BLIND SEPARATION OF UNCORRELATED SOURCES VIA PRINCIPAL COMPONENT ANALYSIS OF OBSERVATIONS FOR A SYMMETRIC MIXING MATRIX

Deniz Erdogmus[1], Kenneth E. Hild II[1], Jose C. Principe[1], Luis Vielva[2]
[1] Computational NeuroEngineering Laboratory, NEB 454, University of Florida, Gainesville, Florida 32611, USA
[2] Grupo de Tratamiento Avanzado de Señal, Universidad de Cantabria, Santander, Spain
e-mail: [deniz ,hildk, principe]@cnel.ufl.edu, luis@dicom.unican.es

ABSTRACT

A well-known fact in blind deconvolution is that if the unknown source signal is white (temporally) and the unknown channel filter is minimum phase, it is possible to determine the inverse filter (equalizer) by evaluating simply the power spectral density (PSD) of the received signal. For blind source separation, however, a similar special case, equivalent to the situation in blind deconvolution, is not reported. In this paper, we identify the special conditions for which the solution of the blind source separation problem can be identified using only second order statistics of the observed mixtures. In this special case, the equivalent of minimum phase channel turns out to be a symmetric mixing matrix, and the equivalent of temporally white input signal translates to uncorrelated source signals. A fast-converging and robust on-line blind source separation algorithm using a recently introduced principal components analysis (PCA) algorithm named SIPEX-G is also presented and its performance is evaluated in simulations of source separation.

1 INTRODUCTION

In contemporary signal processing applications, blind approaches play a central role. Ranging from communications to speech and image processing, these techniques proved themselves valuable in numerous problems [1,2]. Blind deconvolution (BD) and blind source separation (BSS) are two such approaches investigated in detail especially in the last two decades. These two problems are so similar in nature that it is often possible to form analogies and even utilize algorithms interchangeably to solve these problems [3].

Blind deconvolution problem is schematically shown in Fig. 1, where both the channel impulse response and the input signal are unknown. It is assumed that we have only some knowledge on the statistical properties of the input signal. Common assumptions include stationary-white or iid (independent and identically distributed) as source models [4,5]. The unknown channel $\boldsymbol{h}$ is linear time-invariant.

In the blind deconvolution literature, it is a well-known fact that for a temporally white input and a minimum phase channel, the solution, which is the inverse filter $\boldsymbol{w}$, can be uniquely determined from only the second order statistics of the observations. Since the source is white, its power spectral density (PSD) is flat and the magnitude spectrum of the channel can be determined using the PSD of the observed signal. Using the knowledge that the channel is minimum phase, its transfer function can be determined as the inverse filter [2,3]. For all other situations, higher order statistics about the data need to be considered to arrive at a solution [2,3].

Instantaneous blind source separation is similar to the above problem in nature. The difference is that there are multiple sources,

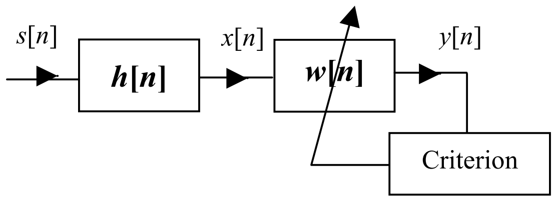

Figure 1. Schematic diagram of blind deconvolution

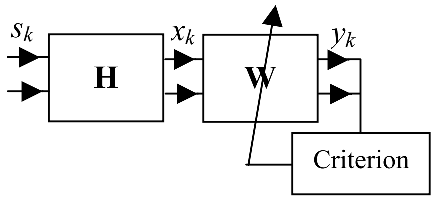

Figure 2. Schematic diagram of blind source separation

which are commonly assumed to be independent to allow utilization of independent component analysis (ICA) algorithms [1,6]. The instantaneous BSS (or the ICA) problem is schematically shown in Fig. 2. It is assumed that the (stationary) source signals are unknown as well as the mixing matrix.

Despite this structural similarity and the fact that ICA algorithms are used for BD as well as BSS, the question of whether there is a special case of BSS requiring only second order statistics of the observed signals has not been addressed. In this paper, we will show that, there is a special case of the mixing matrix H where the wide sense stationary (WSS), *uncorrelated* sources (not necessarily independent) can be separated using only second order statistics. Recall that in the case of non-stationary source signals, algorithms employing only second order statistics can readily be used [7]. In these situations, due to the nonstationarity of the signal and time-invariance of the mixing matrix, decorrelation of the mixtures at multiple lags (of the cross-correlation function) allows the computation of the solution. The procedures that will be presented here, however, will work even if the source signals are stationary, whereas nonstationarity of the sources is crucial to the success of the above mentioned second order methods in BSS.

Our investigation points out that in situations where the mixing matrix is symmetric, the sought property occurs, i.e. the instantaneous blind source separation problem can be solved using second order statistics only (even in the stationary case). Realistic situations where a nearly symmetric mixture may arise include, for example, nearly symmetric physical settings in audio mixtures. Another situation is the *cleaning* of old documents, where ink from a page leaks to the opposite face of that page and also to the other neighboring page in time. Intuitively, one expects the gain of leakage to be symmetric in both directions, yielding a symmetric mixing matrix.

The organization of this paper is as follows: First we present the theoretical background for source separation in the symmetric matrix situation. Then we provide a PCA-based source separation algorithm and the SIPEX-G PCA algorithm. Next, simulation results are provided to verify the performance of the proposed approach. In the conclusions, we discuss relevant applications and possible future research.

2 SYMMETRIC MIXING MATRIX IN BSS

Consider the instantaneous BSS problem depicted in Fig. 2. We have the mixtures as a linear combination of the sources, i.e. $x=Hs$. Typically, the sources are assumed WSS and independent. In contrast, assume the sources are merely uncorrelated, but still WSS. In addition, assume that the mixing matrix H is symmetric. From linear algebra, we know that if H is symmetric, its eigenvalues and eigenvectors are real. Let Q_h be the orthonormal eigenvector matrix of H, ordered in descending order of their corresponding eigenvalues. Let Λ_h be the corresponding diagonal eigenvalue matrix. Then we have $HQ_h=Q_h\Lambda_h$, therefore $H=Q_h\Lambda_hQ_h^T$.

A solution to the source separation problem is given by the inverse of the mixing matrix, $H^{-1}=Q_h^{-T}\Lambda_h^{-1}Q_h^{-1}= Q_h\Lambda_h^{-1}Q_h^T$. The gain factor and permutation indeterminacy of the ICA problem still applies here. Therefore, we can assume the sources are all unit power, without loss of generality. Combining this with the assumption that they are uncorrelated, we get the source vector covariance matrix as $\Sigma_s=I$. The covariance matrix of x is then $\Sigma_x=H\Sigma_sH^T= HH^T$.

Now consider $HH^T=Q_h\Lambda_hQ_h^T Q_h\Lambda_hQ_h^T= Q_h\Lambda_h^2Q_h^T$. From this, we get $(HH^T)Q_h=Q_h(\Lambda_h^2)$. Since $\Sigma_x=HH^T$, we conclude that the eigenvectors of H and the eigenvectors of Σ_x are the same. The corresponding eigenvalues of Σ_x, however, are squared eigenvalues of H. The following lemma summarizes this result.

Lemma 1. Let $s,x \in \Re^{nx1}$ be two random vectors related by the equation $x=Hs$. Assume H is symmetric and the covariance matrix of s is $\Sigma_s=I$. If (Q_h, Λ_h) are the eigenvector-eigenvalue matrices for H, and (Q_x, Λ_x) are the eigenvector-eigenvalue matrices for Σ_s, then these matrices are related to each other by $Q_x=Q_h$, and $\Lambda_x= \Lambda_h^2$.
Proof: In the preceding text.

This result has significant theoretical and practical implications. The primary theoretical and intuitively appealing implication of this result is that the symmetric mixing matrix situation in BSS is the counterpart of the minimum phase mixing filter in BD. In accordance with the structural variations between the two problems, the temporal whiteness assumption on the source signal of the BD problem is replaced by spatial whiteness (uncorrelatedness of sources). On the other hand, the primary practical implication is that it is possible to (approximately) solve the blind source separation problem using only second order statistics for WSS, uncorrelated sources, when the mixing matrix is symmetric (close to being symmetric).

3 SOURCE SEPARATION ALGORITHM

In the previous section, we have established the fact that under certain conditions it is possible to determine the sources blindly using only second order statistics, namely principal components analysis (PCA). Since the mixing matrix and the mixture covariance matrix has the same eigenvectors, applying PCA on the mixture, x, one can determine the eigenvectors of H. The eigenvalues can also be estimated from the variances of the principle components according to the relationship given in Lemma 1. The algorithm, then can be summarized as follows:

BSS Algorithm (Off-Line or On-Line):

- Estimate $Q_x=Q_h$ using any PCA algorithm (perhaps on-line) from samples of x
- Estimate Λ_x from the variances of the principle components
- Calculate $\Lambda_h^{-1}=\Lambda_x^{-1/2}$
- Evaluate $H^{-1}=Q_h\Lambda_h^{-1}Q_h^T$, the separation matrix

This algorithm can be used to separate WSS, uncorrelated sources, even if these sources all have Gaussian densities as opposed to ICA, where at most one source is allowed to exhibit a Gaussian distribution [1,6]. As a consequence, WSS-independent sources and non-WSS-uncorrelated (also independent) sources can also be separated. If the mixing matrix is time varying, then a fast and robust on-line PCA algorithm can be employed with a forgetting factor to track the inverse of the time-varying mixture. This brings us to the choice of the PCA algorithm.

4 THE SIPEX-G ALGORITHM

This fast-converging, robust PCA algorithm has been recently proposed and is shown to outperform benchmark PCA algorithms including Sanger's rule, APEX, and Xu's LMSER [8]. Conventional PCA algorithms approach the problem as a constrained optimization problem, where Oja's first order weight normalization update rule and deflation are common practice to obtain a solution to this *constrained* optimization problem [9,10]. On the contrary, SIPEX-G parameterizes the PCA weight matrix in terms of Givens rotation angles thus guaranteeing that the PCA matrix is orthonormal at all times and the optimization task is *unconstrained*. This approach enables the SIPEX-G algorithm to outperform the conventional methods by restricting the search space of the weight matrix to the set of orthonormal matrices.

Assuming a PCA network of the form $y=Rx$, where R is an orthonormal matrix parameterized in terms of the Givens angles as

$$R = \prod_{p=1}^{n-1} \prod_{q=p+1}^{n} R^{pq} \tag{1}$$

where each Givens rotation matrix in the plane of the i^{th} and j^{th} axes is denoted by R^{ij}, and is given by an identity matrix whose four entries at the intersection of i^{th} and j^{th} rows with i^{th} and j^{th} columns are modified as follows: The $(i,i)^{th}$ and $(j,j)^{th}$ entries are $cos\theta_{ij}$, and the $(i,j)^{th}$ and $(j,i)^{th}$ entries are $-sin\theta_{ij}$ and $sin\theta_{ij}$, respectively [8]. In order to determine the principle components, the following cost function is used.

$$J = \sum_{o=1}^{n-1} \gamma_o Var(y_o) \tag{2}$$

Here y_o is the o^{th} output and $\gamma_1>\gamma_2>...>\gamma_{n-1}>0$ are gain factors to increase convergence speed in case of large eigenspread and close eigenvalues.

SIPEX-G Algorithm (On-Line):

Step 1. Initialize Givens angles, $\Theta = [\theta_{pq}]$

Step 2. Initialize input covariance estimate R_x.

Step 3. In non-WSS environments, update the covariance estimate using the recursion

$$R_x(k) = (1-\alpha)R_x(k-1) + \alpha x_k x_k^T \tag{3}$$

and in WSS environments, use the consistent

$$R_x(k) = \frac{k-1}{k} R_x(k-1) + \frac{1}{k} x_k x_k^T \tag{4}$$

Unbiased covariance estimates could also be used.

Step 4. Calculate the gradient of the cost function with respect to the Givens angles using

$$\frac{\partial J}{\partial \theta_{kl}} = \sum_{o=1}^{n-1} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(R_{oi} \frac{\partial R_{oj}}{\partial \theta_{kl}} + \frac{\partial R_{oi}}{\partial \theta_{kl}} R_{oj} \right) R_{x,ij} \tag{5}$$

where R_{ij} and $R_{x,ij}$ are the $(i,j)^{th}$ entries of the corresponding matrices.

Step 5. Update Givens angles using gradient ascent.

$$\Theta(k+1) = \Theta(k) + \eta \ \frac{\partial J}{\partial \Theta} \tag{6}$$

Step 6. Go back to step 3 and continue until convergence.

As mentioned before, SIPEX-G is a robust and fast-converging on-line PCA algorithm that easily outperforms conventional approaches.

5 SIMULATIONS

In this section, we will evaluate the performance of the proposed blind source separation algorithm for symmetric and nearly symmetric matrices. Recall that the method can successfully separate uncorrelated sources (without any restrictions on their density functions), if they are mixed with an almost symmetric matrix. However, for comparison purposes, we will restrict the sources to be independent and non-Gaussian.

As a case study, we perform a series of Monte Carlo runs with two independent, Laplacian distributed sources, where the mixing matrix is given by $H+H_{pert}$ where H_{pert} is a 2x2 random perturbation matrix whose entries have an average power set to a percentage of the symmetric part of the matrix, H. For each value of the percentage in the range 0-100%, 40 Monte Carlo runs are performed using random perturbation matrices. In each run, on-line separation of the two sources is realized using the PCA-based method and the benchmark ICA algorithm, Infomax (with whitening and Amari's natural gradient) [6,11]. For the PCA algorithm, which uses SIPEX-G, the Givens angles were initialized to all zeros (corresponding to an initial eigenvector matrix estimate of I_n). The average of signal-to-interference-ratios (SIR), as defined in (7), of the last 1000 iterations of each 10000-sample run was taken as the performance of the algorithm for that specific simulation. The mean performance of the Monte Carlo runs is plotted versus the percentage of perturbation in Fig. 3. As expected, when the mixing matrix diverges from being symmetric, the performance of the PCA-based separation algorithm degrades and Infomax is unaffected by the structure of the mixing matrix. However, up to a level of approximately 10% perturbation (in each off-diagonal entry), the PCA separation algorithm maintains its superior performance up to a perturbation level of approximately 25% in each entry. PCA is still sufficiently successful (>20dB) even at close to 100% perturbation, since in audio applications, usually 20dB and over corresponds to acceptable separation as far as human hearing capabilities are concerned [12].

Consider the physical setting in Fig. 4, with two speakers and a listener (two observations, one from each ear), which is located at the center. Assuming that the sound pressure decreases inversely proportional to the distance from the source and the mixing is instantaneous (a more realistic model would be convolutive), each entry of the mixing matrix will be proportional to $1/d$, where d is the distance between the corresponding source-sensor pair. Suppose that one source is at a distance d from one of the sensors (right ear in this case), then the shaded area in Fig. 4 (which is centered at the left ear) corresponds to the region in which the second source could be located, in order for the performance of the proposed PCA-based algorithm to surpass that of Infomax.

In terms of a physical setting where sound pressure decreases inversely proportional to the distance between the source and the sensor, the 10% perturbation of the matrix corresponds approximately to an 18% perturbation in distance. As depicted in Fig. 4, the distance of the second speaker may vary in the shaded region and the mixing matrix may deviate from being symmetric, yet the PCA-based separation algorithm would still perform better.

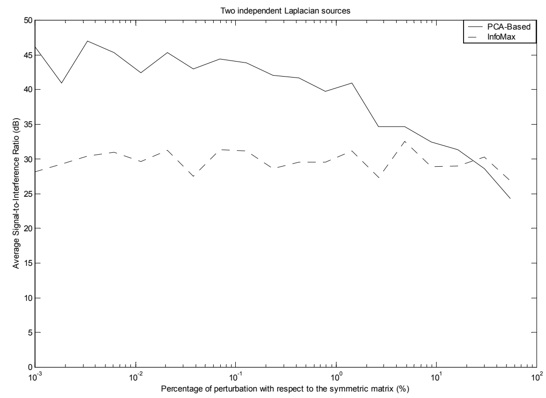

Figure 3. Average SIR(dB) vs percentage perturbation from a symmetric matrix

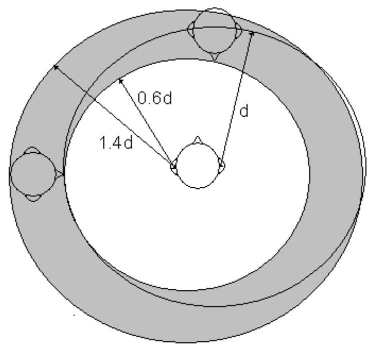

Figure 4. Illustration of permissible deviation from a symmetric mixing matrix translated to a physical environment in a two-source-two-sensor case.

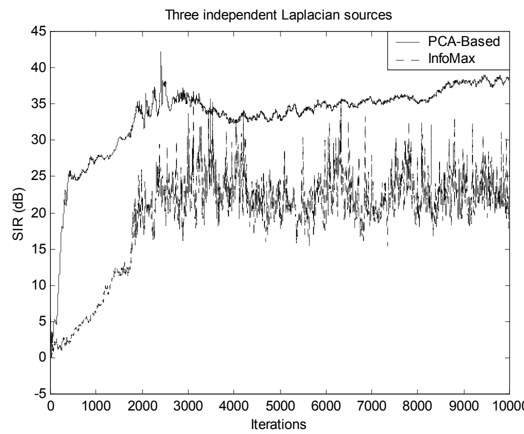

Figure 5. A sample convergence plot for PCA-based and InfoMax algorithms for the separation of three Laplacian sources

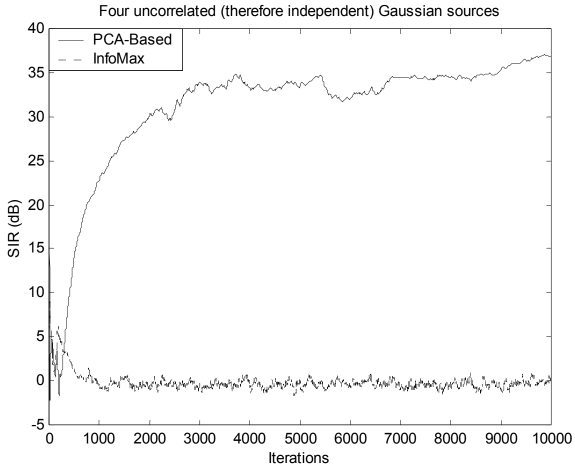

Figure 6. A sample convergence plot for PCA-based and InfoMax algorithms for the separation of four Gaussian sources

The SIR performance measure we have utilized is defined as follows. Given a mixing matrix A and a demixing matrix estimate B, the overall matrix is BA, which should be as close as possible to a permutation times a diagonal scaling matrix. SIR measures this distance between BA and a permuted scaling matrix by

$$SIR(dB) = \frac{1}{n}\sum_{i=1}^{n} 10\log_{10}\frac{(\max q_i)^2}{q_i^T q_i - (\max q_i)^2} \qquad (7)$$

where q_i is the i^{th} row of BA.

The final superior performance of the PCA-based separation when the necessary assumptions are met is not its only appeal. It also converges faster, in general, compared to the ICA methods that do not *constrain* their separation matrices in regard to the extra information about the mixing process. Usually, these methods undertake a first-whiten-then-separate approach. Clearly, in this case, the convergence of the whitening and the separation matrices are simultaneous, since the separation matrix (which is a rotation) is immediately determined from the eigenvectors of the whitetning matrix. This fast-convergence is demonstrated in Fig. 5, in a three-source separation problem. Infomax uses the whitening solution generated by SIPEX-G also.

It was mentioned in the theoretical section that this PCA-based separation method only requires that the sources are uncorrelated and no specific distribution type is excluded as in ICA. As a final example, we present a typical convergence plot from the separation of four uncorrelated Gaussian sources for a symmetric mixing matrix. The result is shown in Fig. 6. As expected, Infomax fails to separate these sources as they are all Gaussian distributed, however, the proposed approach successfully finds the inverse of the mixing matrix to achieve a very good SIR of approximately 35dB.

6 CONCLUSIONS

Temporal white input and minimum phase channel is a special case in blind deconvolution, where only second order statistics are sufficient to obtain a unique solution. In this paper, we have demonstrated that such a special case also exists for blind source separation (instantaneous mixing), where spatially white, i.e. uncorrelated, source signals and a symmetric mixing matrix replaces their counterparts in blind deconvolution.

Starting from basic assumptions, we have established the limitations and capabilities of the proposed approach; given a symmetric mixing matrix, it has been demonstrated that with this method, WSS / non-WSS and uncorrelated / independent sources can be separated successfully using only PCA in both time-invariant and time-varying mixing matrix situations. In addition, it was shown that separation of sources irrespective of their densities is possible with this method even if more than one source is Gaussian.

Finally, we proposed an on-line blind source separation algorithm based on a robust and fast-converging on-line PCA algorithm called SIPEX-G and demonstrated its performance by Monte Carlo runs on the separation of two and three independent Laplacian sources. Simulation results demonstrated that even if the mixing matrix is not perfectly symmetric, the algorithm tolerates this situation and successfully determines the inverse of the mixing matrix in that it provides a more than sufficient accuracy. Comparisons with Bell-Sejnowski's Infomax revealed that for a range of mixing matrices that are nearly symmetric, the proposed PCA approach is both faster and better in terms of final signal-to-interference ratio attained.

This work provides interesting intuitions and raises questions about the relationship between minimum phase filters and symmetric matrices. It also opens an alternative door to solving symmetric and minimum-phase multi-channel blind deconvolution problems (also called convolutive BSS) using only second order statistics (PCA and PSD). Further research in these promising topics may result in efficient and effective approaches to solve the convolutive blind source separation problem in the above-mentioned particular situations.

Acknowledgments: This work is partially supported by NSF grant ECS-9900394.

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