

# Linearization Technique for Stereophonic Reproduction Systems with Nonlinearity

*Yoshinobu Kajikawa and Yasuo Nomura*

Dept. of Electronics, Faculty of Engineering, Kansai University,  
3-3-35, Yamate-cho, Suita-shi,  
Osaka 564-8680, Japan  
Tel: +81-6-6368-0828; fax: +81-6-6330-3770  
e-mail: [kaji@joho.densi.kansai-u.ac.jp](mailto:kaji@joho.densi.kansai-u.ac.jp)

## ABSTRACT

In this paper, we propose a novel linearization technique for stereophonic reproduction systems with nonlinearity by using the MINT and Volterra filters. In the proposed technique, the linearization is achieved by incorporating Volterra filters into the MINT, which can realize exact linear inverse filtering. The linearization performance of the proposed technique is consequently very high. The proposed technique can simultaneously linearize two loudspeaker systems in the stereophonic reproduction systems, also. On the other hand, the conventional linearization technique for monaural reproduction systems cannot realize exact linear inverse filtering. The linearization performance consequently deteriorates remarkably. Simulation results demonstrate that the proposed technique has about 20dB higher performance than the conventional one. The proposed technique also has smaller computational complexity than the conventional one.

## 1 INTRODUCTION

Recently, digitization of audio systems has been progressing. The digitization has reduced some distortions occurring in the transmission paths significantly. Consequently, the sound quality has been improved considerably. However, loudspeaker systems, which are a human interface in the digital audio systems, have a lot of distortions, especially, nonlinear distortions. The performance of the whole digital audio systems consequently deteriorates. Hence, the compensation of the nonlinear distortions (linearization of loudspeakers) is a very important issue in the digital audio systems.

The linearization can be achieved by using a Volterra filter [1, 2], which identifies the nonlinearity of a target loudspeaker system, and a linear inverse filter, which compensates the linear distortion [3, 4, 5]. One of some factors influencing the linearization performance is the estimation accuracy of the Volterra filter. However, this estimation accuracy can be made high by using an identification method employing multi-sinusoidal waves [5]. Another factor is the design accuracy of the linear inverse filter to compensate linear distortions. In other

words, whether exact linear inverse filtering can be realized influences the linearization performance. However, the exact linear inverse filtering cannot be realized because loudspeaker systems have nonminimum phases. In this case, only an approximate inverse filtering is realized. If the approximate accuracy is low, the linearization performance deteriorates remarkably. Moreover, when used for two loudspeaker systems in stereophonic reproduction systems, the conventional linearization technique is separately introduced into those two loudspeaker systems. We therefore propose a novel linearization technique for stereophonic reproduction systems. In the proposed technique, we use the MINT [6], which can realize an exact linear inverse of a target acoustic system. The linearization performance is consequently very high. Moreover, the proposed technique can simultaneously linearize two loudspeaker systems in the stereophonic reproduction systems.

## 2 Conventional Linearization Technique and Its Problem

Figure 1 shows a block diagram in case of introducing the conventional linearization technique into stereophonic reproduction systems. In Fig. 1,  $D_{1L}(z)$ ,  $D_{1R}(z)$ ,  $D_{2R}(z_1, z_2)$ , and  $D_{2L}(z_1, z_2)$  represent the transfer functions of the first- and second-order Volterra kernels of the left and right loudspeaker systems, respectively.  $\hat{D}_{2L}(z_1, z_2)$  and  $\hat{D}_{2R}(z_1, z_2)$  are Volterra filters to model the second-order Volterra kernels of the loudspeaker systems, and  $D_{1L}^{-1}(z)$  and  $D_{1R}^{-1}(z)$ , which are linear inverse filters of  $D_{1L}(z)$  and  $D_{1R}(z)$ , are designed so as to satisfy the following condition, respectively.

$$D_{1i}(z)D_{1i}^{-1}(z) = z^{-\Delta}, \quad i \text{ is } L \text{ or } R \quad (1)$$

The second-order nonlinear transfer function of the whole system is consequently represented by the following equation.

$$\begin{aligned} & D_{2i}(z_1, z_2)z^{-\Delta} - D_{1i}(z)D_{1i}^{-1}(z)\hat{D}_{2i}(z_1, z_2) \\ &= D_{2i}(z_1, z_2)z^{-\Delta} - z^{-\Delta}\hat{D}_{2i}(z_1, z_2) \\ &= \{D_{2i}(z_1, z_2) - \hat{D}_{2i}(z_1, z_2)\}z^{-\Delta} \end{aligned} \quad (2)$$

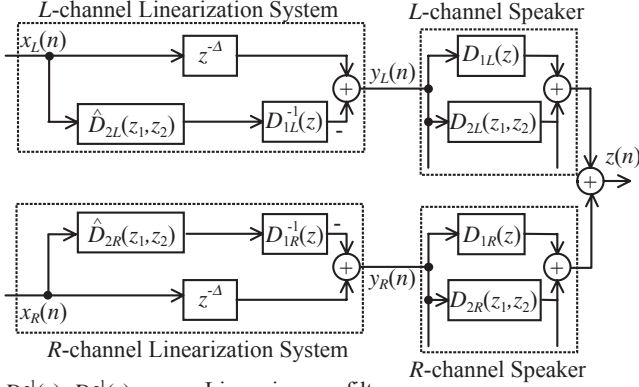


Figure 1: Block diagram of the conventional linearization system.

If  $\hat{D}_{2i}(z_1, z_2)$  is equal to  $D_{2i}(z_1, z_2)$  of the loudspeaker system and  $D_{1i}^{-1}(z)$  is designed so as to satisfy the condition shown in (1), the linearization systems can completely compensate the second-order nonlinear distortions on the two loudspeaker systems. The high accuracy  $\hat{D}_{2i}(z_1, z_2)$  can be obtained if narrow band signals are used to model  $D_{2i}(z_1, z_2)$ . On the contrary,  $D_{1i}^{-1}(z)$  to satisfy the condition of (1) can exist if and only if  $D_{1i}(z)$  is a minimum phase function. However, the acoustical transfer function  $D_{1i}(z)$  is generally considered to be a nonminimum phase function. Hence, only approximate inverse filters are obtained. It is therefore very difficult to compensate (cancel)  $D_{2i}(z_1, z_2)$  completely because  $D_{1i}^{-1}(z)$  does not satisfy (1). Accordingly, the performance of linearization systems is greatly influenced by whether exact linear inverse filtering can be realized.

### 3 Linearization Technique Using the MINT

#### 3.1 MINT [6]

In this section, we explain the MINT (Multiple-input/output INverse-filtering Theorem), which can realize an exact linear inverse of a target acoustic system. Consider the acoustic system shown in Fig. 2. In Fig. 2, the transfer function  $D_{1L}(z)$  from loudspeaker  $S_L$  to receiving point  $C$  is defined by

$$D_{1L}(z) = z^{-u}d_{1L}(z) \quad (3)$$

where  $z^{-u}$  is the time delay between  $S_L$  and  $C$ ,  $d_{1L}(z)$  the  $M$ 'th order polynomial of  $z^{-1}$ , which represents reflection sound effects. The transfer function  $D_{1R}(z)$  from loudspeaker  $S_R$  to receiving point  $C$  is also defined

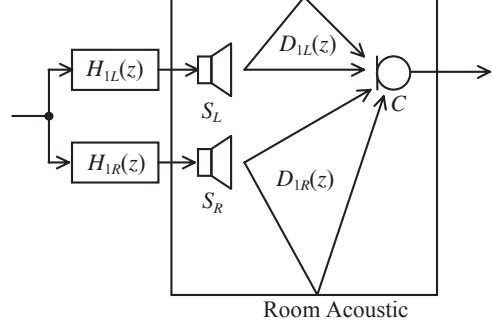


Figure 2: Sound field inverse filtering using the MINT.

by

$$D_{1R}(z) = z^{-(u+w)}d_{1R}(z) \quad (4)$$

where  $z^{-(u+w)}$  is the time delay between  $S_R$  and  $C$ ,  $d_{1R}(z)$  the  $N$ 'th order polynomial of  $z^{-1}$ . To realize inverse filtering of the system,  $H_{1L}(z)$  and  $H_{1R}(z)$  must satisfy the expression

$$1 = d_{1L}(z)H_{1L}(z) + z^{-w}d_{1R}(z)H_{1R}(z) \quad (5)$$

This relationship is called Diophantine equation. The solutions for this equation exist if and only if  $d_{1L}(z)$  and  $z^{-w}d_{1R}(z)$  are relatively prime (in other words, do not have any common zero in the  $z$ -plane). The solutions is expressed by

$$\begin{aligned} H_{1L}(z) &= H_{1L,min}(z) + z^{-w}d_{1R}(z)Q(z) \\ H_{1R}(z) &= H_{1R,min}(z) - d_{1L}(z)Q(z) \end{aligned}$$

where  $Q(z)$  is an arbitrary polynomial.  $H_{1L,min}(z)$  and  $H_{1R,min}(z)$  are the only pair of the minimum order solution that satisfies (5) and the orders have the following relation.

$$\begin{aligned} \deg H_{1L,min}(z) &< \deg z^{-w}d_{1R}(z) = N + w \\ \deg H_{1R,min}(z) &< \deg d_{1L}(z) = M \end{aligned}$$

The property of the Diophantine equation is not concerned with whether  $d_{1L}(z)$  and  $z^{-w}d_{1R}(z)$  are nonminimum phase functions. If some symmetrical positions of loudspeakers and a microphone are avoided,  $d_{1L}(z)$  and  $z^{-w}d_{1R}(z)$  does not have a common zero. Hence, exact inverse filtering is realized.

Next, we describe the computation of  $H_{1L,min}(z)$  and  $H_{1R,min}(z)$ . Figure 3 shows a system arrangement to obtain  $H_{1L,min}(z)$  and  $H_{1R,min}(z)$  by using adaptive filters. First, the transfer functions of  $D_{1L}(z)$  and  $D_{1R}(z)$  are modeled beforehand. Next, as shown in Fig. 3, two adaptive filters  $\hat{H}_{1L,n}(z)$ ,  $\hat{H}_{1R,n}(z)$  are connected to the outputs of the modeled transfer functions. Finally, the coefficients of the two adaptive filters are updated as minimizing the following error signal.

$$e(n) = x(n - \Delta) - \{\hat{H}_{1L,n}(z)D_{1L}(z) + \hat{H}_{1R,n}(z)D_{1R}(z)\} \quad (6)$$

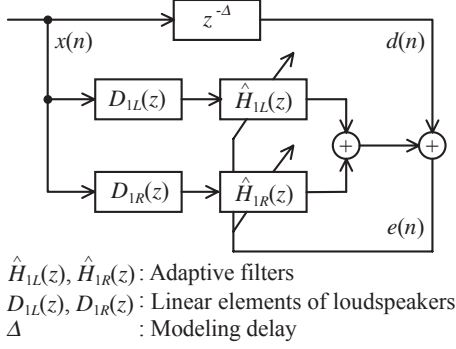


Figure 3: Block diagram of identification method for  $H_{1L,min}(z)$  and  $H_{1R,min}(z)$  by using adaptive filters.

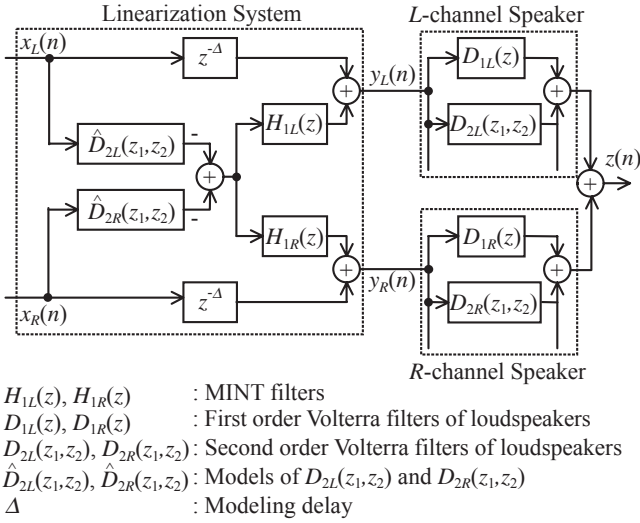


Figure 4: Block diagram of the proposed linearization system using the MINT.

With the above procedure, you can obtain the filters to realize exact linear inverse filtering.

### 3.2 Linearization Technique for Stereophonic Reproduction Systems

In this section, we introduce a system arrangement to apply the MINT to the linearization system for stereophonic reproduction systems.

Figure 4 shows the block diagram of the proposed system. In Fig. 4,  $H_{1L}(z)$  and  $H_{1R}(z)$  are FIR filters in the MINT as explained in the previous section. The relation of these filters is shown in the following equation again.

$$D_{1L}(z)H_{1L}(z) + D_{1R}(z)H_{1R}(z) = z^{-\Delta} \quad (7)$$

Hence, the second-order nonlinear property of the whole

Table 1: Simulation conditions in the proposed and the conventional systems.

Sampling frequency	44100[Hz]
Tap length of $D_{1L}(z)$	128
Tap length of $D_{1R}(z)$	128
Tap length of $H_{1L}(z)$	127
Tap length of $H_{1R}(z)$	127
Tap length of $D_{1L}^{-1}(z)$	512
Tap length of $D_{1R}^{-1}(z)$	512
Tap length of 2nd-order models	128
Delay of the proposed system	64
Delay of the conventional system	256

system in Fig. 4 is represented by

$$\begin{aligned}
 & \{D_{2L}(z_1, z_2) + D_{2R}(z_1, z_2)\}z^{-\Delta} \\
 & - \{D_{1L}(z)H_{1L}(z) + D_{1R}(z)H_{1R}(z)\} \\
 & \cdot \{\hat{D}_{2L}(z_1, z_2) + \hat{D}_{2R}(z_1, z_2)\} \\
 & = z^{-\Delta}\{D_{2L}(z_1, z_2) + D_{2R}(z_1, z_2) \\
 & - \hat{D}_{2L}(z_1, z_2) - \hat{D}_{2R}(z_1, z_2)\} \quad (8)
 \end{aligned}$$

If

$$D_{2L}(z_1, z_2) = \hat{D}_{2L}(z_1, z_2), D_{2R}(z_1, z_2) = \hat{D}_{2R}(z_1, z_2), \quad (9)$$

that is, the second-order Volterra kernels of two loudspeakers are identified accurately, the nonlinear distortion can be compensated completely.

## 4 Simulation Results

To verify the applicability of the proposed technique, some simulations were conducted. In the simulations, we employed the characteristics of actual loudspeakers, which were measured by the identification method in Ref. [5]. In the simulation, sinusoids with different frequency ( $f_1$  or  $f_2$ ) are input to two loudspeakers separately, then the two output spectra before and after compensation are compared. Table 1 shows the simulation conditions. Figures 5~7 show the output spectra before compensation, after compensation by the conventional technique, and after compensation by the proposed technique, respectively. In these figures, 0[dB] means the maximum linear output level.

It can be seen from these figures that the proposed technique can considerably reduce the second-order nonlinear distortions compared with the conventional one. This is due to the difference in the design accuracy of inverse filters. On the other hand, the proposed method generates larger 3rd and 4th nonlinear distortions than the conventional one. However, the level is fully low and the effect to output signals can be consequently neglected in actual environments.

Compare the computational complexity and system delay of the conventional and the proposed linearization

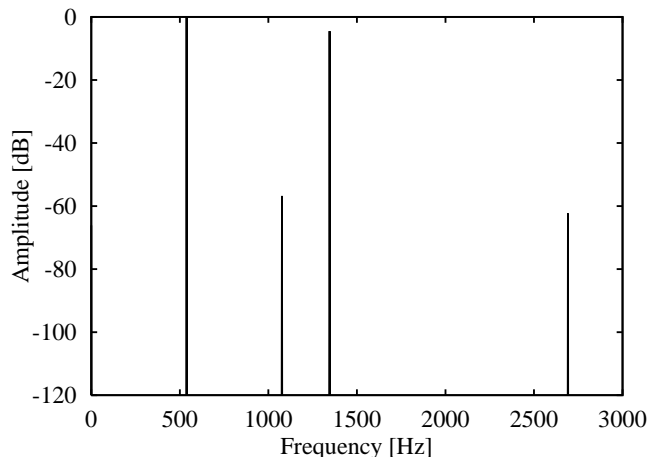


Figure 5: Output spectrum before compensating the 2nd-order nonlinear distortion ( $f_1 = 538.33[Hz]$ ,  $f_2 = 1345.83[Hz]$ ).

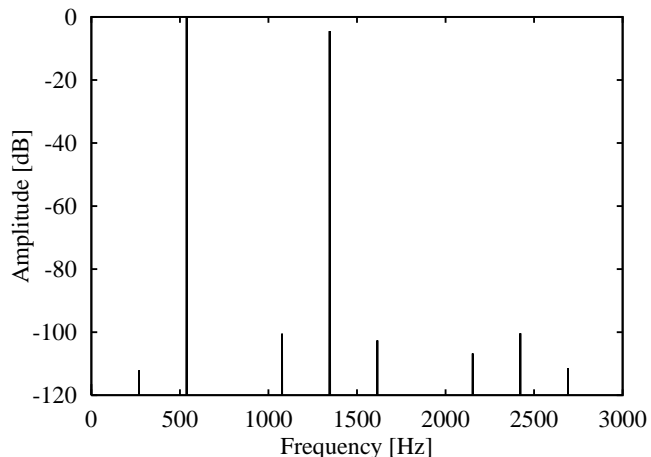


Figure 7: Output spectrum after compensating the 2nd-order nonlinear distortion by using the proposed method ( $f_1 = 538.33[Hz]$ ,  $f_2 = 1345.83[Hz]$ ).

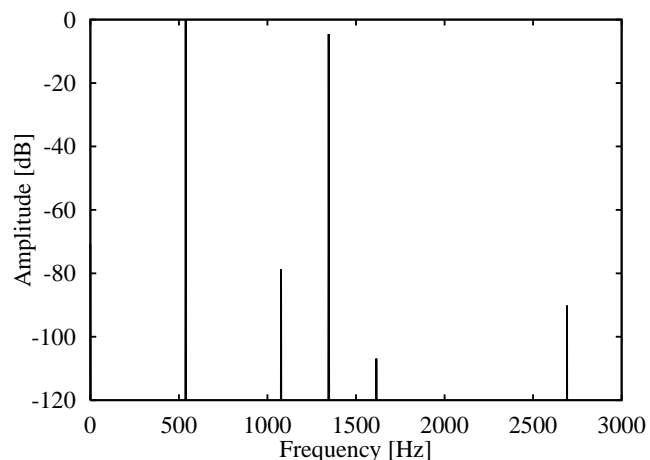


Figure 6: Output spectrum after compensating the 2nd-order nonlinear distortion by using the conventional method ( $f_1 = 538.33[Hz]$ ,  $f_2 = 1345.83[Hz]$ ).

systems. From table 1, the proposed system has 1/4 computational complexity and system delay of the conventional one. Hence, the proposed technique can be easily implemented.

## 5 Conclusions

In this paper, we have proposed a novel linearization system for stereophonic reproduction systems. Since exact inverse filtering can be realized by using the MINT, the proposed technique has higher linearization ability than the conventional one. Moreover, the whole computational complexity of the proposed technique is 1/4 as large as that of the conventional one and the system delay is also small.

## References

- [1] V. J. Mathews, "Adaptive polynomial filters", *IEEE Signal Processing Magazine*, vol. 8, no. 3, pp. 10-26, Jul. 1991.
- [2] M. Schetzen, *Volterra and Wiener theories of nonlinear systems*, Krieger, Florida, 1989.
- [3] A. J. M. Kaizer, "Modeling of the nonlinear response of an electrodynamic loudspeaker by a Volterra series expansion", *Journal of Audio Engineering Society*, vol. 35, no. 6, pp. 421-432, Jun. 1987.
- [4] W. A. Frank, "An efficient approximation to the quadratic Volterra filter and its application in real-time loudspeaker linearization", *Signal Processing*, vol. 45, pp. 97-113, 1995.
- [5] M. Tsujikawa, T. Shiozaki, Y. Kajikawa, and Y. Nomura, "Identification and elimination of second-order nonlinear distortion of loudspeaker systems using Volterra filter", *Proc. of the 2000 IEEE International Symposium on Circuits and Systems*, Geneva, Switzerland, Vol. V, pp. 249-252, May. 2000.
- [6] M. Miyoshi and Y. Kanada, "Inverse Filtering of Room Acoustics", *IEEE Trans. on Acoustics, Speech, and Signal Processing*, vol. 36, no. 2, pp. 145-152, Feb. 1988.