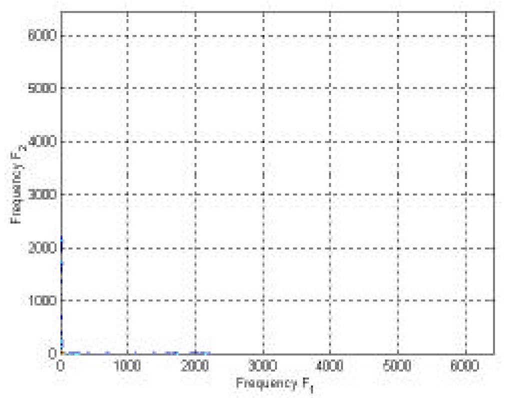
4.2 Cyclic bispectrum of cumulants

In this example, the *purity* of estimation is taken into account. This would be the case if the analysis of signals were to focus on each order of cyclostationarity. That is equivalent to separate the contributions of each order for diagnosis purposes, for example. Thus, the cyclostationary component of first order is extracted from the signal and the cyclic bispectrum is computed for the residual signal without the periodic components. Fig. 2(a) shows the result of the contour plot of the cyclic bispectrum of cumulants of the same vibration signal. The observation of the result obtained leads to an important conclusion. The local peaks which appear in Fig.1 are absent when the cyclic bispectrum is computed based on cumulants. It is clear that they result from the cyclostationary components of order lower than 3 and that they mask the contribution of cyclostationary components of order 3. This pure contribution of order 3, which corresponds to cyclic bispectrum of the pure third-order cyclostationary signal appears in Fig.2(a). This proves also that the gear vibration signals from a complex industrial system are cyclic and bilinear, since the cyclic bispectrum for a specific cyclic frequency is not zero. This corresponds to the theoretical calculus of the higher order cumulants in section 1.

To complete the analysis, the possible potential of the third-order cyclostationarity for the analysis and diagnosis of rotating machines is examined. A comparison with a vibrating signal with fault-free components will then be necessary. The contour plot of the cyclic bispectrum for a helicopter gear signal with no fault is shown in Fig. 2(b) . These results show that the gear without any fault generates only very few cyclostationary components of order 3. The appearance of the spalling strongly enhances the level of these components. The unique correlations that appear in Fig. 2(b) concern the correlations between the rotating frequency f_{r_1} and the other frequencies existing in the industrial system. As a conclusion, this study underlines that third order cyclic statistics will be a useful and powerful tool of diagnostics for industrial and complex systems such as the U.S. Navy helicopter gearbox.



(a)



(b)

Fig.2 Contour plot of the magnitude of cyclic bispectrum of cumulants of Helicopter signal for a cyclic frequency $a = f_{r_1}$ (a) with default (b) No fault

5 CONCLUSION

This paper deals with the third-order cyclic statistics of vibration signals in rotating machinery. Until now, the general theory has only been the subject of some studies and applications such as communications. [5] and [6] were the pioneers in extensively studying the theory of cyclostationarity and in proposing consistent estimators to compute statistical parameters. For the second order, the spectral correlation was demonstrated as a powerful tool for the diagnosis of gear faults. For the third order, no research has previously been proposed for mechanical applications. The originality of this paper consists in proving that vibration signals from rotating machinery are bilinear and cyclic and display third-order cyclostationarity. Application of the cyclic bispectrum to vibration signals shows the difference between the estimation based on moments and on cumulants and demonstrates the performance of the third-order cyclic statistics for diagnostic purposes.

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