FINE FREQUENCY OFFSET ESTIMATION FOR OFDM SIGNALS IN FREQUENCY SELECTIVE CHANNELS

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ABSTRACT

In this paper we derive a fine frequency estimator for OFDM (Orthogonal Frequency Division Multiplexing) systems in frequency selective channels using an approximation to the ML (Maximum Likelihood) principle. Advantage is taken from the redundancy introduced by cyclic extensions -the common strategy used to avoid ISI (Inter Symbol Interference)-. In [1], Sandell, Van de Beek and Borjesson derived one such joint time and frequency ML estimator using cyclic extensions. In their work they assumed an ideal channel for the OFDM signal, so that performance is degraded in frequency selective channels. A reasonable approximation to the ML criterion is used to derive a fine frequency estimation algorithm based on the second order statistics of the received signal.

1 INTRODUCTION

In OFDM transmission, a single data stream is transmitted over several lower rate orthogonal subcarriers. OFDM is an efficient technique to deal with multipath propagation because for a given channel delay spread, its implementation complexity is lower than for a single carrier system with an equalizer [8]. OFDM is robust against narrow band interference and avoids ISI when a cyclic prefix longer than the channel impulse response length is included. OFDM is used in single-frequency networks for broadcasting applications. It has been adopted for some standards such as DVB (Digital Video Broadcasting), DAB (Digital Audio Broadcasting) and appears as an important technology in wideband data communications, such as ADSL, VDSL and HiperLAN. One potential disadvantage of OFDM-based systems is its sensitivity to frequency offset which originates ICI (Inter-Carrier Interference) at the receiver. OFDM systems only tolerate a carrier frequency offset corresponding to a small fraction of the subcarrier spacing without incurring in large system performance degradation [2]. In continuous data stream transmissions, frequency synchronization is achieved in two steps. First, the frequency offset is reduced to $\frac{1}{2}$ of the subcarrier spacing and second, in the tracking process, fine frequency offset estimation and correction are accomplished. There are many methods that allow to extract the fine frequency offset from the OFDM signal [3]. In [1], it was observed that no extra information need be inserted for fine frequency estimation because it is possible to extract it from the cyclic prefixes. Nevertheless, the ML estimators were obtained under the consideration of ideal channel. Complexity is increased in frequency selective channles. In this work, we propose a fine frequency estimator for frequency selective channels based on cyclic extensions.

2 SIGNAL MODEL: SECOND ORDER STATISTICAL CHARACTERIZATION

Let us consider the samples of the base-band OFDM symbol ordered in vector $\mathbf{s}(k)$, formed by applying the normalized IDFT (Inverse Discrete Fourier Transform) matrix, $\mathbf{F}^{-1}$, to the vector of data symbols $\mathbf{c}(k)$ as follows:

$$\mathbf{s}(k) = \mathbf{F}^{-1}\mathbf{c}(k) \tag{1}$$

The data elements in $\mathbf{c}(k)$ have zero mean and are statistically independent. The cyclic extension is formed by appending the last P samples of $\mathbf{s}(k)$, called $\mathbf{s}_P(k)$,

$$\mathbf{s}(k)^T = [\mathbf{s}_{N-P}(k)^T \mathbf{s}_P(k)^T] \tag{2}$$

at the beginning of $\mathbf{s}(k)$.

$$[\mathbf{s}_P(k)^T \mathbf{s}_{N-P}(k)^T \mathbf{s}_P(k)^T] \tag{3}$$

where the superindex ^{T} indicates transpose. Next, each symbol (k, $k-1$,...) is ordered sequentially in vector $\mathbf{x}$ to form the data stream,

$$\mathbf{x} = \begin{bmatrix} \cdot\cdot & \mathbf{s}_P^T(k-1) & \mathbf{s}^T(k-1) & \mathbf{s}_P^T(k) & \mathbf{s}^T(k) & \cdot\cdot \end{bmatrix}^T \tag{4}$$

Then, we can write the vector of samples $\mathbf{y}$ at the receiver as follows,

$$\mathbf{y} = \mathbf{EHx} + \mathbf{n} \tag{5}$$

where vector $\mathbf{n}$ represents the zero mean AWGN (Additive White Gaussian Noise). Matrix $\mathbf{H}$ models the convolution between $\mathbf{x}$ and the elements of the sampled channel impulse response h_n. The impulse response has a finite length of L samples.We have,

$$\mathbf{H}_{(A,A+L-1)} = \begin{bmatrix} \cdot\cdot & \cdot\cdot & \cdot\cdot & & & \\ & h_{L-1} & \cdot\cdot & h_0 & & \\ & & h_{L-1} & \cdot\cdot & h_0 & \\ & & & \cdot\cdot & \cdot\cdot & \cdot\cdot \end{bmatrix} \quad (6)$$

where null elements are not represented. When necessary, the dimensions of a rectangular matrix will be shown in parentheses. Diagonal matrix $\mathbf{E}$ models the frequency doppler. We have,

$$\mathbf{E} = e^{j\phi_0}\text{diag}\left\{e^{j\phi n}\right\} \text{ per } n = 0, 1, \cdots, \dim(\mathbf{E}) - 1 \quad (7)$$

The parameter ϕ (to be estimated) is the relative frequency offset due to the intercarrier spacing. The term ϕ_0 is irrelevant to the estimation of Doppler. So, without loss of generality, we set $\phi_0 = 0$.

At this point it is important to note that we assume the hypothesis of considering the parameters ϕ and $h_{L-1} \cdots h_0$ deterministic but not known. Now, let us look at vector $\mathbf{c}(k)$ in (1). As its elements are unncorrelated and $\mathbf{F}^{-1}$ is an orthogonal operator, the elements of $\mathbf{s}(k)$, $s_i(k)$, are also uncorrelated. If N, the number of subcarriers, is high, the values of $s_i(k)$ can be modeled as a zero mean Gaussian probability density function. For $N = 16$, this is an accurate approximation if data-symbols $s_i(k)$ are binary [5]. This leads to modeling the samples of $\mathbf{y}$ as a multivariate normal distribution with first- and second-order moments:

$$E[\mathbf{y}] = \mathbf{EH}E[\mathbf{x}] + E[\mathbf{n}] = \mathbf{0} \quad (8)$$

$$\mathbf{R}_{yy} = E\left[\mathbf{y}\ \mathbf{y}^H\right] = \mathbf{EHR}_{xx}\mathbf{H}^H\mathbf{E}^H + \mathbf{R}_{nn} \quad (9)$$

Here, it is interesting to observe how cyclic extensions appended to each symbol introduce correlation in the process $\mathbf{x}$ and therefore in $\mathbf{y}$. This can be appreciated if $\mathbf{R}_{yy}$ is written as:

$$\begin{aligned} \mathbf{R}_{yy} &= \sigma_x^2\mathbf{EHH}^H\mathbf{E}^H + \sigma_n^2\mathbf{I} \\ &+\sigma_x^2\mathbf{EH}\begin{bmatrix} \cdot\cdot & & & & \\ & \mathbf{J} & & & \\ & & \mathbf{J} & & \\ & & & \mathbf{J} & \\ & & & & \cdot\cdot \end{bmatrix}\mathbf{H}^H\mathbf{E}^H \end{aligned} \quad (10)$$

where submatrix $\mathbf{J}$ is defined by means of a $P \times P$ identity matrix $\mathbf{I}_{(P)}$ and a $N-P \times N-P$ null matrix $\mathbf{O}_{(N-P)}$ as

$$\mathbf{J} = \begin{bmatrix} & & \mathbf{I}_{(P)} \\ & \mathbf{O}_{(N-P)} & \\ \mathbf{I}_{(P)} & & \end{bmatrix} \quad (11)$$

This expression shows in which way the discontinuous lateral diagonals appear. The contribution of matrix $\mathbf{H}$ in $\mathbf{R}_{yy}$ can be seen as a spreading effect over the main and the two discontinuous lateral diagonals. Note than $\mathbf{R}_{yy}$ is periodic, and the process $\mathbf{y}$ is cyclostationary. Another interesting matrix expression of the non zero elements in both discontiuous lateral diagonals can be obtained after a subblock partition analysis of $\mathbf{R}_{yy}$ that gives for the upper and lower diagonals,

$$\mathbf{R}_{ps(A)} = e^{-j\phi N}\mathbf{E}_{(A)}\mathbf{H}_{(A,B)}\mathbf{\Phi}_{(B)}\mathbf{H}^H_{(B,A)}\mathbf{E}^H_{(A)} \quad (12)$$

$$\mathbf{R}_{sp(A)} = e^{j\phi N}\mathbf{E}_{(A)}\mathbf{H}_{(A,B)}\mathbf{\Phi}_{(B)}\mathbf{H}^H_{(B,A)}\mathbf{E}^H_{(A)} \quad (13)$$

respectively, where matrix $\mathbf{\Phi}_{(\cdot)}$ is defined by using the null matrix $\mathbf{O}_{(\cdot)}$ and the identity matrix $\mathbf{I}_{(\cdot)}$ as follows,

$$\mathbf{\Phi}_{(B)} = \begin{bmatrix} \mathbf{O}_{(L-1)} & & \\ & \mathbf{I}_{(P)} & \\ & & \mathbf{O}_{(L-1)} \end{bmatrix} \quad (14)$$

All matrix dimensions in (12)(13) and (14) are expressed in terms of the impulse response length L and the cyclic prefix length P in the following way: $A = P + L - 1$ and $B = P + 2(L-1)$.

3 MAXIMUM LIKELIHOOD FUNCTION

Let us consider a set of $2N + P + L$ received samples in vector $\mathbf{y}$ modeled as a zero mean complex multivariate normal distribution.

$$f(\mathbf{y}|\phi, \mathbf{H}, \sigma_x^2, \sigma_n^2) = \frac{1}{\pi^{2N+P+L-1}\left|\mathbf{R}_{yy}\right|}e^{-\mathbf{y}^H\mathbf{R}_{yy}^{-1}\mathbf{y}} \quad (15)$$

In our model we assume that the parameters ϕ and $\mathbf{H}$ are deterministic but unknown.

A complex Gaussian process is completely defined by its first and second-order moments. After considering the second order expression in (10) and observing that this moment destroys phase information, we change the space of variables as follows,

$$\mathbf{H}' = \sigma_x\mathbf{EH} \quad (16)$$

This is motivated by the fact that in the maximization process we will take derivatives in the new space of parameters, so replacing $\frac{\partial}{\partial\phi}$, $\frac{\partial}{\partial\mathbf{H}}$ for $\frac{\partial}{\partial\phi}$, $\frac{\partial}{\partial\mathbf{H}'}$. This new space of parameters brings some new considerations into play. The first, that the main continuous diagonals in $\mathbf{R}_{yy}$ do not depend on ϕ. The second, that only the discontinuous lateral diagonals convey information on ϕ. As can be seen,

$$\mathbf{R}_{ps(A)} = E\left[\mathbf{y}_p\mathbf{y}_s^H\right] = e^{-j\phi N}\mathbf{H}'_{(A,B)}\mathbf{\Phi}_{(B)}\mathbf{H}'^H_{(B,A)} \quad (17)$$

$$\mathbf{R}_{sp(A)} = E\left[\mathbf{y}_p\mathbf{y}_s^H\right] = e^{j\phi N}\mathbf{H}'_{(A,B)}\mathbf{\Phi}_{(B)}\mathbf{H}'^H_{(B,A)} \quad (18)$$

Here, vectors $\mathbf{y}_p$ and $\mathbf{y}_s$ have dimension $P + L - 1 \times 1$. The first samples of $\mathbf{y}_p$ and $\mathbf{y}_s$ are spaced a number of N samples of distance into vector $\mathbf{y}$.

When the vector $\mathbf{y}$ in expression (15) is the observation vector, expression (15) becomes the maximum likelihood function. After some tedious calculations involving a representation of $\mathbf{y}$ in terms of its subvectors

$\mathbf{y}_p$ and $\mathbf{y}_s$ and a $\mathbf{R}_{yy}$ subblock decomposition in which $\mathbf{R}_{ps}$ and $\mathbf{R}_{sp}$ explicitly appear, it is possible to prove that we can approximate the ML function as,

$$f(\mathbf{y}|\phi,\mathbf{H}',\sigma_x^2,\sigma_n^2) \approx f(\mathbf{y}_p,\mathbf{y}_s|\phi,\mathbf{H}',\sigma_x^2,\sigma_n^2) \quad (19)$$

We can show some details on this approximation in [5]. In the following, we use expression (19) to derive the frequency estimator. We refer to it as the approximate ML function. This expression, in terms of the $\mathbf{R}_{yy}$, is represented in figure 1. We have:

$$\begin{aligned} & f(\mathbf{y}_p,\mathbf{y}_s|\phi,\mathbf{H}',\sigma_x^2,\sigma_n^2) \\ = & \frac{1}{\pi^{2(P+L+1)}|\mathbf{R}|} e^{-\left[\begin{array}{cc} \mathbf{y}_p^H & \mathbf{y}_s^H \end{array}\right]\mathbf{R}^{-1}\left[\begin{array}{c} \mathbf{y}_p \\ \mathbf{y}_s \end{array}\right]} \end{aligned} \quad (20)$$

Where,

$$\mathbf{R} = E\left[\begin{array}{c} \mathbf{y}_p \\ \mathbf{y}_s \end{array}\right]\left[\begin{array}{cc} \mathbf{y}_p^H & \mathbf{y}_s^H \end{array}\right] = \left[\begin{array}{cc} \mathbf{R}_{pp} & \mathbf{R}_{ps} \\ \mathbf{R}_{sp} & \mathbf{R}_{ss} \end{array}\right] \quad (21)$$

$$\mathbf{R}_{pp} = \mathbf{R}_{ss} = \mathbf{H}'_{(A,B)}\mathbf{H}'^H_{(B,A)} + \sigma_n^2\mathbf{I} \quad (22)$$

$$\mathbf{R}_{ps} = \mathbf{R}_{sp}^H = e^{-j\phi N}\mathbf{H}'_{(A,B)}\mathbf{\Phi}_{(B)}\mathbf{H}'^H_{(B,A)} \quad (23)$$

In order to maximize this function with respect to the estimation parameter ϕ, we can write the approximate log-ML function $\Lambda(\mathbf{y}_p,\mathbf{y}_s|\mathbf{H}',\phi,\sigma_n^2)$ taking the logarithm of (20). Next, we show some partial results to achieve this goal. It is necessary to put the determinant and the inverse of $\mathbf{R}_{yy}$ in terms of its subblock matrices $\mathbf{R}_{pp}$, $\mathbf{R}_{ss}$, $\mathbf{R}_{ps}$ and $\mathbf{R}_{sp}$. Using some determinant properties [7] we have,

$$\det\left|\begin{array}{cc} \mathbf{R}_{pp} & \mathbf{R}_{ps} \\ \mathbf{R}_{ps}^H & \mathbf{R}_{pp} \end{array}\right| = \det|\mathbf{R}_{pp}|\det\left|\mathbf{R}_{pp} - \mathbf{R}_{ps}\mathbf{R}_{pp}^{-1}\mathbf{R}_{ps}^H\right| \quad (24)$$

Where it can be proved [5] that $\mathbf{R}_{pp}^{-1}$ is non singular. Using the matrix inversion lemma [7] we can formulate the inverse of $\mathbf{R}_{yy}$ in the following way:

$$\left[\begin{array}{cc} \mathbf{R}_{pp} & \mathbf{R}_{ps} \\ \mathbf{R}_{sp} & \mathbf{R}_{ss} \end{array}\right]^{-1} = \left[\begin{array}{cc} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{array}\right] \quad (25)$$

$$\mathbf{A} = \left(\mathbf{R}_{pp} - \mathbf{R}_{ps}\mathbf{R}_{ss}^{-1}\mathbf{R}_{ps}^H\right)^{-1} \quad (26)$$

$$\mathbf{B} = -\mathbf{R}_{pp}^{-1}\mathbf{R}_{ps}\left(\mathbf{R}_{pp} - \mathbf{R}_{ps}^H\mathbf{R}_{pp}^{-1}\mathbf{R}_{ps}\right)^{-1} \quad (27)$$

$$\mathbf{C} = -\left(\mathbf{R}_{pp} - \mathbf{R}_{ps}^H\mathbf{R}_{pp}^{-1}\mathbf{R}_{ps}\right)^{-1}\mathbf{R}_{ps}^H\mathbf{R}_{pp}^{-1} \quad (28)$$

$$\mathbf{D} = \left(\mathbf{R}_{pp} - \mathbf{R}_{ps}\mathbf{R}_{ss}^{-1}\mathbf{R}_{ps}^H\right)^{-1} \quad (29)$$

Here, it is possible to find different equivalences for $\mathbf{A},\mathbf{B},\mathbf{C}$ and $\mathbf{D}$. At this point it is very important to prove that there are no singularity problems. Using some properties of Hermitian matrices this can be done [7] (the matrix products $\mathbf{H}'_{(\cdot)}\mathbf{H}'^H_{(\cdot)}$ i $\mathbf{H}'_{(\cdot)}\mathbf{\Phi}_{(\cdot)}\mathbf{H}'^H_{(\cdot)}$ are hermitian and $\mathbf{R}_{ps} = \mathbf{R}_{sp}^H$) Now we define $\mathbf{\Delta}$, as:

$$\mathbf{\Delta} = \mathbf{R}_{pp} - \mathbf{R}_{ps}\mathbf{R}_{pp}^{-1}\mathbf{R}_{ps}^H \quad (30)$$

Matrix $\mathbf{\Delta}$ and its inverse, which can be proved to be non-singular, are Hermitian, independent on ϕ in the new parameter space and only dependent on subblock matrices $\mathbf{R}_{pp}$, and $\mathbf{R}_{ps}$. After some algebra, the approximate log-ML function $\Lambda(\mathbf{y}_p,\mathbf{y}_s|\mathbf{H}',\phi,\sigma_n^2)$ in terms of (30) is:

$$\begin{aligned} & \Lambda(\mathbf{y}_p,\mathbf{y}_s|\mathbf{H}',\phi,\sigma_n^2) \\ & = -\ln\pi^{2(P+L-1)} - \ln\det|\mathbf{R}_{pp}|\det|\mathbf{\Delta}| \\ & -\mathbf{y}_p^H\mathbf{\Delta}^{-1}\mathbf{y}_p - \mathbf{y}_s^H\mathbf{\Delta}^{-1}\mathbf{y}_s \\ & + \mathbf{y}_p^H\mathbf{R}_{pp}^{-1}\mathbf{R}_{ps}\mathbf{\Delta}^{-1}\mathbf{y}_s + \mathbf{y}_s^H\mathbf{\Delta}^{-1}\mathbf{R}_{ps}\mathbf{R}_{pp}^{-1}\mathbf{y}_p \end{aligned} \quad (31)$$

Note that this function depends on ϕ and $\mathbf{H}'$.

4 DERIVATION PROCESS

With the intention of maximizing the approximate log-ML function, we determine the corresponding score function $\mathbf{s}(\mathbf{y}_p,\mathbf{y}_s|\mathbf{H}',\phi,\tau,\sigma_n^2)$ by differentiating (31) with respect to the known parameters,

$$\begin{aligned} & s_\phi(\mathbf{y}_p,\mathbf{y}_s|\mathbf{H}',\phi,\tau,\sigma_n^2) \\ = & \frac{\partial}{\partial\phi}\ln f(\mathbf{y}_p,\mathbf{y}_s|\mathbf{H}',\phi,\tau,\sigma_n^2) \\ = & \frac{\partial}{\partial\phi}\Lambda(\mathbf{y}_p,\mathbf{y}_s|\mathbf{H}',\phi,\tau,\sigma_n^2) \end{aligned} \quad (32)$$

Taking into account the implicit dependency of (32) on ϕ and $\mathbf{H}'$ this expression can be written in terms of $\mathbf{R}_{pp}$ (22), and $\mathbf{R}_{ps}$ (23) and $\mathbf{\Delta}$ (30) as:

$$\begin{aligned} & s_\phi(\mathbf{y}_p,\mathbf{y}_s|\mathbf{H}',\phi,\tau,\sigma_n^2) = \\ & -jN\mathbf{y}_p^H\mathbf{R}_{pp}^{-1}\mathbf{R}_{ps}(\phi)\mathbf{\Delta}^{-1}\mathbf{y}_s + jN\mathbf{y}_s^H\mathbf{\Delta}^{-1}\mathbf{R}_{ps}(\phi)\mathbf{R}_{pp}^{-1}\mathbf{y}_p \\ & = 2N\,\mathrm{Im}\left[\mathbf{y}_p^H\mathbf{R}_{pp}^{-1}\mathbf{R}_{ps}(\phi)\mathbf{\Delta}^{-1}\mathbf{y}_s\right] \end{aligned} \quad (33)$$

Where the dependency of $\mathbf{R}_{ps}$ on ϕ is indicated, $\mathbf{R}_{ps} = \mathbf{R}_{ps}(\phi)$**.** Next**,** from our model, the estimator can be obtained by solving the following equations.

$$\mathbf{s}_{\mathbf{H}'}(\mathbf{y}_p,\mathbf{y}_s|\mathbf{H}',\phi,\sigma_n^2) = 0 \quad (34)$$

$$s_\phi(\mathbf{y}_p,\mathbf{y}_s|\mathbf{H}',\widehat{\phi},\sigma_n^2) = 0 \quad (35)$$

Channel estimation involving equation (34) is avoided for reasons of mathematical complexity. We focus our attention in (35), getting,

$$\widehat{\phi} = \frac{1}{N}\mathrm{angle}\left[\mathbf{y}_p^H\mathbf{R}_{pp}^{-1}\mathbf{H}'\mathbf{\Phi}\mathbf{H}'^H\mathbf{\Delta}^{-1}\mathbf{y}_s\right] \quad (36)$$

Where matrix $\mathbf{\Phi}$ is defined in (14). Before expression (36) becomes an estimator, some questions need be solved. First, as we assume perfect time information we are able to compute a ML estimation of $\mathbf{R}_{yy}$ and consequently we can obtain a ML estimation for $\mathbf{R}_{pp}$ and for $\mathbf{R}_{ps}$. This allows us, using the invariance principle [7][4], to replace $\mathbf{R}_{pp}$ and $\mathbf{R}_{ps}$ for its ML estimation, $\mathbf{R}_{pp\_ML}$ and $\mathbf{R}_{ps\_ML}$, in expression (36). The second and most important question is that if equation (34) is not solved, we do not have any estimation of $\mathbf{H}'$. To

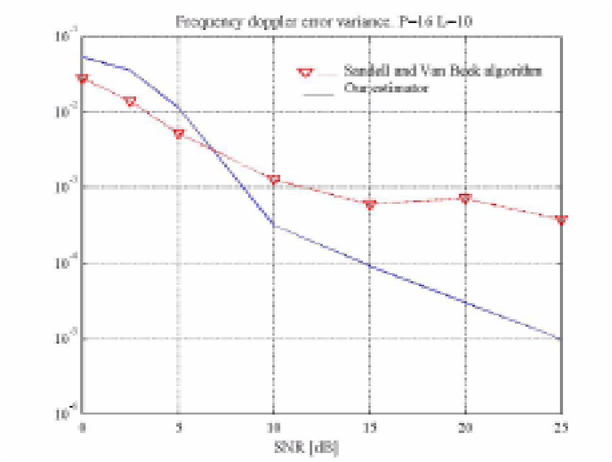

Figure 1: Comparation between Sandell and our estimator.

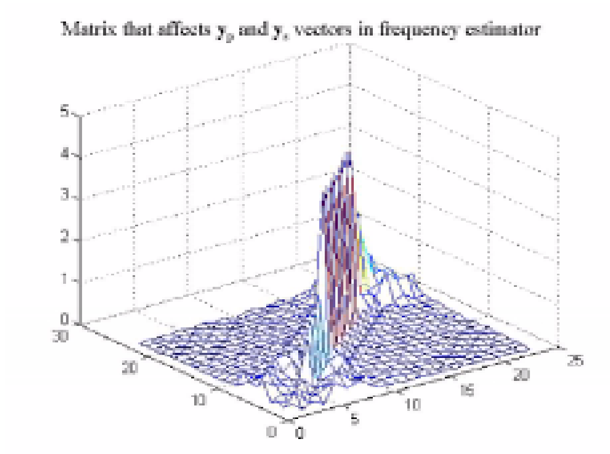

Figure 2: Matrix $\mathbf{R}_{pp\_ML}^{-1}\mathbf{\Psi}\mathbf{\Delta}_{ML}^{-1}$ when P=16 and L=10. The channel has a Rayleigh distribution.

overcome this, we need to make an approximation. The following step is to consider:

$$\mathbf{H}'\mathbf{\Phi}\mathbf{H}'^{H} \approx \mathbf{\Psi} \tag{37}$$

$$\mathbf{\Psi} = \begin{bmatrix} \mathbf{O}_{(L-1)} & & \\ & \mathbf{I}_{(P-L+1)} & \\ & & \mathbf{O}_{(L-1)} \end{bmatrix} \tag{38}$$

This approximation can be justified [5] after observing that the elements of the main diagonal matrix $\mathbf{H}'\mathbf{\Phi}\mathbf{H}'^{H}$ in the same positions that those in $\mathbf{\Psi}$ are real and have a larger modulus than those in $\mathbf{H}'\mathbf{\Phi}\mathbf{H}'^{H}$. This allows as to formulate the following estimator:

$$\widehat{\phi} = \frac{1}{N}\text{angle}\left[\mathbf{y}_p^H \mathbf{R}_{pp\_ML}^{-1}\mathbf{\Psi}\mathbf{\Delta}_{ML}^{-1}\mathbf{y}_s\right] \tag{39}$$

Where:

$$\mathbf{\Delta}_{ML} = \mathbf{R}_{pp\_ML} - \mathbf{R}_{ps\_ML}\mathbf{R}_{pp\_ML}^{-1}\mathbf{R}_{ps\_ML}^{H} \tag{40}$$

5 SIMULATION RESULTS

In this section we have compared our algorithm with the performance of Sandell's algorithm for OFDM symbols with N=64 information samples and a cyclic extension of 16 samples. In both cases the same sampled channel has been used. It has an impulse response length of 10 samples statistically modeled with a Rayleigh distribution.

6 SUMMARY

A Non Data-Aided fine frequency OFDM estimator for frequency selective channels has been derived by using the cyclic extension. When the channel is slowly time warying our method outperforms Sandell's estimator, based in cyclic extensions but derived assuming an ideal channel. The proposed algorithm requires the estimation of the autocorrelation matrix. When we particularize our expressions for the ideal channel, that is L=1, Sandell's estimator is reproduced.

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