STATISTICALLY OPTIMAL CONTROL OF A BLENDING - TYPE FILTER WITH ITS APPLICATION TO OLD FILM RESTORATION

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ABSTRACT

The blending-type filter, widely used for several image-processing tasks, is composed of multiple component filters, and it gives out a dividing value between the component filters' outputs in a certain ratio, referred to as a blending coefficient, that can be defined as a function of some observable parameters measured for a given input image. The propriety of the blending-type filter depends on how to define the blending coefficient as a function of the observable parameters. So far, to define the blending-coefficient function, heuristic design methods such as a steepest descent method using a training sequence of data are often employed. On the contrary, we present a new well-grounded design method that optimizes the blending coefficient statistically as a function of the observable parameters by taking into account their statistics and the posterior probability model of the events for which the component filters are prepared. Our new design method provides the theoretical framework that nicely explains the propriety of the blending-type filter to the practical image-processing task. We apply our new design method to the statistically optimal control of the blending-type blotch repair filter for old film restoration.

1. INTRODUCTION

As a filter for several image-processing tasks such as noise removal, image restoration, scan conversion and so on, blending-type filters have been widely used [1], [2]. The blending-type filter is composed of multiple component filters each of which is prepared for its corresponding particular statistical event, and as an output it gives a dividing value between the component filters' outputs in a certain ratio, referred to as a blending coefficient, that can be defined as a function of some observable parameters measured for a given input image. The propriety of the blending-type filter depends on how to define the blending coefficient as a function of the observable parameters. So far, in most cases, to define the blending-coefficient function, either an inductive steepest descent design method using a training sequence of data or a priori/heuristic rules are employed [2].

Instead of these previous somewhat baseless design methods, in this paper we deductively derive a new well-grounded design method that optimizes the blending coefficient statistically as a function of the observable parameters by taking into account the statistics of the observable parameters and the posterior probability model of the events for which the component filters are prepared.

We apply our new design method to the real image-processing task of the old film restoration, i.e. the film blotch removal. Previously, we have presented the blotch removal method, in which first blotch distortion regions are softly characterized and then repaired by a blending-type filtering approach [3]. As a method for characterizing blotches softly we have developed the approach of the robust spatio-temporal continuity analysis. We apply our new design method to the statistically optimal design of the blending coefficient of the blending-type blotch repair filter, and give the theoretical foundation of the blending filter's applicability to the blotch repair according to a statistical model of blotch distortions.

2. STATISTICAL OPTIMAL CONTROL

This paper concentrates on the simplest blending-type filter that is composed of two component filters, and presents a new design method to optimize its blending coefficient statistically. Applying the following discussion repeatedly to the blending-type filter composed of more than two component filters, we will be able to derive the more general design method for it. In the simplest case of the two component filters, we assume that one component filter is prepared for the statistical event $\boldsymbol{E}$, whereas the other component filter is prepared for its complement event $\boldsymbol{E}^C$. In addition, we denote the set of some observable parameters by $\boldsymbol{O}$.

2.1 Posterior Probability Model of the Event

Let denote the joint probability that the event $\boldsymbol{E}$ occurs and at the same time the observable parameters $\boldsymbol{O}$ are observed by $P(\boldsymbol{E},\boldsymbol{O})$; on the contrary, let denote the joint probability that the event $\boldsymbol{E}^C$ occurs and at the same time the observable parameters $\boldsymbol{O}$ are observed by $P(\boldsymbol{E}^C,\boldsymbol{O})$. The joint probabilities $P(\boldsymbol{E},\boldsymbol{O})$, $P(\boldsymbol{E}^C,\boldsymbol{O})$ are given by

$$P(\boldsymbol{E},\boldsymbol{O}) = P(\boldsymbol{O})\cdot P(\boldsymbol{E}|\boldsymbol{O}) \;;\; P(\boldsymbol{E}^C,\boldsymbol{O}) = P(\boldsymbol{O})\cdot P(\boldsymbol{E}^C|\boldsymbol{O}) \qquad (1)$$

where the posterior probabilities $P(\boldsymbol{E}|\boldsymbol{O})$, $P(\boldsymbol{E}^C|\boldsymbol{O})$ can be inferred from a typical training sequence of image data containing the occurrence of the event $\boldsymbol{E}$. From a typical training sequence we measure the joint frequency distributions $N(\boldsymbol{E},\boldsymbol{O})$, $N(\boldsymbol{E}^C,\boldsymbol{O})$ of the event $\boldsymbol{E}/\boldsymbol{E}^C$ and the observable parameters $\boldsymbol{O}$, and then we estimate the posterior probabilities, $P(\boldsymbol{E}|\boldsymbol{O})$, $P(\boldsymbol{E}^C|\boldsymbol{O})$, according to the equations:

$$P(\boldsymbol{E}|\boldsymbol{O}) = \frac{N(\boldsymbol{E},\boldsymbol{O})}{N(\boldsymbol{E},\boldsymbol{O}) + N(\boldsymbol{E}^C,\boldsymbol{O})}$$
$$P(\boldsymbol{E}^C|\boldsymbol{O}) = \frac{N(\boldsymbol{E}^C,\boldsymbol{O})}{N(\boldsymbol{E},\boldsymbol{O}) + N(\boldsymbol{E}^C,\boldsymbol{O})} \qquad (2)$$

2.2 Statistically Optimal Design Method

The propriety of the adaptive control for the blending-type filter depends on how to define the blending coefficient μ as a function of the observable parameters $\boldsymbol{O}$. This paper deductively derives a new design method to optimize the blending coefficient μ statistically as a function of the observable parameters $\boldsymbol{O}$ by taking the posterior probability model { $P(\boldsymbol{E}|\boldsymbol{O})$, $P(\boldsymbol{E}^C|\boldsymbol{O})$ } of the statistical event $\boldsymbol{E}/\boldsymbol{E}^C$ into considerations.

In this section, we employ the generalized expression of the blending-type filter. We assume that the blending-type filter is composed of the two component filters and as an output it gives a dividing value between the two component filters' outputs in a ratio of μ to 1-μ, as follows:

$$
\begin{aligned}
& I_O = \mu \cdot I_{E^C} + (1-\mu)\cdot I_E \\
& \quad , \ I_E \quad ; \textit{component filter for the event } \boldsymbol{E} \\
& \quad \ \ I_{E^C} \quad ; \textit{component filter for the event } \boldsymbol{E^C} \\
& \quad \ \ 0 \le \mu \le 1
\end{aligned} \tag{3}
$$

The ideal control of the blending coefficient μ for the filter is as follows: in the case of the occurrence of the event $\boldsymbol{E}$, the filter's output will be $I_{\boldsymbol{E}}$, whereas in the case of the occurrence of the complement event $\boldsymbol{E}^C$, it will be $I_{\boldsymbol{E}^c}$. On the other hand, as an output the blending-type filter gives a dividing value between $I_{\boldsymbol{E}}$ and $I_{\boldsymbol{E}^c}$. The deviation of the actual blending-type filter's output I_o from the ideal output I_d is given by

$$
\begin{aligned}
& e_{\boldsymbol{E};\boldsymbol{O}} = I_E - I_O = \mu \cdot \Delta_f \qquad ; \textit{for the event } \boldsymbol{E} \\
& e_{\boldsymbol{E^C};\boldsymbol{O}} = I_{E^C} - I_O = (\mu - 1)\cdot \Delta_f \quad ; \textit{for the event } \boldsymbol{E^C} \\
& \quad , \ \Delta_f = I_E - I_{E^C}
\end{aligned} \tag{4}
$$

Introducing the γ-th power cost function $\psi(e)$,

$$
\psi(e) = \frac{1}{\gamma}\cdot |e|^\gamma \quad , \ \gamma > 1 \tag{5}
$$

we estimate the average cost T_μ. Assuming that in equation 4 the probability distribution of the difference Δ_f between the two component filters' outputs depends on the event index $\{\boldsymbol{E}, \boldsymbol{E}^C\}$ and the observable parameters $\boldsymbol{O}$, the average cost T_μ will be estimated by

$$
\begin{aligned}
T_\mu = & \sum_{\boldsymbol{O}} P(\boldsymbol{O})\cdot \Bigg[\lambda \cdot \sum_{\Delta_f} \Psi\big[(\mu-1)\cdot\Delta_f\big]\cdot P\big(\Delta_f \,|\, \boldsymbol{E}^C, \boldsymbol{O}\big) \\
& + (1\text{-}\lambda)\cdot \sum_{\Delta_f} \Psi\big[\mu\cdot\Delta_f\big]\cdot P\big(\Delta_f \,|\, \boldsymbol{E}, \boldsymbol{O}\big) \Bigg] \\
& , \lambda \triangleq P\big(\boldsymbol{E}^C \,|\, \boldsymbol{O}\big)
\end{aligned} \tag{6}
$$

where λ denotes the posterior probability $P(\boldsymbol{E}^C|\mathbf{O})$. Minimizing the function $\boldsymbol{\Phi}(\mu|\boldsymbol{O})$ within the bracket [*] in the above equation

$$
\begin{aligned}
\Phi(\mu|\boldsymbol{O}) = & \lambda \cdot \sum_{\Delta_f} \Psi\big[(\mu-1)\cdot\Delta_f\big]\cdot P\big(\Delta_f \,|\, \boldsymbol{E}^C, \boldsymbol{O}\big) \\
& + (1\text{-}\lambda)\cdot \sum_{\Delta_f} \Psi\big[\mu\cdot\Delta_f\big]\cdot P\big(\Delta_f \,|\, \boldsymbol{E}, \boldsymbol{O}\big)
\end{aligned} \tag{7}
$$

separately with respect to the variable μ under the condition that the values of the observable parameters $\boldsymbol{O}$ are given, we can determine the blending coefficient μ as a function $\mu(\lambda)$ as follows:

$$
\begin{aligned}
& \mu(\lambda) = \frac{\left[\alpha_\gamma \cdot \lambda\right]^{\frac{1}{\gamma-1}}}{\left[\alpha_\gamma \cdot \lambda\right]^{\frac{1}{\gamma-1}} + \left[1-\lambda\right]^{\frac{1}{\gamma-1}}} \\
& \quad , \ \alpha_\gamma = \mathbf{E}\Big[|\Delta_f|^\gamma \,|\, \boldsymbol{E}^C, \boldsymbol{O}\Big] \Big/ \mathbf{E}\Big[|\Delta_f|^\gamma \,|\, \boldsymbol{E}, \boldsymbol{O}\Big]
\end{aligned} \tag{8}
$$

where the operator $\mathbf{E}[*|*]$ means conditional expectation and the coefficient α_γ is dependent on both the observable parameters $\boldsymbol{O}$ and the characteristics of the component filters. If the γ-th moment of $|\Delta_f|$ in the case of the occurrence of the event $\boldsymbol{E}$ is much greater than that in the case of the occurrence of the complement event $\boldsymbol{E}^C$, then the coefficient α_γ will be much smaller than 1. Moreover, we can acquire the coefficient α_γ and the posterior probability $\lambda = P(\boldsymbol{E}^C|\mathbf{O})$ from a training image sequence containing the occurrence of the event $\boldsymbol{E}$.

2.3 Forte of the Design Method

Unlike the above-mentioned derivation of the blending coefficient $\mu(\boldsymbol{O})$, let's assume that the probability distribution of the difference Δ_f between the two component filters' outputs depends only on the observable parameters $\boldsymbol{O}$, but does not depend on the event index $\{\boldsymbol{E}, \boldsymbol{E}^C\}$. This assumption leads to the state that the coefficient α_γ in equation 8 is fixed at 1.0 irrespective of the observable parameters $\boldsymbol{O}$. In this case, the characteristics of the component filters have no effect on the control rule for the blending-type filter, and hence satisfactory filtering performance cannot be expected. To achieve satisfactory filtering performance, we should assume that the probability distribution of the difference Δ_f between the two component filters' outputs depends on not only the observable parameters $\boldsymbol{O}$, but also the event index $\{\boldsymbol{E}, \boldsymbol{E}^C\}$.

As for the γ-th power cost function $\psi(e) = |e|^\gamma / \gamma$ (γ>1) of equation 5, the usual squared cost function, i.e. γ=2, will not necessarily provide an excellent filtering performance. To design an excellent blending coefficient function, we optimize the value of γ.

3. APPLICATION TO FILM BLOTCH REMOVAL

Blotch areas are characterized by the following properties [4]. First, blotch areas build temporal discontinuity; blotch areas randomly appear in an image sequence, and they are rarely or never located in the same place over successive image frames. Second, blotch areas are coherent image areas with almost the same brightness intensity. Between the two properties, the first property referred to as the temporal discontinuity is more important for the blotch characterization. However, if we do not take image motion into account, it will confuse the blotch detection. Hence, we should discriminate temporal discontinuity due to blotches from that due to image motion. To solve this problem, we have presented an approach of robust local analysis of spatio-temporal discontinuity, and a practical method for the blotch removal. Our blotch removal method first characterizes blotch distortion regions softly and then repairs them with a blending-type filter [3]. In this paper, we apply our new design method to the statistically optimal control of the blending-type blotch repair filter, and gives the theoretical foundation of the blending filters' applicability to the blotch repair according to a statistical model of blotch distortions.

3.1 Blotch Removal Method [3]

Our blotch removal method is composed of the two serial separate stages: the analysis stage and the restoration stage. Because of limitations of space, we describe only its outline.

3.1.1 Analysis Stage

(1) Performing the robust continuity analysis, for each examined pixel $I(x,y,t)$ located at (x,y,t), we determine the smoothest spatio-temporal direction (k', l'), and compute a continuity parameter $C(x,y,t)$ defined as follows:

$$C(x,y,t) = \min\left[\left(d_f(k',l'),\ d_b(k',l')\right) \right]$$
$$, d_f(k',l') = I(x+k', y+l', t+1) - I(x,y,t) \quad (9)$$
$$d_b(k',l') = I(x,y,t) - I(x-k', y-l', t-1)$$

The continuity parameter C means a first-order derivative property of a brightness change around a pixel indexed by (x,y,t).
(2) For each pixel denoted by (x,y,t), instead of performing the hard blotch-detection, we estimate the blending coefficient $\mu(x,y,t)$ used for the blending-type blotch-repair filter in the successive restoration stage, as a function of the continuity parameter $C(x,y,t)$ as follows:

$$\mu(x,y,t) \triangleq \mu\left(C(x,y,t)\right) \ , \quad 0 \le \mu(x,y,t) \le 1 \quad (10)$$

The blending coefficient $\mu(x,y,t)$ has a value within the range between 0 and 1. The blending coefficient $\mu(x,y,t)$ is defined in a way that the less the examined pixel is corrupted, the larger the blending coefficient μ will be close to 1. The function $\mu(C)$ can be designed in advance using a training sequence. Previously we have presented an heuristic steepest descent design method using a training sequence of data [3]. Instead of this heuristic design method, in this paper we apply our new well-grounded design method to the statistical optimization of the function $\mu(C)$ with taking into account all the statistics of blotch distortions and the continuity parameter C.

3.1.2 Restoration Stage

We restore the missing data due to blotches with the blending-type filter whose response we control according to the blending coefficient $\mu(x,y,t)$ as follows:

$$I_O(x,y,t) = \mu(x,y,t) \times I(x,y,t) + [1-\mu(x,y,t)] \times M(x,y,t) \quad (11)$$

where $I_o(x,y,t)$ denotes the blending-type filter's output. The blending-type filter puts out an intermediate value between the anisotropic smoothing filter's output $M(x,y,t)$ and the input brightness $I(x,y,t)$ according to the blending coefficient $\mu(x,y,t)$. As the anisotropic smoothing filter $M(x,y,t)$, we employ the 3-point temporal median filter, whose response is defined as the median brightness among the examined pixel indexed by (x,y,t) and the two pixels designated on the preceding and succeeding frames by the selected smoothest direction (k', l'). The input-output property of 3-point temporal median is defined by

$$M(x,y,t) = \text{Median}\left\{ \begin{array}{c} I(x-k', y-l', t-1) \\ I(x,y,t) \\ I(x+k', y+l', t+1) \end{array} \right\} \quad (12)$$

3.2 Application of the Design Method

In the above-mentioned blotch removal method, the continuity parameter C corresponds to the observable parameter $\boldsymbol{O}=\{C\}$, and the occurrence of a blotch pixel ($\boldsymbol{B}$) corresponds to the occurrence of the statistical event $\boldsymbol{E}$ for which the component filter $I_E=M$ is prepared. The complementary event $\boldsymbol{E}^C$ means that a pixel is not corrupted by a blotch, and its corresponding component filter is an identity filter $I_{E^C}=I$. Under the above setting, we can directly apply our new design method to the blending-type blotch-repair filter.

4. EXPERIMENTAL SIMULATIONS

Using a training sequence of images contaminated by artificial blotches, we design the rule of the adaptive control for the blending coefficient μ as a function of the observable parameter $\{C\}$. Moreover, we applied the blending-type filter equipped with the designed control rule to the real blotch-restoration problem.

Figure 1 shows the designed control rules for several values of the parameter γ in the γ-th power cost function of equation 5. When the parameter γ of the γ-th power cost function is set within the range from $1.0+\varepsilon$ ($\varepsilon>0$) to 2.0, the designed control rule shows a proper soft-blending function that achieves the desirable restoration performance. If we decrease the parameter γ under 1.1, i.e. very close to 1.0, then the designed control rule gradually approaches a simple threshold function, which results in a switchover-type filter based on the hard blotch-detection. Thus we can also optimize the threshold value for a switchover-type filter statistically. On the contrary, if we increase the parameter γ over 2.0, the designed control rule approaches a uniform function of a constant value of 0.5, which is meaningless for the control rule for the blending-type filter.

Figure 2 shows the blotch-repaired results provided by the blending-type filter equipped with the designed control rule for several values of the parameter γ in the γ-th power cost function. In figure 2, the control rule designed in setting γ within the range below 2.0 gives the best blotch-restoration result whose picture quality is no less excellent than that provided by the blending-type filter using our previously presented heuristically designed control rule [3].

5. CONCLUSIONS

The blending-type filter composed of multiple component filters is widely used for several image-processing tasks. The propriety of the adaptive control for the blending-type filter depends on how to define the blending coefficient as a function of the observable parameters. So far, in most cases, to define the blending-coefficient function, either a steepest descent design method using a training sequence of data or a priori/heuristic rules are employed. On the contrary, this paper presents a new well-grounded design method that optimizes the blending coefficient statistically as a function of the observable parameters by taking into account their statistics and the posterior probability model of the events for which the component filters are prepared.

We apply our new design method to the statistically optimal design of the blending coefficient of the blending-type blotch repair filter for old film restoration. The results of experimental simulations show that our new design method provides the theoretical framework that successfully explains the propriety of the individual blending-type filter to its image-processing target. Our new design method, of course, is applicable to the general design problem of blending-type filters for other applications such as de-interlacing and format conversion and so forth.

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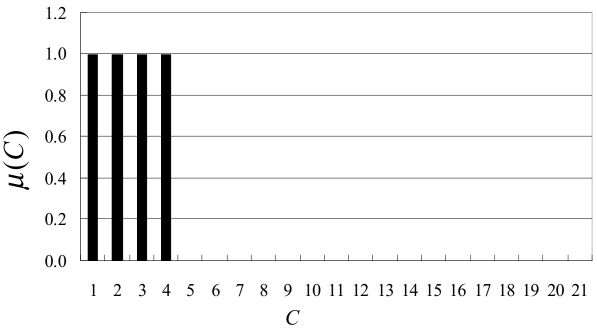
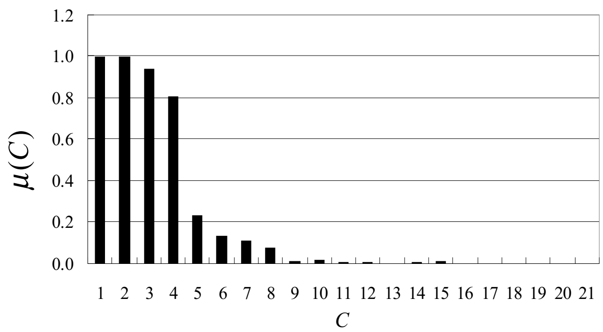
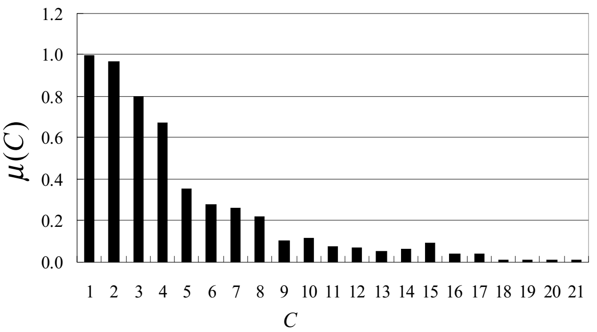


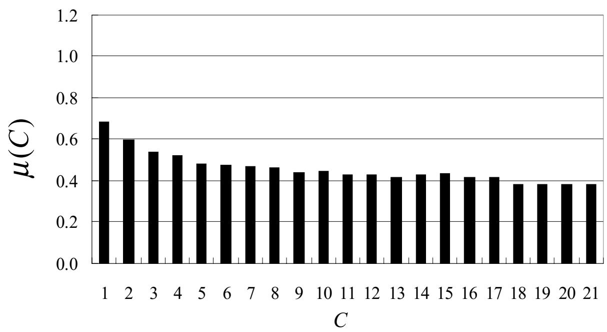
(a) $\gamma = 1.01$



(b) $\gamma = 1.50$



(c) $\gamma = 2.00$



(d) $\gamma = 10.0$

Figure 1: Designed control rules $\mu(C)$ for several values of the parameter γ in the γ-th power cost function of equation 5.

(a) Test image corrupted by artificial blotches

(b) Blotch-repaired image ($\gamma = 1.01$)

(c) Blotch-repaired image ($\gamma = 1.50$)

(d) Blotch-repaired image ($\gamma = 10.0$)

Figure 2: Test image corrupted by artificial blotches and blotch-repaired images provided by the blending-type blotch repair filter equipped with the designed control rule $\mu(C)$ for several values of the parameter γ in the γ-th power cost function of equation 5.