A NOVEL METHOD IN REDUCING THE COMPLEXITY OF FRACTAL ENCODING

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ABSTRACT

Fractal coding is a promising technique for image compression. However, one of the challenges for cost effective implementation is to reduce the huge computational complexity of the encoder. In this paper, we propose a novel algorithm to address this issue. Firstly, we replace mean square error with mean absolute error as distortion measure to reduce multiplication. Secondly, we use statistical normalization to eliminate the need to compute the scaling factor and offset during the search. Thirdly, we change the domain block search to range block search to reduce memory requirement. Simulation results suggest that our algorithm can reduce computation by three order of magnitude for a QCIF image with negligible visual degradation.

1. INTRODUCTION

In recent years, there has been much interest in applying fractals to encode images and video sequences due to the potential of achieving very large compression ratios. Instead of coding the image itself, fractal coding achieves compression by encoding the self-similarity structure of the image[1, 2]. One major drawback of fractal encoding is its large computational complexity due to the search for suitable affine contractive mappings. While much work has been done to reduce the computation complexity of fractal encoding, most of them only try to reduce the number of comparisons but each comparison itself requires considerable computation. In this paper, we propose a novel algorithm to reduce the amount of computation by computing a different distortion measure, a different scaling factor and a different offset during the search. Also, domain block search is changed to a range block search to reduce memory requirement.

In the traditional fractal encoding[2], an image is divided into non-overlapping blocks called range blocks. For each range block, an exhaustive search is performed to find the optimal domain block which minimizes the mean square error (*MSE*) under certain affine contractive mapping. For each possible range block, the brute force domain-block search involves computing the optimal scaling factor and offset for each of the eight possible rotation and flipping isometrics. Let the image size be PxQ, the range block size be NxN and the domain block size be $2Nx2N$. Assuming P and Q to be much larger than N, the brute force search requires approximately

$$\left(\frac{P}{N}\right)\left(\frac{Q}{N}\right)(P-2N+1)(Q-2N+1)(9N^2+48) \approx 9P^2Q^2$$

multiplications and

$$\left[\left(\frac{P}{N}\right)\left(\frac{Q}{N}\right)+2(P-2N+1)(Q-2N+1)\right]N^2$$

$$+8\left(\frac{P}{N}\right)\left(\frac{Q}{N}\right)(P-2N+1)(Q-2N+1)N^2 \approx 8P^2Q^2$$

additions. For a QCIF sized image ($P=176$, $Q=144$) with $N=8$, this corresponds to $5.8x10^9$ multiplications and $5.1x10^9$ additions, which is excessive for encoding an image.

2. MOTIVATION

Traditionally mean square error (*MSE*) is used as distortion measure in the domain block search because of the relatively simple mathematical analysis. One drawback of *MSE* is the need of much multiplication in the encoding. To reduce multiplication, we propose to use mean absolute error (*MAE*) instead as the distortion measure. In general, neither *MSE* nor *MAE* are good measure of perceptual distortion but both are reasonably good in measuring distortion.

Traditionally, the optimal scaling factor s and offset o in the affine transform are derived by minimizing the *MSE*, which is the distortion measure used in the domain block search. These optimal scaling factor s and offset o must be computed for each possible pair of domain block and range block, which is computationally very expensive. As we change the *MSE* to *MAE*, the minimization is more difficult and the resulting equations even more difficult to

compute.
As a result, we choose not to minimize the *MAE*. Instead, we use s and o to match the mean and variance of the domain block and range block. This is reasonable because, when a domain block is perceptually close to a range block, they should have similar mean and variance. However, straight forward implementation would still require significant computation during the search. To reduce computation, we normalize all the blocks to zero mean and unit variance before the search. This way no scaling and offset adjustment are needed during the search and the block searching is similar to motion estimation in Motion Picture Expert Group (*MPEG)* encoding.

One problem with the normalization is that it takes excessive memory to store all the normalized domain blocks. To control the memory requirement, we change the domain block search to a range block search. Before the search, all the range blocks are normalized and stored. During the search, only one domain block needs to be normalized and stored at a time, which greatly reduces the memory requirement.

3. Proposed Fast Fractal Encoding

Here is the proposed fast fractal encoding algorithm:

1. (Statistical normalization) For each range block r, compute the mean m_r and variance σ_r^2 and transform each pixel $x_{r,i}$ in r by

$$f(x_{r,i}) = \frac{x_{r,i} - m_r}{\sigma_r}$$

   to yield a zero-mean, unit-variance normalized range block r_{nor}. Store all normalized range blocks r_{nor} with associated m_r and σ_r^2. For each range block, initialize the "best domain block" to be the first domain block with infinitely large mean absolute error MAE_r.

2. (Range block search) For each domain block d (decimated from *2Nx2N* to *NxN)*, compute the mean m_d and variance σ_d^2 and transform each pixel in d by

$$f(x_{d,i}) = \frac{x_{d,i} - m_d}{\sigma_d}$$

   to yield a zero-mean, unit-variance normalized domain block d_{nor}. For each isometric of d, compute the mean absolute error MAE_1 and MAE_2 between d_{nor} and each of the stored range block r_{nor} as follows:

$$MAD_1 = \sum_{i=1}^{N^2} |x_{r,i} - x_{d,i}|$$

$$MAD_2 = \sum_{i=1}^{N^2} |x_{r,i} + x_{d,i}|$$

   If either MAE_1 or MAE_2 is less than MAE_r of r_{nor}, set MAE_r to be the smaller of MAE_1 and MAE_2 and set the "best domain block" of r_{nor} to be d with mean m_r and variance σ_r^2. Store the current isometric, and whether MAE_1 or MAE_2 is used.

3. Stop. The best affine contractive mapping for each range block r is the associated "best domain block" d, scaled and offset adjusted as follows:

$$r = \left(\frac{\sigma_r}{\sigma_d}\right)(d_s - m_d)(-1)^i + m_r$$

   where d_s is d with the associated isometric, $i=0$ if MAE_1 is used and $i=1$ if MAE_2 is used.

In the algorithm, both MAE_1 and MAE_2 are considered because both d_{nor} and $-d_{nor}$ are zero-mean, unit variance blocks, equally suitable for matching the normalized range blocks.

The proposed algorithm is found in experiments to yield slightly lower visual quality than the traditional method. To improve the visual quality, we modify step 3 of the algorithm by using the traditional formula for optimal scaling factor s and offset o, minimizing the *MSE* rather than matching the mean and variance. This is done only once after the range block search and thus has negligible contribution to overall computation.

4. Computational And Memory Requirements

Assuming that P and Q are much larger than N, the proposed algorithm requires approximately

$$(P-2N+1)(Q-2N+1)(N^2+N+1) + \left(\frac{P}{N}\right)\left(\frac{Q}{N}\right)(2N^2+1) \approx N^2PQ$$

multiplications and

$$(P-2N+1)(Q-2N+1)(3N^2-2)$$
$$+8(P-2N+1)(Q-2N+1)(4N^2-2)\left(\frac{P}{N}\right)\left(\frac{Q}{N}\right)$$
$$+\left(\frac{P}{N}\right)\left(\frac{Q}{N}\right)(3N^2-2) \approx 32P^2Q^2$$

additions, and

$$8(P-2N+1)(Q-2N+1)(2N^2)\left(\frac{P}{N}\right)\left(\frac{Q}{N}\right) \approx 16P^2Q^2$$

absolute conversions. This corresponds to a reduction of multiplication by a factor of $9PQ/N^2$. For a QCIF sized image ($P=176$, $Q=144$) with $N=8$, the multiplication reduction factor is about *3600* and the number of multiplication needed is only $1.6x10^6$ compared with $5.8x10^9$ of the traditional method. The amount of addition and absolute conversion of the proposed algorithm are $2x10^{10}$ and 10^{10} respectively, which now becomes the dominating terms. However, note that exhaustive search is used in the proposed algorithm. Fast block-base motion estimation techniques for *MPEG* encoding can be applied here to reduce significantly both the number of addition and absolute conversions. Other fast fractal encoding techniques such as block classification and genetic algorithm[3] can also be applied to reduce the computation further.

If domain block search (with step=1) rather than range block search is used in the proposed algorithm, memory is needed to store $(P-2N+1)(Q-2N+1)N^2$ normalized domain block pixel intensity which are floating points numbers. For the QCIF image, this is *5.3* MB assuming *4* bytes for each floating point number. Using the range block search, memory is needed to store $(P/N)(Q/N)N^2=PG$ normalized range block pixels which is only *0.1* MB for the QCIF image. The memory reduction factor is approximately N^2.

5. Results And Conclusions

The proposed fast encoding method (with modification) and the traditional method are applied to "Lenna" (*512x512*), "Peppers" (*512x512*) and "MissA" (*176x144*). The scaling factor and offset are both quantized to 7 bits. The bit rates (*BR*) and peak signal-to-noise ratio (*PSNR*) are shown in Table 1. The reconstructed images using the two algorithms are very similar visually as shown in Figs. 1-4. Although the proposed algorithm is not optimal with respect to *MSE*, it manages to achieve in two out of three cases slightly better *PSNR* and slightly lower bit rate than the traditional method, optimal with respect to *MSE*.

These suggests that the proposed algorithm can indeed reduce the amount of multiplication significantly with negligible visual degradation.

6. Acknowledgement

This work was supported partially by the Research Grant Council and partially by the Hongkong Telecom Institute of Information Technology under contract HKTIIT93/94.EG02.

7. References

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Table 1: Simulation Results

Image	Encoding method	Bit per pixel	PSNR (dB)
Lenna	traditional	0.453	30.54
	proposed	0.452	30.56
Pepper	traditional	0.451	29.86
	proposed	0.450	29.75
MissA	traditional	0.417	33.77
	proposed	0.413	33.78

Fig. 1 "Lenna" (512x512) coded with traditional method. (BR=0.452bpp. PSNR=30.54dB)

Fig. 2 "Lenna" (512x512) encoded with proposed fast algorithm. (BR=0.452 bpp. PSNR=30.56dB)

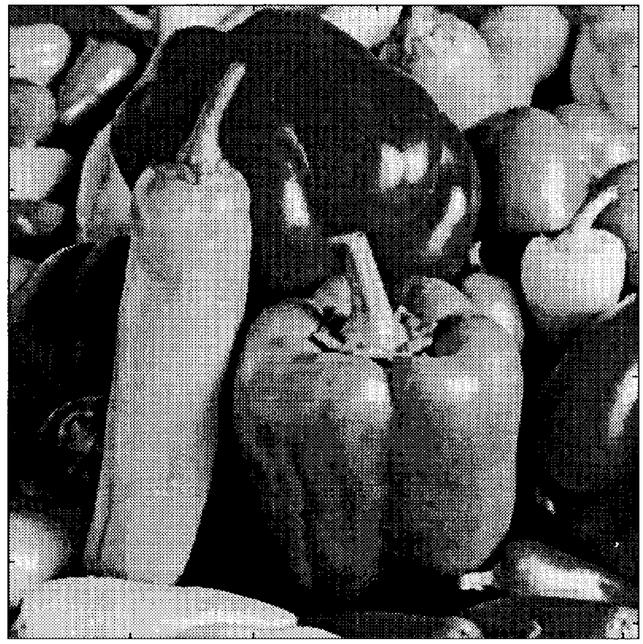

Fig. 3 "Pepper" (512x512) coded with traditional method. (BR=0.451 bpp. PSNR=29.86 dB)

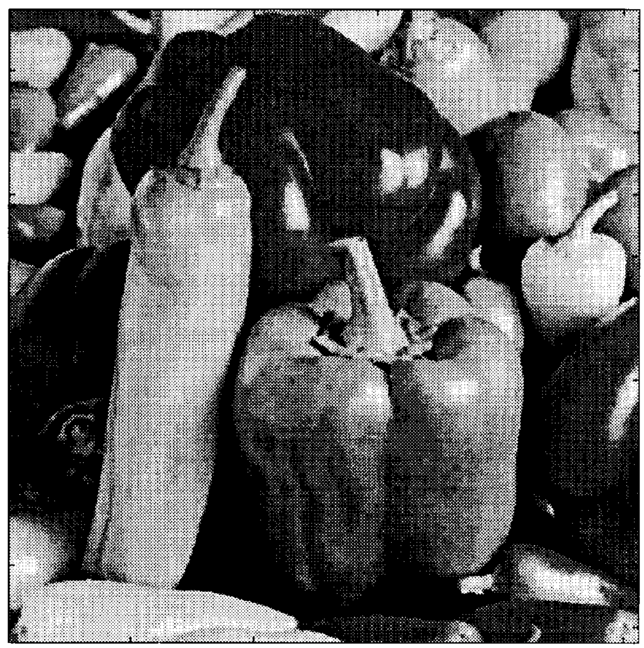

Fig. 4 "Pepper" (512x512) encoded with proposed fast algorithm. (BR=0.450 bpp. PSNR=29.75 dB)