ADAPTIVE LOCAL POLYNOMIAL FOURIER TRANSFORM

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ABSTRACT

Adaptive local polynomial Fourier transform (ALPFT) is proposed in this paper. For multicomponent FM signals with parallel instantaneous frequencies (IF) the ALPFT is chosen as the local polynomial Fourier transforms (LPFT) which produces maximal concentration measure from a set of the LPFTs. For other forms of the FM signals the ALPFT is determined as a weighted sum of the LPFTs. The weighting coefficients are determined as a function of the concentration measure. The proposed transforms produce highly concentrated time-frequency signal representations.

1 INTRODUCTION

In order to get highly concentrated and accurate adaptive TF representations, measures of the time-frequency (TF) distributions concentration have been used in [1]-[7]. Many of the concentration measures have the origin in the information theory [8]. However, problems in the TF representations concentration measuring can be quite different than those in the information theory. Thus, the concentration measures developed in the information theory should be carefully used in TF analysis (usually with necessary modifications). Analysis of some concentration measures in the TF analysis can be found in [8].

Adaptive harmonic fractional Fourier transform has been proposed in [4]. This transform is based on the concentration measure. It has been successively applied to the speech signals. Unfortunately, this method cannot be used in a straightforward manner to other types of FM signals. In this paper we started with similar assumptions as in [4]. The adaptive local polynomial Fourier transform (ALPFT) is defined as a weighted sum of the local polynomial Fourier transforms (LPFT). The adaptive weighting coefficients are obtained by using a concentration measure. Two schemes for weighting coefficients determination are developed.

The paper is organized as follows. Definition and properties of the LPFT are given in Section II. The ALPFT is introduced in Section III. Numerical examples are given in Section IV.

2 LOCAL POLYNOMIAL FOURIER TRANSFORM

The LPFT is defined by Katkovnik in [10] for the IF estimation of polynomial phase signals:

$$LPFT(t,\vec{\omega}) = LPFT(t,\omega_1,...,\omega_M) =$$

$$= \int_{-\infty}^{\infty} x(t+\tau)w^*(\tau)e^{-j\omega_1\tau - j\omega_2\tau^2/2 - ... - j\omega_M\tau^M/M!}d\tau =$$

$$= \int_{-\infty}^{\infty} x(t+\tau)w^*(\tau)e^{-j\sum_{i=1}^{M}\omega_i\frac{\tau^i}{i!}}d\tau, \qquad (1)$$

where $\vec{\omega} = (\omega_1,\omega_2,...,\omega_M)$, and $w(\tau)$ is the window function. The main advantage of LPFT over other TF representations is in its linearity. The local polynomial periodogram (LPP) is used in order to avoid the complex nature of the LPFT:

$$LPP(t,\vec{\omega}) = |LPFT(t,\vec{\omega})|^2. \qquad (2)$$

For $\omega_2 = \omega_3 = ... = \omega_M = 0$ the LPFT is reduced to the short-time Fourier transform (STFT):

$$LPFT(t,\omega_1=\omega,0,...,0) = STFT(t,\omega) =$$

$$= \int_{-\infty}^{\infty} x(t+\tau)w^*(\tau)e^{-j\omega\tau}d\tau. \qquad (3)$$

In this case the LPP is equal to the spectrogram $SPEC(t,\omega) = |STFT(t,\omega)|^2$. The basic drawback of the LPFT is in increase of dimensionality, i.e., increase of the calculation complexity. We will restrict our analysis to the case of $M = 2$, when:

$$LPFT_\alpha(t,\omega) = \int_{-\infty}^{\infty} x(t+\tau)w^*(\tau)e^{-j\omega\tau - j\alpha\tau^2/2}d\tau. \qquad (4)$$

This form of the LPFT is closely related to the fractional Fourier transform (FRFT) [9]. Properties of the LPFT and the LPP in the IF estimation are analyzed in details [10].

3 ADAPTIVE LOCAL POLYNOMIAL FT

3.1 Parallel Instantaneous Frequencies

Consider the sum of linear FM signals with parallel instantaneous frequencies:

$$x(t) = \sum_{i=1}^{Q} A_i e^{jat^2/2+jb_it}. \tag{5}$$

The LPFT of this signal is:

$$LPFT_\alpha(t,\omega) =$$

$$= \int_{-\infty}^{\infty} \sum_{i=1}^{Q} A_i e^{ja\frac{(t+\tau)^2}{2}+jb_i(t+\tau)} w^*(\tau) e^{-j\omega\tau - j\alpha\frac{\tau^2}{2}} d\tau =$$

$$= e^{jat^2/2} \sum_{i=1}^{Q} A_i e^{jb_it} \times$$

$$\int_{-\infty}^{\infty} e^{jat\tau+ja\tau^2/2+jb_i\tau} w^*(\tau) e^{-j\omega\tau-j\alpha\tau^2/2} d\tau =$$

$$= e^{jat^2/2} \sum_{i=1}^{Q} A_i e^{jb_it} \left[W(\omega) *_\omega \sqrt{\frac{2\pi}{\alpha-a}} e^{\frac{(\omega-at-b)^2}{(\alpha-a)}} \right], \tag{6}$$

where $W(\omega)$ is the Fourier transform of the window function $w(t)$. The LPFT (6) achieves maximal concentration for $\alpha = a$, when the LPFT is ideally concentrated along the IF for all signal components $\omega = at + b_i$. Thus, the adaptive TF representation can be obtained by considering the set of the LPFT with various values of $\alpha \in \Lambda$. The adaptive LPFT is the one from the considered set of the LPFTs which produces the highest concentration. The chosen LPFT would have the value α as close as possible to the value of a, for all $\alpha \in \Lambda$. Concentration can be measured by using some of the concentration measures presented in [1]-[4]. Measures that increase their values when the concentration of the TF representation increases its value are used in this paper. Note, that there are several other measures whose values decrease when the TF representation concentration increases [8].

Procedure for determination of the adaptive LPFT can be summarized as follows.

1. Calculate $LPFT_\alpha(t,\omega)$ for $\alpha \in \Lambda$.
2. Calculate the concentration measure:

$$H(\alpha) = g(LPFT_\alpha(t,\omega)), \tag{7}$$

where $g(\cdot)$ is concentration measure of the TF representation calculated for the entire TF plane.

3. The adaptive LPFT is obtained as:

$$ALPFT'(t,\omega) = LPFT_{\hat{\alpha}}(t,\omega) \text{ where}$$

$$\hat{\alpha} = \arg\max_{\alpha} H(\alpha). \tag{8}$$

For signals with time-varying chirp rate a the adaptive LPFT can be calculated for each time-instant:

$$H(t,\alpha) = g_t(LPFT_\alpha(t,\omega)), \tag{9}$$

where $g_t(\cdot)$ is concentration measure for the considered instant t. The adaptive LPFT can be calculated for the considered instant as:

$$ALPFT''(t,\omega) = LPFT_{\hat{\alpha}(t)}(t,\omega) \text{ where}$$

$$\hat{\alpha}(t) = \arg\max_{\alpha} H(t,\alpha). \tag{10}$$

For multicomponent signals that could not be represented as a sum of the signals with parallel IFs, the proposed procedure would produce the ideal TF representation of the strongest signal's component while other components would remain low-concentrated. This is the reason for an alternative definition of the ALPFT as the weighted sum of the LPFTs. The weighting coefficients are calculated according to the concentration measure:

$$ALPFT'''(t,\omega) = \sum_{\alpha} f(H(\alpha,t)) LPFT_\alpha(t,\omega), \tag{11}$$

where $f(\cdot) > 0$ is an increasing function $x_1 > x_2 \rightarrow f(x_1) \geq f(x_2)$. The following function:

$$f(H(\alpha,t)) = \left(\frac{H(\alpha,t)}{H(\hat{\alpha}(t),t)} \right)^p \tag{12}$$

will be used in numerical examples, where $p > 0$, while $\hat{\alpha}(t)$ is given with expression (10). Note that $ALPFT'''(t,\omega)$ is equal to $ALPFT''(t,\omega)$ for $p \rightarrow \infty$.

4 NUMERICAL EXAMPLES

The following form of the concentration measure $H(\alpha,t)$ will be used in the examples [4]:

$$H(\alpha,t) = \frac{\int_{-\infty}^{\infty} |LPFT_\alpha(t,\omega)|^2 d\omega}{\left(\int_{-\infty}^{\infty} |LPFT_\alpha(t,\omega)| d\omega \right)^{3/2}}. \tag{13}$$

Example 1: Consider the signal:

$$x(t) = \exp(j256\pi t^2)[1 + 2\cos(256\pi t)], \tag{14}$$

within $t \in [-3/4, 3/4]$, with the sampling interval $\Delta t = 1/1024$. The Hamming window of the width $N = 512$ is used. The absolute values of the STFT, $ALPFT'''(t,\omega)$ and $ALPFT''(t,\omega)$ are shown in Figure 1. The concentration measure $H(\alpha)$ is shown in Figure 1d. It can be seen that $ALPFT''(t,\omega)$ produces the ideal TF concentration while $ALPFT'''(t,\omega)$ is close to the optimal one. Note that in this case holds $ALPFT'(t,\omega) = ALPFT''(t,\omega)$.

Example 2: Consider the sum of three parallel sinusoidal modulated FM signals:

$$x(t) = \exp(j64\cos(2\pi t))[1 + 2\cos(512\pi t)]. \tag{15}$$

Considered time and sampling intervals are the same as in the previous example. The STFT, $ALPFT'''(t,\omega)$, $ALPFT''(t,\omega)$ and $\hat{\alpha}(t)$ are shown in Figure 2.

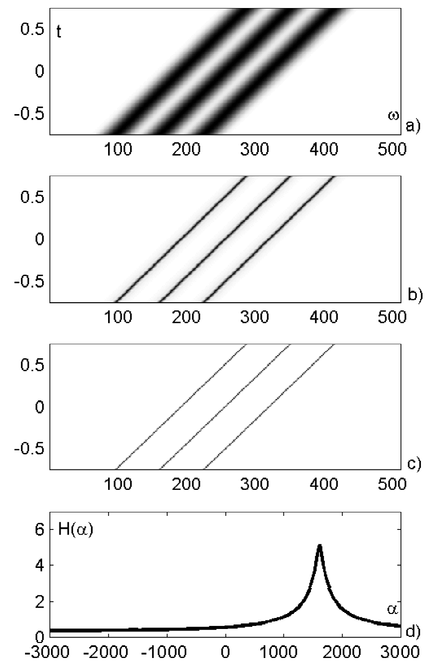

Figure 1: TF representations of three parallel FM signals: a) STFT; b) $ALPFT'''(t,\omega)$; c) $ALPFT''(t,\omega)$; d) $H(\alpha)$.

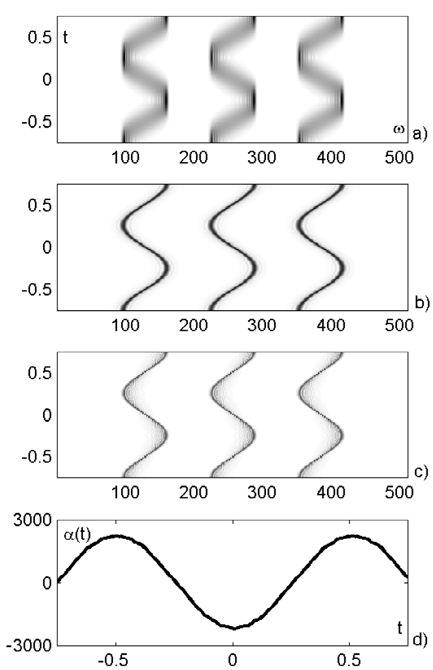

Figure 2: TF representations of three parallel sinusoidal FM signals: a) STFT; b) $ALPFT'''(t,\omega)$; c) $ALPFT''(t,\omega)$; d) $\hat{\alpha}(t)$.

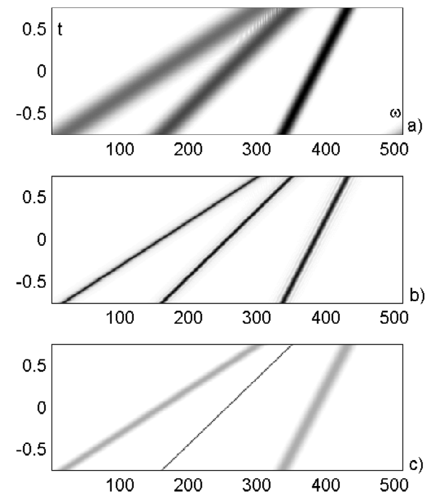

Figure 3: TF representations of three non-parallel FM signals: a) STFT; b) $ALPFT'''(t,\omega)$; c) $ALPFT''(t,\omega)$.

Example 3: Consider the sum of three linear FM signals with nonparallel IFs:

$$x(t) = \exp(j256\pi t^2) + \exp(j128\pi t^2 + j512\pi t) +$$
$$+ \exp(j384\pi t^2 - j384\pi t). \qquad (16)$$

The STFT, $ALPFT'''(t,\omega)$ and $ALPFT''(t,\omega)$ of signal (16) are shown in Figure 3. It can be seen that $ALPFT''(t,\omega)$ is ideally concentrated along the IF of the first component $\exp(j256\pi t^2)$, while other two components remain spread, Figure 3c. The trade-off between concentration of all components is achieved by using $ALPFT'''(t,\omega)$, Figure 3b.

Example 4: Consider the sum of three FM signals (linear modulated, sinusoidal modulated and chirp pulse):

$$x(t) = \exp(j512\pi t^2 + j192\sin(2\pi t)) + \exp(j512\pi t^2) +$$
$$+ \exp(-64(t-0.2)^2 - j128\pi t^2 - j512\pi t). \qquad (17)$$

The STFT, $ALPFT'''(t,\omega)$ and $ALPFT''(t,\omega)$ are shown in Figure 4. It can be seen that $ALPFT'''(t,\omega)$ outperforms $ALPFT''(t,\omega)$.

Example 5: Consider a frequency coded signal with elementary signals of the form [11]:

$$x(t) = A\exp(j8\pi \cdot a_1(t)(t-\bar{t}_m)^2/2) +$$
$$+ A\exp(-j8\pi \cdot a_2(t)(t-\bar{t}_m)^2/2) \text{ for } t \in [t_{m-1}, t_m), \quad (18)$$

where $\bar{t}_m$ is the middle point of the interval of elementary signal $\bar{t}_m = (t_m + t_{m-1})/2$. The values of the chirp rate $a(t)$ represent coded message:

$$a_1(t) = a_m,\ a_2(t) = -a_m \text{ for } t \in [t_{m-1}, t_m). \qquad (19)$$

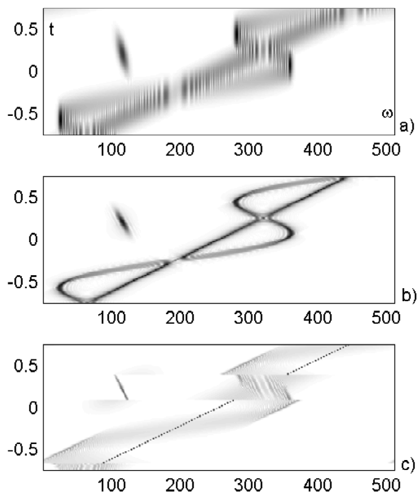

Figure 4: TF representations of three non-parallel FM signals: a) STFT; b) $ALPFT'''(t,\omega)$; c) $ALPFT''(t,\omega)$.

where a_m takes the values $2, 4, 8, 5, 10, 9, 7, 3, 6$ and 1. The signal is corrupted by the complex Gaussian noise with variance $2\sigma^2 = 1$. The STFT, $ALPFT'''(t,\omega)$ and $ALPFT''(t,\omega)$ are shown in Figure 5. From this figure we can easily see the improvement in TF representation that is achieved by using $ALPFT'''(t,\omega)$.

5 CONCLUSION

The adaptive LPFT based on the concentration measure is proposed. Two different types of this transformation are proposed and analyzed. The procedure for determination of the adaptive LPFT is presented. Theory is illustrated on several numerical examples, including nonlinear FM signals and signals whose components are not parallel in the TF domain. In all cases highly concentrated representations are achieved.

6 ACKNOWLEDGMENT

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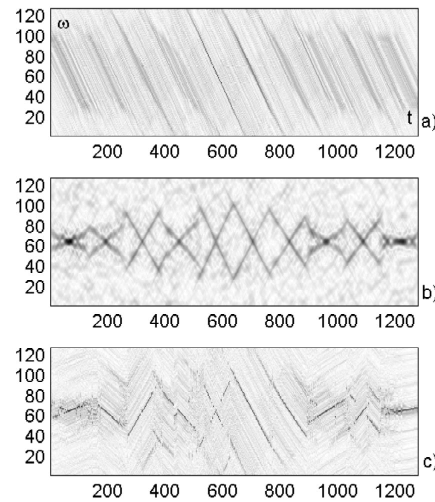

Figure 5: TF representations of frequency coded signals: a) STFT; b) $ALPFT'''(t,\omega)$; c) $ALPFT''(t,\omega)$.

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