NONLINEAR FUZZY FILTERS: AN OVERVIEW

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ABSTRACT

Emergent techniques based on Fuzzy Logic have successfully entered the area of nonlinear filters. Indeed, a variety of methods have been recently proposed in the literature which are able to perform detail-preserving smoothing of noisy image data yielding better results than classical operators. The aim of this paper is to present a selection of the most significant contributions in this field focussing on their similarities and differences. A brief introduction to the theory of fuzzy sets and systems is presented in order to make these results available to non-fuzzy researchers too.

1. INTRODUCTION

Since the first introduction of Fuzzy Set Theory [1] fuzzy techniques for image processing applications have mainly dealt with high-level computer vision and pattern recognition [2]. Indeed, a variety of different approaches including fuzzy clustering, fuzzy integral, fuzzy entropy and fuzzy aggregation networks have been proposed in order to address key tasks such as image segmentation, object identification and scene interpretation [3-10]. Only recently, however, fuzzy techniques have successfully entered the area of low-level computer vision for general purpose applications becoming competitive with classical methods in some very important pre-processing tasks. In particular, focussing on the area of nonlinear filtering of noisy image data, many original approaches have been proposed in the last few years. The number of new applications is now rapidly growing as a result of the great effort in the research work all over the world. It opens up new vistas in the design of nonlinear filters.

This paper aims at presenting a selection of the most significant contributions in this field focussing on their application to the enhancement of noisy image data. Fuzzy operators dealing with images mainly from a pattern recognition point of view, or not directly dealing with images, will not be described here due to space limitations. In order to make the description of fuzzy methods understandable to non-fuzzy researchers too, a brief introduction to fuzzy systems is provided.

2. A BRIEF INTRODUCTION TO FUZZY SYSTEMS

We shall describe in this section the basic structure of a fuzzy system. An outstanding presentation of the theory of fuzzy systems can be found in [11].

Fuzzy sets are extensions of classical sets. Unlike classical sets, fuzzy sets permit partial membership. Indeed, a fuzzy set, say A, defined in the domain X, is described by a *membership function* m_A which maps X to the real interval [0,1]. For each $x \in X$, $m_A(x)$ yields the degree of membership of x to the fuzzy set A (Fig.1)

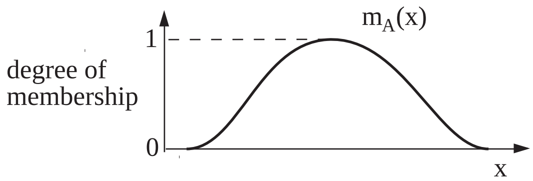

Fig.1 - Example of a fuzzy set.

An element x can belong to different fuzzy sets with different degrees of membership. This offers a well-suited approach to solve problems intrinsically affected by uncertainty. An example is shown in Fig.2, where x represents a pixel luminance in the range [0, 255] and *dark*, *medium* and *bright* are triangular-shaped fuzzy sets defined in the domain of the luminances.

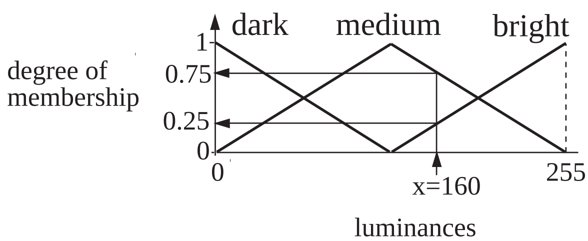

Fig.2 - Example of triangular-shaped fuzzy sets labeled *dark*, *medium* and *bright* .

Fuzzy sets are key components of *fuzzy rules*. Fuzzy rules allow a processing strategy to be expressed in the form of approximate reasoning. As an example, let us consider the following rule:

$$\text{IF } (x_1, A_1) \text{ AND } (x_2, A_2) \text{ AND } (x_3, A_3) \text{ THEN } (y, B)$$

where A_1, A_2, A_3 and B are fuzzy sets associated to the quantities x_1, x_2, x_3 and y, respectively. Observing such a rule we can notice three *antecedent clauses* which define

conditions and a *consequent clause* which defines the corresponding action.
A *fuzzy system* is a nonlinear system that adopts fuzzy rules to map a set of inputs to an output. More formally, let us consider a fuzzy system which maps N input variables $x_1, x_2, ..., x_N$ to one output variable y by using M rules. Such a system can be represented in the following form:

IF (x_1,A_{11}) AND....AND (x_N, A_{N1}) THEN (y, B_1)
IF (x_1,A_{12}) AND....AND (x_N, A_{N2}) THEN (y, B_2)
....
IF (x_1,A_{1M}) AND....AND (x_N, A_{NM}) THEN (y, B_M)

where A_{ij} ($1 \le i \le N$, $1 \le j \le M$) is the fuzzy set associated to the i-th input variable in the j-th rule and B_j is the fuzzy set associated to the output variable in the same rule.
The information contained in the fuzzy rulebase *(knowledge base)* is numerically processed by the *inference mechanism.* For a given set of input data, the inference mechanism first evaluates the activations of fuzzy rules and then combines their resulting effects. More precisely, let λ_j be the degree of activation of the j-th rule. This degree can be evaluated by using the following relation:

$$\lambda_j = \text{MIN} \{ m_{A_{ij}}(x_i); i=1,...,N \}$$

The output y of the system is obtained by superimposing the effects of all fuzzy rules and by resorting to a *defuzzification* method. Let the consequent fuzzy set B_j be described by a triangular-shaped membership function m_{B_j} defined as follows:

$$m_{B_j}(x) = \begin{cases} 1 - \dfrac{|x - c_{B_j}|}{w_{B_j}} & c_{B_j} - w_{B_j} \le x \le c_{B_j} + w_{B_j} \\ 0 & \text{otherwise} \end{cases}$$

Thus, by adopting the so called *correlation-product inference* [11], a very simple expression for the output can be obtained:

$$y = \frac{\sum_{j=1}^{M} \lambda_j \, c_{B_j} \, w_{B_j}}{\sum_{j=1}^{M} \lambda_j \, w_{B_j}}$$

The above described structure of a fuzzy system is widely adopted in the literature. It should be observed, however, that different structures have been also proposed, such as, for example, the Takagi-Sugeno model [12]. A variety of fuzzy aggregation mechanisms are also available to evaluate the degree of activation of a fuzzy rule [13].

3. FUZZY TECHNIQUES FOR DETAIL-PRESERVING SMOOTHING OF NOISY IMAGES

A brief overview of fuzzy techniques for detail-preserving smoothing of noisy images is presented by adopting a common mathematical notation.

Let x(**n**) represent the pixel luminance at location **n**=[n_1,n_2]; let W_0(**n**)={ x_i ; i =0,..,N } be the set of pixel values which belong to a window centered on x(**n**), where x_0=x(**n**); let W(**n**)={ x_i ; i =1,..,N } be the corresponding set with the exception of the element x_0; let x_{med} be the median of the pixel values in W_0.

3.1 FIRE filters (1992)

FIRE (Fuzzy Inference Ruled by Else-action) operators are a family of nonlinear operators which adopt fuzzy rules to process image data. Originally proposed in [14], the special IF-THEN-ELSE structure of these operators has been progressively improved. Typical application areas include image smoothing, sharpening and edge extraction [15-19].
A FIRE filter is a totally fuzzy operator. Its input variables are the luminance differences: $\Delta x_i = x_i - x_0$; i=1,...,N. The output variable Δy(**n**) is the correction term which must be added to x(**n**) in order to obtain the resulting pixel value y(**n**)=x(**n**)+Δy(**n**). The rulebase of a FIRE operator typically includes two symmetrical subrulebases and one ELSE-rule. As an example, a rulebase adopting three fuzzy sets labeled *positive* (PO), *zero* (ZE) and *negative* (NE) can be defined as follows:

IF (Δx_1,A_{11}) AND...AND (Δx_N,A_{N1}) THEN (Δy,PO)
IF (Δx_1,A_{12}) AND...AND (Δx_N,A_{N2}) THEN (Δy,PO)
....
IF (Δx_1, A_{1M}) AND..AND (Δx_N, A_{NM}) THEN (Δy, PO)

IF (Δx_1,A^*_{11}) AND..AND (Δx_N,A^*_{N1}) THEN (Δy,NE)
IF (Δx_1,A^*_{12}) AND..AND (Δx_N,A^*_{N2}) THEN (Δy,NE)
....
IF (Δx_1,A^*_{1M}) AND..AND (Δx_N,A^*_{NM}) THEN (Δy,NE)

ELSE (Δy, ZE)

where A_{ij} denotes the fuzzy set (PO or NE) associated to the i-th input variable in the j-th rule of the first sub-rulebase. Due to the symmetry of the processing, we have: A_{ij}*=NE if A_{ij}=PO and A_{ij}*=PO if A_{ij}=NE. It should be noticed that the ELSE-rule aims at leaving the central pixel unprocessed if no THEN-rule is activated.
A specifically developed inference mechanism yields the output Δy of a FIRE filter.
Such a mechanism can be summarized as follows:

$$\lambda_1 = \text{MAX} \{ \text{MIN} \{ m_{A_{ij}}(\Delta x_i) : i=1,...,N \}; j=1,...,M \}$$
$$\lambda_2 = \text{MAX} \{ \text{MIN} \{ m_{A^*_{ij}}(\Delta x_i) : i=1,...,N \}; j=1,...,M \}$$
$$\Delta y = \rho \, (\lambda_1 - \lambda_2)$$

where ρ is a scaling factor. Since fuzzy rules deal with patterns of luminance differences, they can exploit all the spatial information of an image in order to perform an edge-preserving filtering action. By suitably choosing fuzzy sets and rules, different noise statistics can be addressed. In particular, it is worth pointing out that a median filter can be implemented as a particular case of a FIRE operator. The combination of sharpening and smoothing rules in the same rulebase for the enhancement of noisy images is presented in [16]. New classes of FIRE filters are proposed in [18,19].

3.2 Data dependent fuzzy filters (1994)

A data-dependent filter (DDF) adopting fuzzy reasoning is proposed by Taguchi and Takashima in [20,21]: this approach uses fuzzy rules to combine the basic ideas of the adaptive center weighted average (ACWA) filter and modified trimmed mean (MTM) filter [22-23]. The output $y(\mathbf{n})$ of the DDF operator is yielded by:

$$y(\mathbf{n}) = \frac{\sum_{i=0}^{N} w_i(\mathbf{n})\ x_i(\mathbf{n})}{\sum_{i=0}^{N} w_i(\mathbf{n})}$$

where each w_i (i=0,..,N) parameter is evaluated by a fuzzy system whose inputs are three local characteristics. A fuzzy rulebase including 27 fuzzy rules is used to perform the required nonlinear mapping. Fuzzy set parameters are tuned by resorting to an LMS algorithm.

An adaptive L-filter based on fuzzy reasoning is also proposed by Taguchi and Meguro for the suppression of mixed noise [24]. The data inside the window are classified by fuzzy rules into five classes depending on the values of three different local characteristics. For each class a suitable filter is chosen, as follows:

class 1 - midpoint filter,
class 2 - mean filter,
class 3 - median filter,
class 4 - identity filter,
class 5 - small window median filter.

Thus, the output of the fuzzy L-filter is yielded by:

$$y(\mathbf{n}) = \frac{\sum_{k=1}^{5} m_k(\mathbf{n})\ y_k(\mathbf{n})}{\sum_{k=1}^{5} m_k(\mathbf{n})}$$

where m_k is the degree of membership of the input data to the k-th class and y_k is the output of the corresponding filter.

3.3 Hybrid filters (1994)

A hybrid filter for the removal of mixed gaussian and impulsive noise is proposed by Peng and Lucke in [25-28]. The filter basically combines a non-fuzzy and a fuzzy component: the former is a nonlinear filter devoted to the suppression of noise pulses, the latter is a fuzzy weighted linear filter. The basic structure presented in [25] can be described as follows. First, some pre-processing is performed in order to remove impulse noise: for this purpose, a new set $W'_0=\{ x'_i ; i=0,...,N \}$ of pixel values is obtained from W_0 by replacing the minimum and maximum luminance values with x_{med}. Then, the output of the filter is evaluated by means of the following relationship:

$$y(\mathbf{n}) = \frac{\sum_{i=0}^{N} m_\Pi(\Delta x'_i)\ x'_i(\mathbf{n})}{\sum_{i=0}^{N} m_\Pi(\Delta x'_i)}$$

where $\Delta x'_i = x'_i - x'_0$ and m_Π is the membership function which describes a Π-type (i.e. a bell-shaped) fuzzy set. This set aims at reducing the influence of pixels having large luminance differences with respect to the central one. The tuning of fuzzy set parameters is performed by resorting to an LMS algorithm. Some improvements to the above structure are reported in [26-27].

3.4 Image enhancement based on Fuzzy Logic (1995)

An image enhancement technique is proposed by Choi and Krishnapuram [29]. Such a technique addresses the following goals: removing impulse noise, smoothing out non-impulse noise and enhancing edges. Three different filters for each task are developed. Their outputs y_1, y_2, y_3 are combined by resorting to a fuzzy system, as follows:

IF M is small, THEN $y = y_1$
IF M is large, THEN $y = y_2$
ELSE $y=y_3$

where M is a local characteristic and *small* and *large* are fuzzy sets. The final output y is numerically evaluated by the following relationship:

$$y = \frac{\sum_{k=1}^{3} \lambda_k\, y_k}{\sum_{k=1}^{3} \lambda_k}$$

where λ_1 and λ_2 are the degrees of activation of the THEN-rules and λ_3 is the degree of activation of the ELSE-rule, evaluated by adopting the IF-THEN-ELSE paradigm. The local characteristic M depends on the luminance differences and spatial distances between the central pixel and its neighbors. A bell-shaped membership function is adopted for this purpose.

3.5 Fuzzy Vector Directional Filters (1995)

A new class of filters for multichannel image processing is introduced by Plataniotis, Androutsos and Venetsanopoulos in [30]. This class constitutes a fuzzy generalization of the class of well known Vector Directional Filters. A membership function based on a minimum angle criterion between vectors is used to obtain the weights of a weighted mean filter:

$$y = \frac{\sum_{i=0}^{N} w_i\, x_i}{\sum_{i=0}^{N} w_i}$$

Experimental results show that the proposed operators perform better than vector median filters and generalized vector directional filters. In particular, applications to color image data are considered.

3.6 Fuzzy median filters (1995)

A totally fuzzy approach is proposed by Chatzis and Pitas in [31]. Since classical arithmetic operators such as arithmetic mean and median have been largely applied to image smoothing, the paper investigates the suitability of the arithmetic of fuzzy numbers (i.e. fuzzy sets) for the same purposes. As a result, a new definition of mean and median of fuzzy numbers is proposed, by resorting to the *extension principle* [12].

3.7 Fuzzy stack filters (1995)

A new class of operators, called Fuzzy Stack Filters, is proposed by Pao-Ta Yu and Rong Chung Chen [32] in order to extend the smoothing capabilities of classical stack filters. The proposed approach is based on the adoption of a *fuzzy positive Boolean function*. A learning method is presented to achieve the optimal filter design by using a set of training data.

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