Noise constrained LMS algorithm in transform domain

Radu Ciprian Bilcu[1], *Pauli Kuosmanen*[2], *Karen Egiazarian*[3]
Signal Processing Laboratory, Tampere University of Technology,
Hermiankatu 3 A, FIN-33101, Tampere, Finland
e-mail: [1]bilcu@cs.tut.fi, [2]pqo@cs.tut.fi, [3]karen@cs.tut.fi

ABSTRACT

In a recent paper the authors proposed a new Variable Step LMS algorithm for transform domain. In that algorithm the step-size structure was analyzed from a new point of view. The step-size has two components: a global component, that is the same for each filter coefficient and a local time-variable component, given by the power normalization. Making the global component also time-variable, the speed of convergence was greatly increased. In this paper we introduce a new method for adaptation of the global component by using some informations about noise variance. The novelty of this algorithm consists in low complexity and also the trade-off between a small misadjustment and a fast convergence speed is completely eliminated.

Key words: adaptive filters, DCT, system identification, transform domain LMS.

1 INTRODUCTION

Probably the most known algorithm in the field of adaptive filtering is the Least Mean Square algorithm (LMS) [1]-[5]. The main disadvantages of the time domain LMS algorithm are: the trade-off between a small misadjustment and a fast convergence, and the convergence problems that arises when the input signal is highly correlated. The class of the Variable Step LMS algorithms (VS-LMS) was introduced in order to deal with the first drawback [2]-[5]. In order to eliminate the convergence problems when the LMS operate with correlated input signals, the Transform Domain LMS (TDLMS) was introduced [6]-[8].

In a recent published paper [8], the authors introduced a new class of transform domain LMS algorithms namely the Transform Domain Variable Step LMS (TD-VSLMS), where in the update of the step-size some informations about the output error are included. The simulations presented in [8] shows a highly improvement in the convergence rate while maintaining a small steady-state misadjustment. The new algorithm proposed in this paper belongs to the new class of TD-VSLMS algorithms. Our main goal in this paper is to reduce the computational complexity of the algorithm proposed in [8], and to obtain a new algorithm, that has also a simplified setup of the coefficients.

2 EXISTING ALGORITHMS

The block diagram of the TDLMS algorithm is depicted in Fig. 1, where $x(n)$ is the input signal, $\hat{\mathbf{w}}(n)$ is the $N \times 1$ vector of the adaptive filter coefficients, $\mathbf{w}_{opt}$ is the vector of the unknown system coefficients, $y(n)$, $\hat{y}(n)$ and $e(n)$ are the output of the unknown system, the output of the adaptive filter and the output error, respectively. By $v(n)$ we have denoted the output noise. The block denoted by $\mathbf{T}_N$ is the $N \times N$ matrix of the transform applied to the input signal, and $\mathbf{s}(n)$ represents the vector of the transformation coefficients.

Each coefficient of the TDLMS algorithm is updated as follows:

$$\hat{h}_i(n+1) = \hat{h}_i(n) + \frac{\mu}{\gamma + \sigma_i^2(n)} s_i(n) e(n); \qquad (1)$$

where $\sigma_i^2(n)$ is the power estimate of the transform coefficient $s_i(n)$, γ is a small constant that eliminates the overflow when the $\sigma_i^2(n)$ is small and μ is the step-size.

Usually $\sigma_i^2(n)$ is computed as [6], [7]:

$$\sigma_i^2(n+1) = \beta \sigma_i^2(n) + (1-\beta) |s_i(n)|^2 \qquad (2)$$

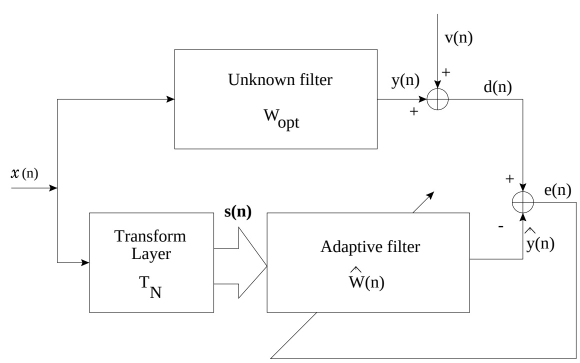

Figure 1: Block diagram of the TDLMS algorithm for system identification.

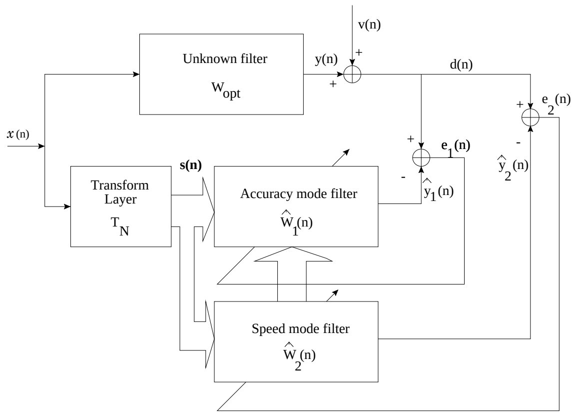

Figure 2: Block diagram of TDVSLMS algorithm for system identification

Analyzing (1) and (2), it is common to consider the TDLMS algorithm having a time-variable step-size $\mu_i(n) = \frac{\mu}{\sigma_i^2(n)}$ for each coefficient. For a stationary input signal $x(n)$, the power estimates $\sigma_i^2(n)$ becomes constants after few iterations and the step-sizes $\mu_i(n)$ will be also constants (actually they will have small variations). Furthermore, it is well known in the literature [4], [5] that the output error $e(n)$ can give very important informations about the optimum behavior of the step-size. For large errors (the algorithm is far from steady-state) the step-size should be large to speed-up the convergence, whereas for small $e(n)$ the step-size should be decreased in order to get the desired misadjustment. The above idea was implemented in the algorithm proposed in [8], where the value of each step-size was considered to be composed from two components: the global component μ, and the local component $\sigma_i^2(n)$ (see (1)). The global component was made also time-variable depending on the output error and the new algorithm obtained this way has a highly improved convergence speed. The block diagram of the algorithm proposed in [8] is depicted in Fig. 2, where the notations are those from Fig. 1 and the blocks denoted by "Speed mode filter" and "Accuracy mode filter" are two adaptive TDLMS filters one with a large and constant step-size (the speed mode filter), and the second one with variable step-size. The speed mode filter was used in order to increase the convergence speed. The accuracy mode filter is actually the filter of interest.

The coefficients of the accuracy mode filter was updated as follows:

$$\hat{\mathbf{w}}_1(n+1) = \begin{cases} \hat{\mathbf{w}}_2(n+1) \text{if} & \begin{cases} \prod_{i=1}^{L} Q(i) = 1 \\ \text{and} \\ n = kT \end{cases} \\ \hat{\mathbf{w}}_1(n) + \mu_1(n)\mathbf{s}(n)e(n) & \text{otherwise} \end{cases} \tag{3}$$

The step-size was updated as follows:

$$\mu_1(n+1) = \begin{cases} \frac{\mu_2+\mu_1(n)}{2} & \text{if} \begin{cases} \prod_{i=1}^{L} Q(i) = 1 \\ \text{and} \\ n = kT \end{cases} \\ max\{\mu_{min}, \alpha\mu_1(n)\} & \text{if} \begin{cases} \prod_{i=1}^{L} Q(i) = 0 \\ \text{and} \\ n = kT \end{cases} \\ \mu_1(n) & \text{otherwise} \end{cases} \tag{4}$$

where T and L are some constants and Q is given by:

$$Q(i) = \begin{cases} 1, & \text{if} \sum_{k=n-iT}^{n-(i-1)T} e_1^2(k) > \sum_{k=n-iT}^{n-(i-1)T} e_2^2(k); \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

3 THE NEW ALGORITHM

The main disadvantage of the algorithm proposed in [8] is the use of the *Speed mode filter* that increase the complexity of the algorithm. In the present paper, we propose a new algorithm, that uses some informations about the noise variance σ_v^2 for updating the step-size. In fact there are a lot of practical applications in which the noise variance or some approximation can be obtained.

The block diagram used for our new algorithm is presented in Fig. 1 when just an adaptive filter is used, and the algorithm becomes:

- the update of each coefficient of the adaptive filter

$$\hat{w}_i(n+1) = \hat{w}_i(n) + \frac{\mu(n)}{\gamma + \sigma_i^2(n)} s_i(n)e(n) \tag{6}$$

- step-size update

$$\mu(n+1) = \begin{cases} \frac{\mu_{max}+\mu(n)}{2} & \text{if} \\ & \begin{cases} \sum_{i=n-T+1}^{n} e(i)^2 > C(n) \\ \text{and} \\ n = k\dot{T} \end{cases} \\ max\{\mu_{min}, \alpha\mu(n)\} & \text{if} \\ & \begin{cases} \sum_{i=n-T+1}^{n} e(i)^2 \le C(n) \\ \text{and} \\ n = k\dot{T} \end{cases} \\ \mu(n) & \text{otherwise} \end{cases} \tag{7}$$

The behavior of the new NCTDLMS algorithms is as follows: for T consecutive iterations, the global component $\mu(n)$ of the step-size is kept constant and the algorithm performs as a standard TDLMS algorithm with $\mu(n) = const$. With this constant value of $\mu(n)$ the algorithm would have a certain steady-state MSE denoted by $C(n)$. At the end of the test interval (after T iterations), the average of the squared error is computed. If the average of the squared error is larger than $C(n)$, then the step-size $\mu(n)$ is increased. This mean that the algorithm is far from the intermediate steady-state and in order to speed-up the convergence the global component of the step-size has to be increased. Otherwise, if the algorithm reaches the intermediate steady-state within the test interval of length T, the step-size is decreased in order to obtain a desired steady-state level. As we can see, the value of $C(n)$ is changed each time the step-size is changed and represents the MSE obtained if the algorithm would have a constant step-size until convergence.

The minimum value allowed for the global component $\mu(n)$ is μ_{min} and at the steady-state ($\mu(n) = \mu_{min}$) the algorithm perform as a TDLMS with constant step-size. Therefore the final level of the misadjustment is given by μ_{min}. The maximum value of $\mu(n)$ can be very close to μ_{max} but is always smaller than μ_{max}.

4 THEORETICAL ANALYSIS AND COEFFICIENTS SETUP

In this section we will discuss about how to choose the coefficients of the NCTDLMS algorithm. We will begin with the condition that ensure the stability of the algorithm. As we can see from (7), the global component $\mu(n)$ of the new algorithm is bounded by $\mu_{min} \leq \mu(n) < \mu_{max}$. Since the new algorithm performs as the standard TDLMS algorithm inside of each test interval, the stability analysis can be done using well known methods (see [6], [7] and the references therein). Following the derivations in [7] and making the usual assumptions, if $0 < \mu_{max} < \frac{2}{3tr[\mathbf{R}_{ss}]}$ then the NCTDLMS algorithm is convergent ($tr[*]$ means the trace of the matrix inside the brackets and $\mathbf{R}_{ss}$ is the autocorrelation matrix of the transform coefficients after power normalization).

In order to compute the value of $C(n)$ we will consider T consecutive iterations on which the overall step-size is constant. The intermediate values of $C(n)$ can be computed as follows (see [7]):

$$E[e^2(n)] = E\left[(y(n) - \hat{y}(n) + v(n))^2\right]$$
$$= E\left[\left(v(n) - \Delta\mathbf{w}^T(n)\mathbf{s}(n)\right)\left(v(n) - \Delta\mathbf{w}^T(n)\mathbf{s}(n)\right)\right]$$
$$= \sigma_v^2 + \Delta\mathbf{w}^T(n)\mathbf{R}_{ss}\Delta\mathbf{w}(n) = \sigma_v^2 + tr\left[\mathbf{R}_{ss}\mathbf{H}(n)\right] \tag{8}$$

where: $\Delta\mathbf{w}(n) = \mathbf{w}_{opt} - \mathbf{T}_N\hat{\mathbf{w}}(n)$ is the weight error vector, and $\mathbf{H}(n) = \Delta\mathbf{w}(n)\Delta\mathbf{w}(n)^T$ is the covariance matrix of the weight error vector.

Writing $\mathbf{R}_{ss} = Q\Lambda_{ss}Q^T$, where Λ_{ss} is a diagonal matrix having on the main diagonal the eigenvalues of $\mathbf{R}_{ss}$ and expanding (8), the intermediate values of $C(n)$ can be approximated as [7]:

$$C(n) = \left(1 + \frac{1}{2}\sum_{i=1}^{N}\mu(n)\lambda_i\right)\sigma_v^2 \tag{9}$$

where λ_i are the eigenvalues of $\mathbf{R}_{ss}$. Since $\mathbf{R}_{ss}$ is the autocorrelation matrix of the transform coefficients after power normalization, and if we assume that the decorrelation was ideal and also the power estimates are exact and furthermore the step-size is constant on the test interval of length T then (9) can be simplified as:

$$C(n) = \left(1 + \frac{1}{2}N\mu(n)\right)\sigma_v^2 \tag{10}$$

If the noise variance cannot be accurate estimated then a penalty term Φ can be introduced in (10) as follows:

$$C(n) = \Phi\left(1 + \frac{1}{2}N\mu(n)\right)\sigma_v^2 \tag{11}$$

where $\Phi \geq 1$ and close to 1.

The value of T in (7) has to be large enough such that the MSE can be approximated and also it has to be smaller than the convergence time for an TDLMS with fixed step-size μ_{min} such that a sufficient number of step-size update occur. In a large number of simulations, for different signal to noise ratio, the following selection $T = 10$ and $\alpha = 0.9$ gives good performances.

5 SIMULATIONS AND RESULTS

The simulations were conducted in the system identification framework. The unknown system has $N = 16$ coefficients. The compared algorithms were the standard LMS, VSS from [4], MVSS from [2], the plain TDLMS, the DCT-LMS from [7], TDVSLMS from [8] and the new NCTDLMS. The coefficients of the algorithms were chosen such that the steady-state misadjustments were the same. The learning curves are the results of 200 Monte-Carlo simulations. The transform used was the DCT tramsform. The input signal is given by the model:

$$x(n) = 1.79x(n-1) - 1.85x(n-2) + 1.27x(n-3) - 0.41x(n-4) + \psi(n);$$

where $\psi(n)$ is a Gaussian random variable with zero mean and variance $\sigma_\psi^2 = 0.14817$. The signal to noise ratio at the output of the unknown system was $SNR = 50$ dB. The comparison between the new algorithm and the time domain implementations are given in Fig. 3. The comparison between the new algorithm, TDLMS, DCT-LMS and TDVSLMS is given in Fig. 4. As expected, the new algorithm has better convergence speed than

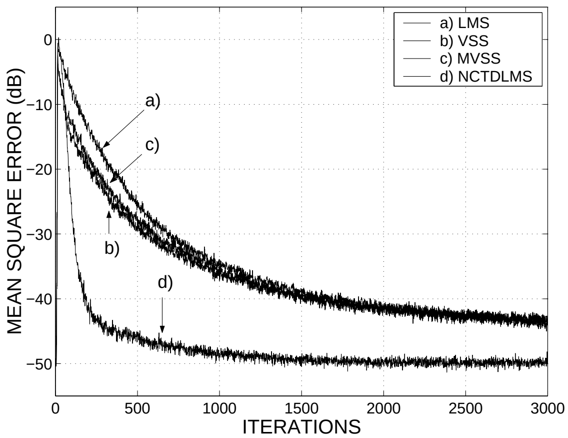

Figure 3: MSE for LMS, VSS, MVSS and NCTDLMS

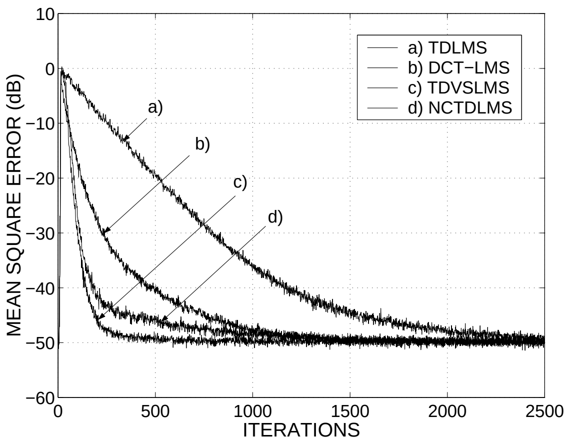

Figure 4: MSE for TDLMS, DCT-LMS, TDVSLMS and NCTDLMS

the time domain implementations for highly correlated input signals. From Fig. 4 we can see that the new algorithm converges faster than the standard TDLMS and DCT-LMS, and has performances close to TDVSLMS from [8]. In Fig. 5 the mean square error and the step-size of the new algorithm in non-stationary environment are shown. In this simulation the coefficients of the unknown system were constants until the iteration 3000 when an abrupt change in the sign of the coefficients occurs. The NCTDLMS algorithm react in this case by increasing the step-size.

6 CONCLUSIONS

In this paper we introduced a new algorithm called Noise Constrained Transform Domain LMS algorithm that belongs to the class of TDVSLMS algorithms introduced in [8]. The main features of the new algorithm are the low computational complexity and the fact that the tradeoff between a small error and a fast convergence is completely eliminated. The desired steady-state misadjustment can be obtained by appropriately chosen value of

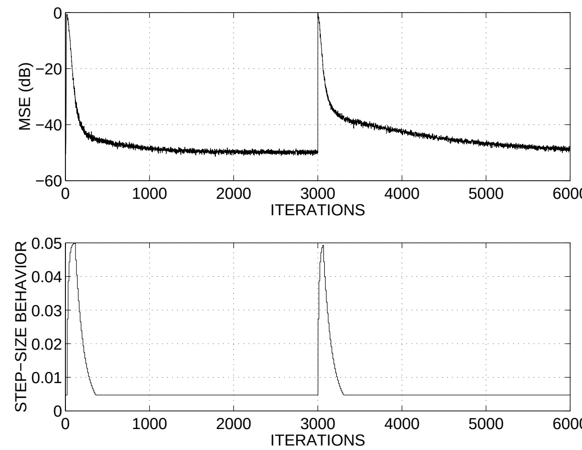

Figure 5: The MSE and the step-size in non-stationary case

μ_{min}, whereas the speed of convergence can be modified by the coefficient α.

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