An Approach to Direction of Arrival Estimation of Multiple Coherently Distributed Sources by Searching Maximum Eigenvalue

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Abstract: To circumvent the problem of performance degradation of direction of arrival (DOA) estimation due to angular spread, it was assumed impracticably that the parameterized shape of the angular distribution is known. Moreover, the disadvantage of most algorithms is the computational complexity as a multi-dimensional numerical search is necessary. By using a simple 1-D maximum eigenvalue search rather than resort to multidimensional parametric search, we propose a blind algorithm for DOA estimation of multiple coherently distributed sources, without prior knowledge about the function form of the angular distribution. We develop the algorithm in uniform linear array (ULA) condition, although our approach is not limited to ULA.

Key Words: distributed sources, maximum eigenvalue, DOA estimation

1、 INTRODUCTION

Most sensor array processing methods are based on the assumption that the signals are propagated from distinct point sources. However, it is just an approximation of the practical environment. Under many circumstances, the application of conventional high-resolution algorithms to estimate the direction of arrival (DOA) of the sources will have erroneous results [1-2].

Recently, previously posed 1-dinensional DOA estimation problems for point source are generalized to a multi-dimensional parameter estimation problem. Four types of distributed sources have been appeared in the literature, including coherently distributed (CD) source, incoherently (ICD) distributed sources, generalized array manifold (GAM) source and partially coherent (PCD) source [3-6]. In this paper, we consider the problem of DOA estimation of multiple CD sources.

Most previous methods are based on the knowledge of all possible generalized directional vectors that are depending on the function form of the angular distribution of the sources [4-7]. The unknown parameters of the angular distribution include the mean DOA and the angular spread parameter. In this paper, we introduce modulus constraint on generalized directional vectors determined by the extent parameters of the distributed sources. Then, without prior knowledge about the concrete function form of the angular signal density, a new high-resolution algorithm is developed for the mean DOA estimation of multiple CD sources. It is applicable for the situations where the distributed sources with different form of distribution coexist, in which case the methods proposed in [6,7] have difficulty in estimating the mean DOA of the distributed sources.

2、 SIGNAL MODEL

Consider a linear array of N equally spaced sensors with spacing d monitoring a wave field of q spatially distributed narrow-band sources in additive background noise. For simplicity, the sensors and sources are assumed on the same plane, although extension to three dimensional space is straightforward. The complex envelope representation of the output of the sensor k can be given by

$$x_k = \sum_{i=1}^{q} \int_{-p/2}^{p/2} a_k(q) s_i(q; j_i) dq + n_k \qquad (1)$$

$k = 1,2,..., N$, where $a_k(q)$ is the k-th element of the directional vector

$$a(q) = [1 \quad e^{-j2p(d/l)\sin q} \quad ... \quad e^{-j2p(M-1)(d/l)\sin q}]^T$$

$s_i(q; j_i)$ is the angular signal density of the i-th source in the direction q , j_i is the unknown parameter vector, and n_k is additive noise.

Assume that the signal components arriving from different angle within the extension are coherent, the angular signal density can be represented as

$$s_i(q; j_i) = g_i g_i(q; j_i) \qquad (2)$$

$i = 1, 2,..., q$, where g_i is a random variable and $g_i(q; j_i)$ is a complex-valued deterministic function of q , which is called the deterministic angular signal density and is unimodal and symmetric about the mean DOA q_i [6]. The output vector of the array can be expressed as

$$\boldsymbol{x} = \sum_{i=1}^{q} g_i \boldsymbol{b}_i(j_i) + \boldsymbol{n} \qquad (3)$$

where $\boldsymbol{x} = [x_1 \quad x_2 \quad ... \quad x_N]^T$, $\boldsymbol{n} = [n_1 \quad n_2 \quad ... \quad n_N]^T$ and

$$\boldsymbol{b}_i(j_i) = \int_{-p/2}^{p/2} \boldsymbol{a}(q) g_i(q; j_i) dq \qquad (4)$$

which is called generalized directional vector (GDV).

We will only consider spatially white noise and assume that signal and noise are uncorrelated from each other, g_i are zero mean complex random variables, g_i and g_k are not fully correlated for all $i \neq k$. The correlation matrix of the array is then given by $\boldsymbol{R} = E(\boldsymbol{x}\boldsymbol{x}^H) = \boldsymbol{BPB}^H + s_n \boldsymbol{I}$, where $\boldsymbol{B} = [\boldsymbol{b}_1(j_i), \boldsymbol{b}_2(j_i), ..., \boldsymbol{b}_q(j_i)]$, $\boldsymbol{P}$ is a correlation matrix with the *ik*-th component defined as $E(g_i g_j^*)$, s_n is the noise variance, I is unit matrix. The singular value decomposition of $\boldsymbol{R}$ is

$$\boldsymbol{R} = \boldsymbol{BPB}^H + s_n \boldsymbol{I} = \boldsymbol{U}_s L_s \boldsymbol{U}_s^H + \boldsymbol{U}_n L_n \boldsymbol{U}_n^H \tag{5}$$

where $L_s = diag(s_1, s_2 ..., s_q)$, $L_n = diag(s_n, s_n ..., s_n)$, $s_1 \geq s_2 \geq ... \geq s_q > s_n$, $\boldsymbol{U}_s$ and $\boldsymbol{U}_n$ are matrix of the column vectors which are singular vectors corresponding to singular values $s_1, s_2 ..., s_q$ and s_n, respectively.

If the function form of deterministic angular signal density $g_i(q; j_i)$ were known, the DOA's and angular spread parameters can be found from the following orthogonality property:

$$\boldsymbol{U}_n^H \boldsymbol{b}_i(j_i) = 0 \quad iff \quad \boldsymbol{b}_i(j_i) \in col(\boldsymbol{B}) \tag{6}$$

As a simple example, a CD source can be characterized by two parameters, mean DOA q_i and distribution parameter r_i. Two-dimensional parametric searching must be required to estimate these parameters.

Under the situations where the function form of the deterministic angular signal density is unknown, we are just interested in the mean DOA and regard extent parameters as redundancy that need not be estimated. Thus, multidimensional parametric search is not necessary. Since the parameterized directional vector $\boldsymbol{b}_i(j_i)$ is unknown, we propose a blind identification method to estimate the mean DOA of the CD sources.

The GDV $\boldsymbol{b}_i(j_i)$ can be expressed as

$$\boldsymbol{b}_i = diag(\boldsymbol{h}_i)\boldsymbol{a}(q_i) \tag{7}$$

$i = 1, 2, ..., q$, where vector $\boldsymbol{h}_i$ is given by

$$\boldsymbol{h}_i = \int_{-p/2}^{p/2} \boldsymbol{a}(q) g_i(q - q_i, j_i) dq \tag{8}$$

and is only dependent on the extent parameters. Note that all the angular distributed function appeared in the literatures are conjugated symmetric about the mean DOA, namely, $g_i(q - q_i, j_i) = g_i^*(q_i - q, j_i)$. It is easy to verify that $\boldsymbol{h}_i$ is a real vector from (8). Below, the new approach to DOA estimation of CD sources is base on this modulus constraint on the GDV $\boldsymbol{b}_i(j_i)$.

3. MAXIMUM EIGENVALUE SEARCHING METHOD

From (6), we can write $\boldsymbol{BPB}^H = \boldsymbol{U}_s(L_s - L_n)\boldsymbol{U}_s^H$. So that matrix B can be expressed as:

$$\boldsymbol{B} = \boldsymbol{U}_s \boldsymbol{W} \tag{9}$$

where $q \times q$ matrix $\boldsymbol{W}$ is nonsingular. Substituting for $\boldsymbol{B}$ in (9) with (7), we have

$$\begin{bmatrix} h_i(1) \\ h_i(2)e^{-j2p(d/l)\sin q_i} \\ \cdots \\ h_i(M)e^{-j2p(M-1)(d/l)\sin q_i} \end{bmatrix} = \begin{bmatrix} u_1^H \\ u_2^H \\ \cdots \\ u_M^H \end{bmatrix} w_i \tag{10}$$

where $h_i(1), h_i(2) ..., h_i(M)$ are the components of vector $\boldsymbol{h}_i$, $\boldsymbol{u}_1^H, \boldsymbol{u}_2^H, ..., \boldsymbol{u}_M^H$ are the row vectors of matrix $\boldsymbol{U}_s$, w_i is the i-th column of matrix $\boldsymbol{W}$, $i = 1, 2 ......, q$. Square the components on both sides of (10), we obtain:

$$\begin{bmatrix} (u_1^H w_i)^2 \\ (u_2^H w_i)^2 \\ \cdots \\ (u_M^H w_i)^2 \end{bmatrix} = \begin{bmatrix} h_i^2(1) \\ h_i^2(2)e^{-j2p(d/l)\sin q} \\ \cdots \\ h_i^2(M)e^{-j2p(M-1)(d/l)\sin q} \end{bmatrix} \tag{11}$$

Since the components of vector $\boldsymbol{h}_i$ are real or imaginary, (11) can be written as

$$\begin{bmatrix} (u_1^H w_i)^2 \\ (u_2^H w_i)^2 \\ \cdots \\ (u_M^H w_i)^2 \end{bmatrix} = \pm \begin{bmatrix} \left|(u_1^H w_i)^2\right| \\ \left|(u_2^H w_i)^2\right| e^{-j2p(d/l)\sin q_i} \\ \cdots \\ \left|(u_M^H w_i)^2\right| e^{-j2p(d/l)\sin q_i} \end{bmatrix} \tag{12}$$

where the sign is positive for the conjugate symmetry case, and negative for the anti-conjugate symmetry case. By left multiplying by $\boldsymbol{U}_s^H$, we get

$$\boldsymbol{w}_i = \boldsymbol{V}(q_i)\boldsymbol{w}_i^* \tag{13}$$

where $\boldsymbol{V}(q_i)$ is a $q \times q$ symmetric matrix, and is given by

$$\boldsymbol{V}(q_i) = \boldsymbol{U}_s^H F(q_i)\boldsymbol{U}_s^* \tag{14}$$

where $F(q_i) = diag(\boldsymbol{a}((q_i))^2$.

Equation (13) is not in a convenient form for computation as the right side is dependent on w_i^*. However, it indicates the condition that the stationary point must satisfy and also suggests the iterative procedure for compute w_i as

$$\boldsymbol{w}(k+1) = \boldsymbol{V}(q)\boldsymbol{w}^*(k) \tag{15}$$

where subscription is omitted as $\boldsymbol{V}(q)$ is dependent on unknown DOA q_i. We can get iterative solutions through 1-D DOA search. It is suffered from the problem associated with iterative algorithm like local convergence or proper initialization. Since the iterative process (15) is determined with matrix $\boldsymbol{V}(q)$ in nature, we can obtain DOA estimation without iterative computation. From (13), we have

$$\boldsymbol{w}_i = \boldsymbol{G}(q_i)\boldsymbol{w}_i \tag{16}$$

where $\boldsymbol{G}(q_i)$ is a matrix of $q \times q$ given by

$$\boldsymbol{G}(q_i) = \boldsymbol{V}(q_i)\boldsymbol{V}^H(q_i) \tag{17}$$

Note that w_i is the eigenvector of $\boldsymbol{G}(q_i)$ at the mean DOA of distributed sources, with the corresponding eigenvalue equals one.

Claim: The eigenvalues of matrix $\boldsymbol{G}(q)$ defined by (17) and (14) are non-negative, and not larger than 1.

Proof: From (17), we can see that $\boldsymbol{G}(q)$ is positive definite Hermitian matrix. So the eigenvalues of $\boldsymbol{G}(q)$ are non-negative. Denote $l(q)$ the maximum eigencalue of matrix $\boldsymbol{G}(q)$, then $l(q)=\max_{x\neq 0}\frac{\boldsymbol{x}^H\boldsymbol{G}(q)\boldsymbol{x}}{\boldsymbol{x}^H\boldsymbol{x}}=\max_{x^Hx=1}\left\|\boldsymbol{V}^H(q)\boldsymbol{x}\right\|_2^2=$ $\max_{x^Hx=1}\left\|\boldsymbol{U}_s{}^TF^H(q)\boldsymbol{U}_s\boldsymbol{x}\right\|_2^2=\max_{y^Hy=1}\left\|\boldsymbol{U}_s{}^T\boldsymbol{y}\right\|_2^2$, where $\boldsymbol{y}=F^H(q)\boldsymbol{U}_s\boldsymbol{x}$, and it is easy to verify that $\boldsymbol{y}^H\boldsymbol{y}=1$ from $\boldsymbol{x}^H\boldsymbol{x}=1$. Note that $\boldsymbol{I}=\boldsymbol{U}_s{}^*\boldsymbol{U}_s+\boldsymbol{U}_n{}^*\boldsymbol{U}_n$, which yields $\left\|\boldsymbol{U}_s{}^T\boldsymbol{y}\right\|_2^2=\boldsymbol{y}^H\boldsymbol{U}_s{}^*\boldsymbol{U}_s{}^T\boldsymbol{y}$ $=\boldsymbol{y}^H(\boldsymbol{I}-\boldsymbol{U}_n{}^*\boldsymbol{U}_n{}^T)\boldsymbol{y}=1-\boldsymbol{y}^H\boldsymbol{U}_n{}^*\boldsymbol{U}_n{}^T\boldsymbol{y}\leq 1$. So that $l(q)\leq 1$.

This property can be applied to obtain DOA estimation without iterative computation. Computing the maximum eigenvalue, denoted as $l(q)$, of $\boldsymbol{G}(q)$ at every possible DOA, the mean DOA of the distributed sources can be estimated by peak value searching of $l(q)$. The DOA algorithm for the coherently distributed sources is summarized in the following.

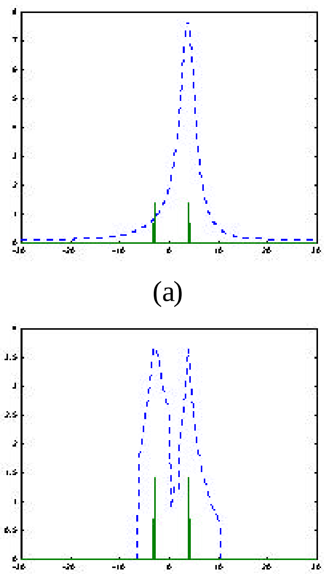
1. Given the sample correlation matrix, find the signal subspace vectors, which are the columns of matrix $\boldsymbol{U}_s$.
2. Using the estimated signal subspace vectors, calculate matrix $\boldsymbol{V}(q)=\boldsymbol{U}_s{}^HF(q)\boldsymbol{U}_s{}^*$ and $\boldsymbol{G}(q)=\boldsymbol{V}(q)\boldsymbol{V}^*(q)$, where q is DOA searching point in $[-p/2, p/2]$.
3. Compute $l(q)$, which is the maximum eigenvalue of $\boldsymbol{G}(q)$, and calculate the generalized spectrum
$$f(q)=-\log(\left|1-l(q)\right|) \qquad (18)$$
4. Estimate the mean DOA of the distributed sources by peak value searching of $f(q)$.

This algorithm requires a SVD of a complex matrix of size $N\times N$ in step 1 and M SVD's of a complex matrix of size $q\times q$ in step 3, where N is the sensor number of the array, M is the number of the DOA searching grids, q is the number of the distributed sources. Due to the term $e^{-j2p(d/l)\sin q}$ in $F(q)$ and regardlessness of the distributed parameters, there probably exists ambiguity of the DOA estimation even if the array manifold $\boldsymbol{a}(q)$ is unambiguous. Taking no account of this ambiguity, the algorithm is applicable for non-uniform array, as no limitation on the geometry of the array is assumed here. Unlike the methods proposed in [6,7], which are based on the knowledge of functional form of the angular signal density, the new method can deal with the complex situations where the sources with different angular signal density coexist.

4. SIMULATION RESULTS

In this section, we will test the validity of the algorithm by three computer simulation examples. Here, the CD sources are corrupted by complex white Gaussian noise with zero mean and variance s_n. The SNR used in the examples is defined as $10\log(1/\sigma_n^2)$. Taking no account of ambiguity, we only consider a linear array of N equally spaced sensors with spacing $d=l/2$ and the mean DOA's of the distributed sources locate in $[-30^\circ, 30^\circ]$. The DOA searching step spacing is 0.1°.

Example 1: The angular signal density of two distributed sources is $g_i(q,j_i)=\frac{1}{1-r_ie^{-j(q-q_i)}}$, i=1, 2, where unknowns are $j_1=[q_1,r_1]=[-3.0^\circ,0.6]$ and $j_2=[q_2,r_2]=[-4.0^\circ,0.9]$. In this example, we assume N=8, SNR=15dB,the number of snapshot is 100. The MUSIC spectrum is shown in fig. 1(a): only one peak is obtained. The generalized spectrum (18) is shown in fig. 1(b): two peaks around the mean DOA of the distributed sources are obtained.



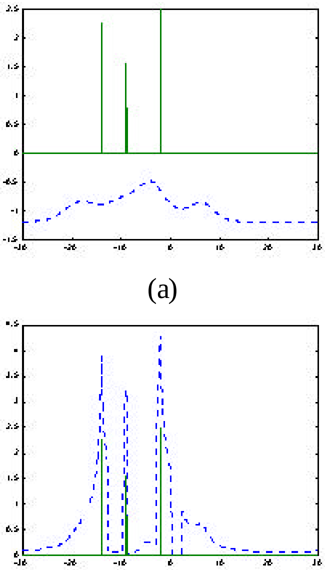
(a)

(b)

Fig. 1 Dash line: (a) MUSIC spectrum and (b) generalized spatial spectrum (18) in Example 1. Solid line: true DOA location.

Example 2: The angular signal density of three distributed sources is $g_i(q,j_i)=\{^{1,|q-q_i|\leq r_i}_{0,|q-q_i|>r_i}$, i=1,2,3, where unknown parameters are $j_1=[q_1,r_1]=[-14,6^\circ]$ and $j_2=[q_2,r_2]=[-9^\circ,8^\circ]$, $j_3=[q_3,r_3]=[-2^\circ,10^\circ]$. In this example, we assume N=16, SNR=10dB,the number of snapshot is 100. The MUSIC spectrum is shown in fig. 2(a): erroneous results are obtained. The generalized spectrum (18) is shown in fig. 2(b): three peaks around the mean DOA of the distributed sources are obtained.

Compared with that in [6,7], which assumed that the angular signal density $g_i(q;j_i)$ is known with unknown parameters and 2-D parametric searching was required, our method does not used this prior information and simple 1-D DOA searching is required. Moreover, our method is applicable for the situations where the sources with different form of distribution coexist, in which case the methods proposed in [6,7] have difficulty in estimating the mean DOA of the distributed sources.



(a)

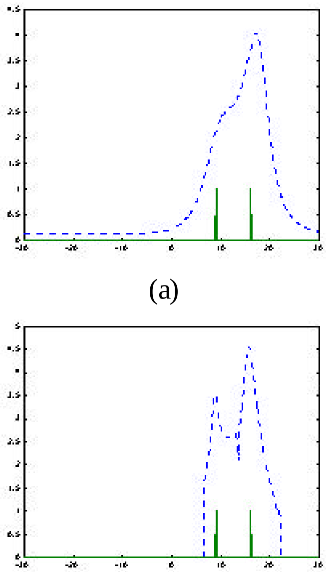
(b)

Fig. 2 Dash line: (a) MUSIC spectrum and (b) generalized spatial spectrum (18) in Example 2. Solid line: true DOA location.

Example 3: The angular signal density of two distributed sources is different, one is $g_1(q,j_1)=\begin{cases}1, & |q-q_1|\le r_1\\ 0, & |q-q_{i1}|>r_1\end{cases}$, another is $g_2(q,j_2)=\dfrac{1}{1-r_2e^{-j(q-q_2)}}$, where unknown parameters $j_1=[q_1,r_1]=[16.0^{\circ},6.0^{\circ}]$ and $j_2=[q_2,r_2]=[9.0^{\circ},0.8]$. In this example, we assume N=8, SNR=15dB,the number of snapshot is 100. The MUSIC spectrum is shown in fig. 3(a): only one peak is obtained. The generalized spectrum (18) is shown in fig. 3(b): two peaks around the mean DOA of the distributed sources are obtained.

5. SUMMARY

Through simple 1-D maximum eigenvalue searching, a blind identification algorithm is proposed to estimate the mean DOA of the coherently distributed sources. The angular signal density is assumed conjugated symmetric about the mean DOA. It is different with the previous methods, which are based on the knowledge of function form of the angular signal density and require multidimensional parametric searching. Moreover, The method is applicable for the situations where the distributed sources with different form of distributed function coexist. We develop the algorithm in the uniform linear array condition, although our approach is not limited to ULA.



(a)

(b)

Fig. 3 Dash line: (a) MUSIC spectrum and (b) generalized spatial spectrum (18) in Example 3. Solid line: true DOA location.

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