MORPHOLOGICAL LIKE OPERATORS FOR COLOR IMAGES

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ABSTRACT

Primarily based on Serra’s framework [5], mathematical morphology has become one of the most used nonlinear processing and analysis techniques. Later work extended the initially set operators to functions, in a general algebraic definition [4] for multidimensional scalar signals. The case of vector valued images (or signals) is not included in this theory.

The extension of mathematical morphology to color images is equivalent to the definition of an ordering relation in a vector space. In this paper we will investigate several ordering relations in the color space, each of them yielding to the definition of morphological operations. The performance of the filtering based on these operations is evaluated in terms of Normalized Mean Square Error (NMSE), Mean Chromaticity Error (MCRE), space topology preservation and visual subjective perception of image quality.

1. INTRODUCTION

Traditionally, the multidimensional signal processing is based on linear system theory and the related integral transforms (Fourier, Laplace, ...); their theoretical bases are well established and in continuous evolution. The image signal, one of the most widely used multidimensional signal, is mainly characterized by shape and geometrical structure concepts, for which the linear analysis is not the most appropriate (losing important and relevant information).

The nonlinear transforms, typically represented by order statistics operations, try to solve the above mentioned problem by offering tools for shape oriented processing and analysis. Among them, the mathematical morphology is successfully used for solving various processing and analysis problems.

2. THE BASICS OF MATHEMATICAL MORPHOLOGY

Serra [5] introduced primary the dilation and the erosion (the basic mathematical morphology operations) as extensions to some set operators, known as the Minkowski (or set) addition and difference. These operations are set transforms in the Boolean lattice, ordered by the set inclusion.

Further developments realized a general theoretical framework for applying mathematical morphology to any multidimensional scalar signal. The general definition of the basic morphological transforms require the existence of some algebraic structure, namely a complete lattice, induced by an ordering relation in the signal values space.

2.1 Definition recall

Definition 1: A lattice is an ordered set for which each pair of elements $(\mathbf{x},\mathbf{y})$ has an upper bound (called sup) and a lower bound (called inf).

Definition 2: A complete lattice is a lattice for which any subset of elements (finite or not) has an upper and a lower bound.

Definition 3: The basic morphological operators, the dilation and the erosion are operators over a complete lattice, that commute with sup and respectively inf.

Indubitably, the main condition to be fulfilled is the existence of a proper ordering relation over the signal values set.

Definition 4: An ordering relation defined on a set (written as $\mathbf{x} \prec \mathbf{y}$) must be: reflexive ($\mathbf{x} \prec \mathbf{x}$), transitive ($\mathbf{x} \prec \mathbf{y}$ and $\mathbf{y} \prec \mathbf{z}$ imply $\mathbf{x} \prec \mathbf{z}$) and antisymmetric ($\mathbf{x} \prec \mathbf{y}$ and $\mathbf{y} \prec \mathbf{x}$ imply $\mathbf{x} = \mathbf{y}$).

2.2 The current case

For scalar multidimensional signals, the definitions above produce the expressions:
$(f \oplus g)(\mathbf{x}) = \sup_{\mathbf{y} \in Supp(g)} \{f(\mathbf{x}-\mathbf{y}) + g(\mathbf{y})\}$ for the dilation and $(f \Theta g)(\mathbf{x}) = \inf_{\mathbf{y} \in Supp(g)} \{f(\mathbf{x}-\mathbf{y}) - g(\mathbf{y})\}$ for the erosion, where $g(\mathbf{y})$ is the structuring element and $Supp(g) = \{\mathbf{x} | g(\mathbf{x}) > -\infty\}$ is its support. The ordering relation is the classical ordering on the real scalar space $\mathbf{R}$, unless for a flat structuring element ($g(\mathbf{x}) = 0,\ \forall \mathbf{x} \in Supp(g)$) used to deduce the set morphological operations, where the ordering relation is

the set inclusion. It is obvious that set mathematical morphology is a particular case of the theory exposed in subsection 2.1.

3. MATHEMATICAL MORPHOLOGY FOR VECTOR SIGNALS

Most of the real world signals are vector valued signals, each sample being described by a vector. The color images are a particular (and the most common) case, each sample (pixel) being described by three values, typically the amounts of pure red, green and blue that compose the local color. Still, there are others signal that can be considered: remote sensing images (described by 5 to 18 dimensional vectors), thermal images (described by 3 to 5 dimensional vectors), stereo and quadraphonic sound (described by 2 or 4 dimensional vectors).

As previously described, the introduction of morphological operators requires the definition of an ordering relation in the signal value space, i.e. an ordering in a vector space. The problem is that no direct ordering relation exists in such a space, or at least it cannot be deduced by any natural extension from the scalar ordering.

3.1 The lexicographic ordering

The only possible ordering relation (according to Definition 4) for a vector space is the lexicographic ordering, described by the relation $x \prec y \Leftrightarrow \exists k \in [1,n] \text{ such that } \begin{cases} x_i = y_i,\ \forall i = \overline{1,k-1} \text{ and} \\ x_k < y_k \end{cases}$.
This ordering relation is the extension of the conditional ordering described by Barnett [2]. The main problems appear in the practical implementation: first, a certain importance criterion must be defined, according to which the vector components are arranged and second, the space topology structure is no longer preserved.

The first problem implies to decide which component of the vector is the most important, and several techniques can be used: decision by eigenvalues, variances, relative contrast (mean to variance ratio), relative range (mean to range ratio). This decision process can be made adaptive by choosing the importance of the vector components in every analysis window (given by the shape of $Supp(g)$). For color images, the result of morphological filtering is not very good, due to the poor space topology preservation.

Since no satisfactory ordering relation can be found, we must relax the theoretical constraints and admit a preordering relation (which is just reflexive and transitive). The operators defined according to Definition 3 will be then morphological like operators.

3.2 Preordering relations

Barnett's paper [2] on multivariate ordering is a classical reading in this domain. Later, in [3] some of these principles are announced, related to color image rank-order processing. Three main ordering criteria (preordering relations) are discussed: the marginal ordering, the partial ordering and the reduced (aggregate) ordering.

The marginal ordering uses the common scalar ordering for each component of the vector (the ordering is performed separately on each component). The ordered vectors are in general different from the original ones. Using this ordering principle for color image processing by mathematical morphology operators, significant different colors than the original ones will be obtained in the resulting image.

The partial ordering is based on the partition of the data vectors in convex hull like sets; this problem is equivalent to an Euclidean geometry problem in an n-dimensional space. Still, the computational amount is significant and there is no way to extract the extreme points from the convex hull sets (all the extreme points will be gathered by the first convex hull, and no difference can be made between the sup and the inf).

The reduced (aggregated) ordering is the only ordering relation already successfully used in the median filtering of color images [1], [6]. The idea of reduced ordering is the computation of a scalar value for each vector; the obtained scalars are sorted and the same ordering is assumed for the corresponding vectors. Commonly, these scalars can be considered as a generalized distance from the vector to some fixed point, given by Definition 5.

Definition 5: The generalized distance from the vector $\mathbf{x}$ to the fixed point $\mathbf{x}_0$ is defined by the expression $d_{\mathbf{x}} = (\mathbf{x}-\mathbf{x}_0)A(\mathbf{x}-\mathbf{x}_0)^T$, A being a positive semidefined matrix. This is a quadratic form.

A large class of particular ordering relations can be obtained, choosing different fixed points (the marginal minimum or maximum on each structuring element, the global minimum (0,0,0) and maximum (255,255,255)) and various matrices (the identity matrix for the Euclidean distance, the covariance matrix for the Mahalanobis distance, diagonal matrices having as elements the variances, relative variances, ranges and relative ranges of the color components).

The computational amount is not excessive, since for each point selected by the structuring element support a single distance must be computed.

Another possible approach, not covered by Definition 5, is to consider a linear form of the vector components, as the luminance (brightness) of the color. This could be viewed as a scalar perceptual measure of the color.

The morphological operations can be defined very simple by the two following definitions.

Definition 6: The morphological like vector dilation is the vector lying the closest of some maximum point (global or local marginal), in terms of quadratic (Definition 5) or linear (brightness) distance.

Definition 7: The morphological like vector erosion is the vector lying the closest of some minimum point (global or local marginal), in terms of quadratic (Definition 5) or linear (brightness) distance.

4. EXPERIMENTAL RESULTS

The tests were conducted on two 256X256 true color images: “fish” and “waterfall”. For each image, some impulsive (salt & pepper) noise degraded versions have been considered. Only the impulsive noise was taken into account, since it is the only one that can be successfully removed by the basic (scalar) morphological filters. The noise was superimposed an each color component with the same probability and with no correlation; noise probabilities from 1% per component (small noise) to 4% per component (hard noise) have been considered.

Each image was filtered by morphological open and open-close operators, defined from basic dilations and erosions according to Definition 6 and Definition 7. The results are measured in terms of Normalized Mean Square Error (NMSE) , Mean Chromaticity Error (MCRE) and subjective perception.

Definition 8: The NMSE is defined as

$$NMSE = \frac{\sum_{i=1}^{H}\sum_{j=1}^{W}\left\|\bar{f}(i,j) - f(i,j)\right\|^2}{\sum_{i=1}^{H}\sum_{j=1}^{W}\left\|f(i,j)\right\|^2}\cdot 100.$$

Definition 9: The MCRE is defined as:

$$MCRE = \frac{\sum_{i=1}^{H}\sum_{j=1}^{W}\left\|\frac{\bar{f}(i,j)}{\left|\bar{f}(i,j)\right|} - \frac{f(i,j)}{\left|f(i,j)\right|}\right\|^2}{HW}\cdot 100.$$

In the definitions above, H and W are the image dimensions in pixels, $\| \;\|$ is the L^2 norm and $| \;|$ is the L^1 norm, $f(i,j)$ and $\bar{f}(i,j)$ are the original and filtered values of the pixel at location (i,j).

The experimental results are presented in Tables 1 and 2 (for the “waterfall” image) and Figures 1 to 6. The ordering types are labeled from 1 to 10 as follows: marginal ordering (1), reduced ordering with fixed points the global black and white and Euclidean distance (2), reduced ordering with fixed points the marginal extremes and Euclidean distance (3), reduced ordering with fixed points the marginal extremes and weighted Euclidean distance by range (4), relative range (5), variance (6), relative variance (contrast ratio) (7), reduced ordering in brightness (8), lexicographic ordering by range (9) and relative variance (contrast ratio) (10) as importance criterion.

Table 1: Open filtering of “waterfall” image

Noise	Order type 1		Order type 2		Order type 3		Order type 4		Order type 5	
	MCRE	NMSE	MCRE	NMSE	MCRE	NMSE	MCRE	NMSE	MCRE	NMSE
1%	1.968	13.813	3.154	28.974	2.086	13.886	2.107	13.556	2.164	13.752
2%	2.088	14.578	3.149	28.899	2.181	14.544	2.195	13.964	3.168	22.371
4%	2.179	15.179	3.129	28.900	2.212	14.843	2.307	14.403	3.805	32.394

Noise	Order type 6		Order type 7		Order type 8		Order type 9		Order type 10	
	MCRE	NMSE	MCRE	NMSE	MCRE	NMSE	MCRE	NMSE	MCRE	NMSE
1%	2.168	13.652	2.175	13.779	1.939	12.381	5.743	53.259	5.944	51.708
2%	2.300	14.367	3.184	22.425	2.006	12.728	6.016	49.527	6.907	50.072
4%	2.544	16.096	3.843	32.482	2.052	13.002	7.105	47.979	9.777	51.708

Table 2: Open-close filtering of “waterfall” image

Noise	Order type 1		Order type 2		Order type 3		Order type 4		Order type 5	
	MCRE	NMSE	MCRE	NMSE	MCRE	NMSE	MCRE	NMSE	MCRE	NMSE
1%	2.181	14.854	3.698	39.634	2.311	14.818	2.363	15.001	4.845	43.927
2%	2.229	14.860	3.700	39.783	2.323	14.980	2.378	14.987	5.468	51.562
4%	2.264	14.804	3.733	40.077	2.339	14.998	2.431	14.965	5.197	53.984

Noise	Order type 6		Order type 7		Order type 8		Order type 9		Order type 10	
	MCRE	NMSE	MCRE	NMSE	MCRE	NMSE	MCRE	NMSE	MCRE	NMSE
1%	2.420	16.466	4.947	44.111	3.808	39.839	6.042	64.151	7.889	41.605
2%	2.469	15.347	5.179	49.494	3.936	40.377	6.552	59.676	11.808	51.222
4%	2.653	16.466	5.197	53.984	3.966	40.666	6.793	46.279	19.438	60.849

Figure 1: 4% impulsive noise degraded “fish”; MCRE=4.996%, NMSE=36.112%

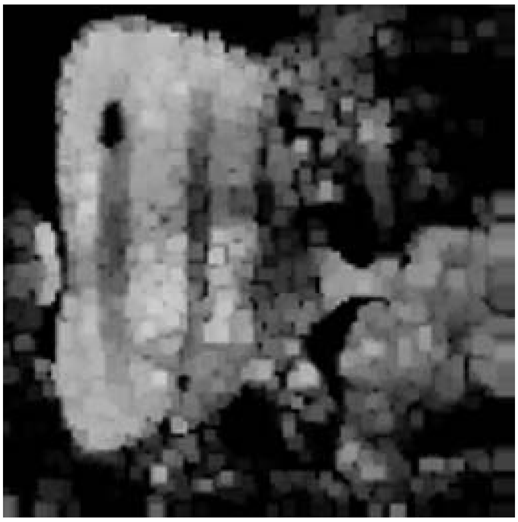

Figure 2: Open filtering of Figure 1 by reduced ordering with marginal extremes as fixed points and Euclidean distance; MCRE=7.067%, NMSE=29.614%

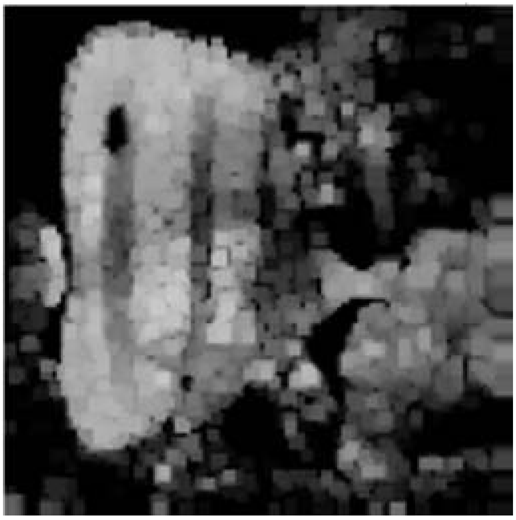

Figure 3: Open filtering of Figure 1 by reduced ordering by brightness; MCRE=6.794%, NMSE=28.401%

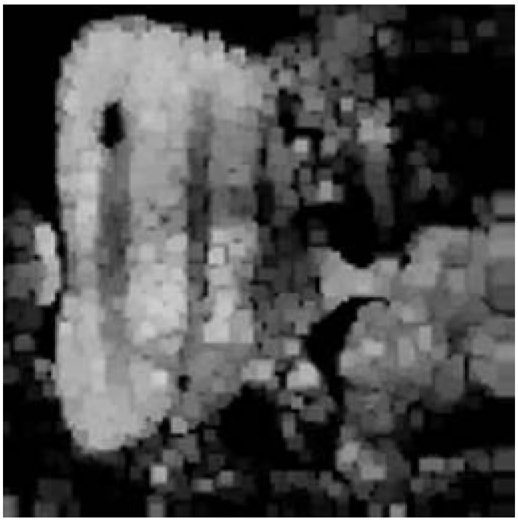

Figure 4: Open filtering of Figure 1 by reduced ordering with marginal extremes as fixed points and weighted Euclidean distance by components range; MCRE=7.106%, NMSE=29.069%

5. CONCLUSIONS

Different types of implementation of vector mathematical morphology operators have been considered. The results of applying these operators for color image filtering show that, in general, the task of impulsive noise reduction is accomplished. The results are comparable with those produced by separate component processing (some reduced ordering criteria yielding slightly better results in terms of NMSE or MCRE). The visual subjective quality of the images is rather good, but no spectacular improvement has been noticed.

The computational amount is not prohibitive and is comparable with some classical color median filtering techniques (as the Vector Median Filter). Better results may be obtained if considering some other representation of the color than the RGB components, that could maximize the relative intercolor distances.

6. REFERENCES

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