A MRF BASED APPROACH TO COLOR IMAGE RESTORATION

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ABSTRACT

In this paper, a Markov Random Field (MRF)-based method is presented. MRF methods are based on a probabilistic representation of a image processing problem; the problem is represented as the maximization of a probability measure computed starting from input data for all possible solutions. The optimization process is often computationally expensive. The coupled problem of restoring and extracting edges from an image is here considered. An extension to the color case of the deterministic mean-field annealing method presented in [1] is presented. The main advantage of this approach is its capability of obtaining a sub-optimum solution in a faster way with respect to optimal stochastic methods (e.g., Simulated Annealing).

1 INTRODUCTION

Color is an important feature for both human and computer vision. As compared to a monochrome image, a color image provides additional information on the image content: such information can be used by image processing modules to extract image features in a more robust way [2].

Several works in the recent literature deal with the problem of color image segmentation as an important step of low-level vision. Image segmentation techniques cover both region-based and edge-based segmentation. They can be roughly classified into three types: (a) histogram-based segmentation [3], (b) physically-based segmentation [4] and (c) neighbourhood-based segmentation methods [1]. The histogram-based approach assumes that homogeneous objects in the image can be detected by searching for clusters in the measurement space (3D color histogram). Physically-based segmentation techniques are mainly focused on the physical model of color image formation and on its used in image estimation processes; neighbourhood-based methods take into account a-priori knowledge about the shape of the solution to regularize the image processing problem. The approach based on Markov Random Fields (MRFs), which is used in this paper, is a neighbourhood-based regularization method for coupled image restoration and edge extraction. Histogram-based approaches do not need a priori information of the image; however, the loss of locality inherent to the method does not allow a precise scene segmentation. Physical-based methods, if considered alone, are based on a ill-posed formulation of the edge detection problem [5], so that they can provide unstable solutions. Neighbourhood-based approaches are able to represent global constraints at a local level; however, they can be computationally expensive, specially when considering that multidimensional images must be processed.

In Section 2 the presented MRF-based approach for coupled edge-detection and image restoration is discussed, by first introducing problem representation. Then, the extension of deterministic mean field annealing [1] to the color case is discussed. In Section 3, results are presented showing the performances of the proposed method for different parameter settings. In Section 4 conclusions are sketched out.

2 MODEL DESCRIPTION

The restoration process of an image requires generally the solution of two related problems: (a) to preserve the discontinuities among different image regions and (b) to smooth the inner areas of such regions. In the case of color images, the smoothness operation must be done for each single component. The MRF based approach allows one to perform at the same time edge detection and smoothing processes by means of two coupled fields: a) the image intensity field $F=\{f_{i,j}\}$, defined over a lattice $S=\{m=(i_m,j_m):(i,j)=(1,1), \ldots, (x_{dim},y_{dim})\}$, where x_{dim}, and y_{dim} are the image dimensions; b) the line process, $L=(H,V)$, where H and V represent the fields of horizontal and vertical discontinuities, e.g., $H=\{h_{mn}: m,n \in S, i_m=i_n, j_m=j_n-1\}$, and $h_{mn}=1$ (=0) represent the presence (the absence) of a horizontal discontinuity between site m and site n, see (Fig. 1) [1].

A color image is usually described by means of three images representing color components; R (red), G (green) and B (blue) representation is used in the model presented in this paper. Extending the MRF model for monochrome images can be done by first considering a vectorial field **F** whose components are fields related to the three basic components:

$$\mathbf{F} = \{F_R, F_G, F_B\}$$

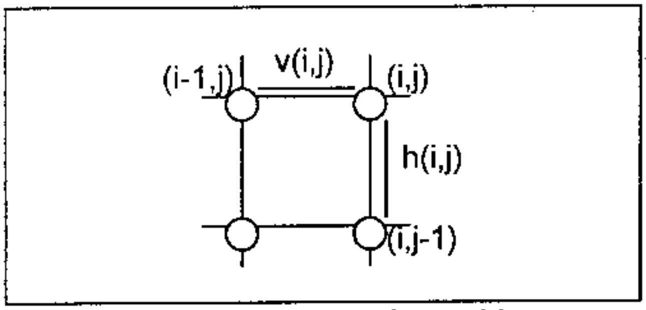

Fig.1 *Horizontal line process and vertical line process represented in (i,j) site.*

Edges to be detected can be represented also in the color case as scalar fields of random variables, i.e., by using the line process L=(H,V). Following the approach described in [1], the coupled image restoration and edge detection problem can be solved, according to the *Maximum A-Posteriori (MAP)* criterion, by finding the configuration **F***, H*,V* satisfying the following condition:

$$(\mathbf{F}^*, H^*, V^*) = \max_{\{\mathbf{F},H,V\}} P(\mathbf{F}, H, V/\mathbf{G}) \qquad (1)$$

where $\mathbf{G}=\{g_m;\ m \in S\}$ represents the observed image, consisting of $\{G_R, G_G, G_B\}$ components.

Bayes theorem allows one to write the posterior probability density function used for color images in (1) as :

$$P(\mathbf{F}, H, V/\mathbf{G}) = \frac{P(\mathbf{F}, H, V)\cdot P(\mathbf{G}/\mathbf{F})}{P(\mathbf{G})} \qquad (2)$$

Some considerations can be done on (2); first, the term $P(G_R, G_G, G_B)$ can be omitted in the maximization process being independent on the configuration of (**F**,H,V).

Then, color components can be hypothesized to be independent; consequently the so-called solution model is provided by (3), while the observation model is given by (4):

$$P(\mathbf{F}, H, V) = P(F_R, H, V)\cdot P(F_G, H, V)\cdot P(F_B, H, V) \qquad (3)$$

$$P(\mathbf{G}/\mathbf{F}) = P(G_R/F_R)\cdot P(G_G/F_G)\cdot P(G_B/F_B) \qquad (4)$$

Then the three color components can be considered as identically distributed; the solution and observation models of each color component *k={R, G, B}* become:

$$P(F_k, H, V) = \frac{1}{C_1}\cdot exp(-\sum_{i,j}\{\alpha[(f_{i,j_k} - f_{i,j-1_k})^2(1-h_{i,j}) +$$

$$+(f_{i,j_k} - f_{i-1,j_k})^2(1-v_{i,j}) + \gamma^h_{i,j}\cdot h_{i,j} + \gamma^v_{i,j}\cdot v_{i,j}\} \qquad (5)$$

and

$$P(G_k/F_k) = \frac{1}{C_2} exp\left[-\sum_{i,j}\frac{(f_{i,j_k} - g_{i,j_k})}{2\sigma^2}\right] \qquad (6)$$

This is equivalent to say that the field (F_k,H,V) is a coupled MRF model [6] describing piecewise constant solutions, while an additive Gaussian noise $G(0, \sigma^2)$ is assumed to affect observations. The approach developed in [1] for grey-levels image, involve the introduction of a β parameter, which is the deterministic counterpart of temperature in Simulated Annealing [6]. The above assumptions leave us to write the posterior probability as a Gibbs distribution, i.e. the Hammersley-Clifford theorem allows us to say that the conditioned field **F**,H,V/**G** is a MRF:

$$P(\mathbf{F}, H, V/\mathbf{G}) = \frac{1}{Z} exp\{[-\beta\cdot V(\mathbf{F}, H, V/\mathbf{G})]\} \qquad (7)$$

where Z is the partition function and:

$$V(\mathbf{F}, H, V/\mathbf{G}) = \sum_{i,j}\{\sum_k[\frac{1}{2\sigma^2}(f_{i,j_k} - g_{i,j_k})^2 + \alpha((f_{i,j_k} - f_{i,j-1_k})\cdot(1-h_{i,j}) +$$

$$+(f_{i,j_k} - f_{i+1,j_k})(1-v_{i,j}) + \gamma^h_{i,j}\cdot h_{i,j} + \gamma^v_{i,j}\cdot v_{i,j})]\} \qquad (8)$$

The mean-field annealing method proposed in [1] can be used to obtain the optimum configuration also in the color case. According to this method, the partition function must be first obtained:

$$Z = \sum_{\{F_R,F_G,F_B,H,V\}} exp[-\beta\cdot V(\mathbf{F}, H, V/\mathbf{G})] = \qquad (9)$$

$$= \sum_{\{F_R,F_G,F_B\}} exp\{-\beta\sum_{i,j}[\sum_k(f_{i,j_k} - g_{i,j_k})^2 + \gamma^h_{i,j} + \gamma^v_{i,j}]\}\cdot$$

$$\sum_{\{H,V\}} exp\{-\beta\sum_{i,j}[(1-h_{i,j})\cdot G^h_{i,j} + (1-v_{i,j})\cdot G^v_{i,j}]\}$$

where:

$$G^h_{i,j} = \sum_k\left[\alpha\left(f_{i,j_k} - f_{i-1,j_k}\right)^2 - \gamma^h_{i,j}\right] \qquad (10a)$$

$$G^v_{i,j} = \sum_k\left[\alpha\left(f_{i,j_k} - f_{i,j-1_k}\right)^2 - \gamma^v_{i,j}\right] \qquad (10b)$$

After some computations, similar to those explained in [1] for grey-level images, Z can be written as:

$$Z = \sum_{\{F_R,F_G,F_b\}} exp\left[-\beta\left(V_{f,g}(f) + V_{eff}(f)\right)\right] \qquad (11)$$

where:

$$V_{eff}(f) = \sum_{i,j}\{\gamma^{*h}_{i,j} + \gamma^{*v}_{i,j} - \frac{1}{\beta}ln[(1+exp(-\beta\cdot G^h_{i,j}))\cdot$$

$$(1+exp(-\beta\cdot G^v_{i,j}))]\} \qquad (12a)$$

$$V_{fg}(f) = \sum_{i,j}\frac{1}{2\sigma^2}\sum_k\left(f_{i,j_k} - g_{i,j_k}\right)^2 \qquad (12b)$$

and

$$\gamma^{*h}_{i,j} = 3\gamma^{h}_{i,j} \qquad \gamma^{*v}_{i,j} = 3\gamma^{v}_{i,j}$$

α and $\gamma_{i,j}$ are positive parameters.

A saddle-point approximation [1] allows one to write Z as:

$$Z = c \cdot exp\left[-\beta\left(V_{fg}(\bar{f}) + V_{eff}(\bar{f})\right)\right] \qquad (13)$$

where $\bar{f}$ satisfies the relation:

$$\frac{\partial}{\partial f_{i,j}}\left(V_{fg}(f) + V_{eff}(f)\right)\Bigg|_{f=\bar{f}} = 0 \qquad (14)$$

Mean field equations for the line process variables can be obtained by solving (14) as:

$$\bar{h}_{i,j} = \frac{1}{1+exp\left[\beta\left(\gamma - \alpha\sum_{k}\left(f_{i,j_k} - f_{i-1,j_k}\right)^2\right)\right]} \qquad (15a)$$

$$\bar{v}_{i,j} = \frac{1}{1+exp\left[\beta\left(\gamma - \alpha\sum_{k}\left(f_{i,j_k} - f_{i,j-1_k}\right)^2\right)\right]} \qquad (15b)$$

where $\gamma = \gamma^{*h}_{i,j} = \gamma^{*v}_{i,j}$,

These expressions are equivalent to the grey scale case ones, except for the sum over color components which is the definition of the square distance between two vectors.

Indeed, in the grey level case $th=\sqrt{\frac{\gamma}{\alpha}}$ is the threshold for creating a line thus changing γ value will change the results of applying the method: this parameter must be set to guarantee that the highest noise effect is smoothed out without loosing edges [1]. If the local gradient vector module, is higher (lower) than *th*, an edge (no edge) is detected. For real values of β the detection is characterized by a "strength" depending on the sigmoid behaviour.

As the α parameter can be chosen equal to the grey-level case, it can be said that the γ parameter has the role of taking into account the 3D characteristic of the signal. In this paper, we suggest to use a value γ equal to the value chosen for grey-level images. This is equivalent to say that it is sufficient that a single component of the module of the gradient vector is higher than the threshold used for scalar images to create an edge (*OR* rule).

Finally, equation (14) can be solved by using a *gradient descent* method, by obtaining:

$$\bar{f}^{n+1}_{i,j_k} = \bar{f}^{n}_{i,j_k} - \omega\{(\bar{f}_{i,j_k} - g_{i,j}) + 2\alpha\sigma^2[(\bar{f}_{i,j_k} - \bar{f}_{i,j-1_k})\cdot (1-\bar{v}_{i,j}) + (\bar{f}_{i,j+1_k} - \bar{f}_{i,j_k})(1-\bar{v}_{i,j+1}) + (\bar{f}_{i,j_k} - \bar{f}_{i-1,j_k})\cdot (1-\bar{h}_{i,j}) + (\bar{f}_{i+1,j_k} - \bar{f}_{i,j_k})(1-\bar{h}_{i+1,j})]\}^{n} \qquad (16)$$

in which ω is the step of the algorithm and n represents the iteration number. Deterministic annealing [1] consists of obtaining the solution of (15) and (16) for increasing values of β, starting at each iteration n from the solution (**F,H,V**) obtained at the previous step n-*1*. The solution of (15) and (16) can be obtained by iterating the application of the expression in (16) and the computation of (15) until **F** becomes stable.

3 EXPERIMENTAL RESULTS

The method was applied both to synthetic images and to real images and the results were good in both cases.

In Fig.2 a graph is reported representing SNR vs. noise standard deviation σ: when noise standard deviation is greater then 20, algorithm parameters can be changed to obtain an image quality improvement.

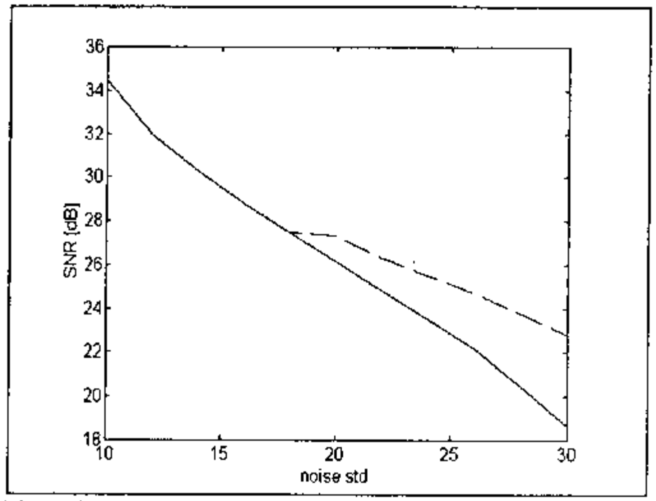

Fig.2: *SNR vs. noise standard deviation* σ*; the continuous line was obtained with* α*=0.1* ,γ*=23 and the dashed line was obtained with* α*=0.061* ,γ*=23* .

β parameter scheduling consist on ten increasing values from 0.000125 to 125000 ($\beta_{s+1} = 10\beta_s$).

In Fig.3 the results of applying the algorithm to a synthetic image are shown; we notice that the algorithm provides good results though the noise level is not low (σ=20)

In Fig.4 the results of applying the algorithm to a real image are shown; a good noise filtering of the considered image (SNR=21.26 *dB*) is obtained and also quality of edges is good.

Automatic parameters selection goes beyond the scope of this paper.

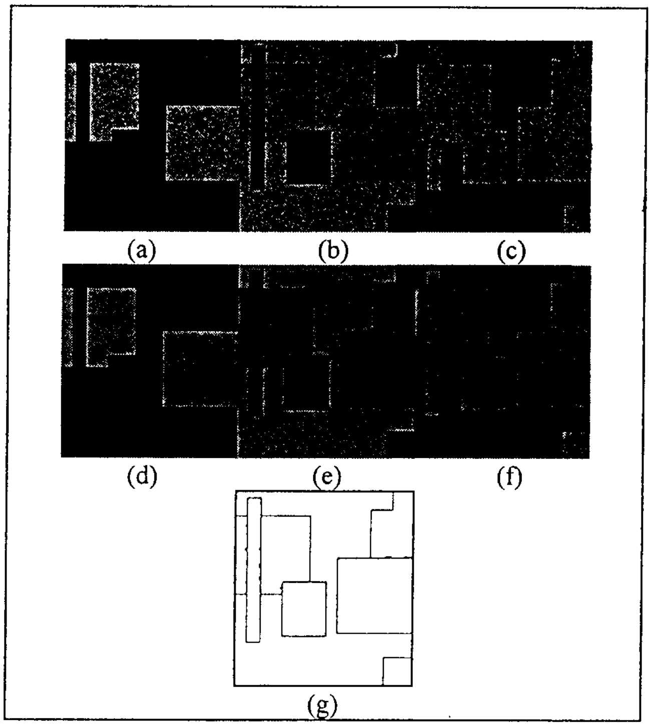

Fig.3: *Algorithm application on synthetic image (α =0.061, γ=23). (a),(b) and (c) represent the color components R, G and B of the images corrupted by Gaussian G(0,400) noise; (d),(e), and (f) show the corresponding restored images. (g) represents detected edges after algorithm convergence.*

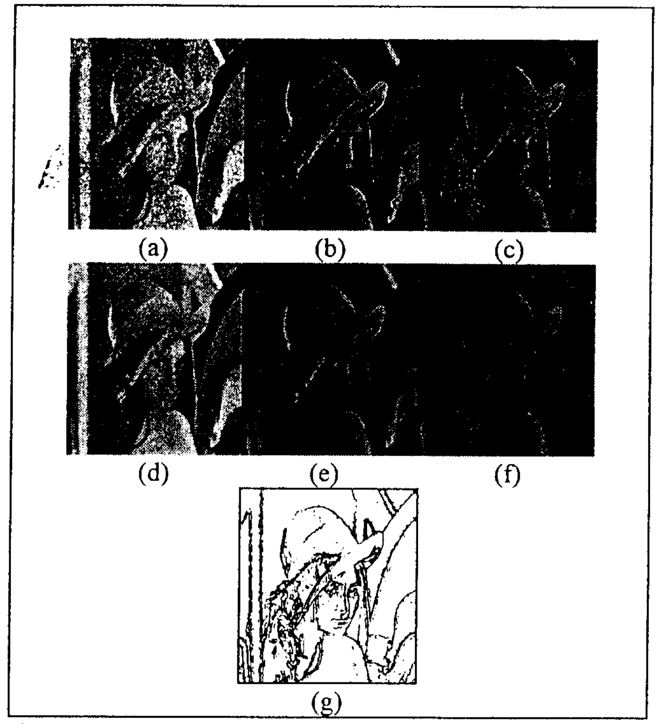

Fig.4: *Algorithm application on real image (α=0.1, γ =23). (a),(b) and (c) represent the color components R, G and B of the images corrupted by Gaussian G(0,100) noise; (d),(e), and (f) show the corresponding restored images. (g) represents detected edges after algorithm convergence.*

4 CONCLUSIONS

A MRF-based approach is presented which is based on an extension of monochromatic deterministic annealing to the color case. It is shown that the main modifications to the monochromatic case are: a) an estimation of edges based on a scalar integration (i.e., the norm of the gradient vector) of multidimensional data related to neighbourhood pixels. b) the application of the gradient descent resolution of the mean-field equation to the three color components. These considerations suggest us to exploit the proposed approach to deal with norms which take into account also phase, besides module information, and thus are more suitable for the color case; for example, in [7] a norm based on the concept of space filling curves has been introduced on which research is currently under development.

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