4. Iterative Constrained Image Deconvolution Algorithm in a Multiresolution Scheme.

We use the iterative constrained scheme with an optimized rate of convergence described in [7].

$$\hat{\mathbf{x}}_u^{(n)} = C_u \hat{\mathbf{x}}_u^{(n-1)} + \mu_u^{(n)} \mathbf{V}_u^{(n)} \qquad (20)$$

C_u is a linear or non-linear constraint operator. It is at least nonexpansive or contractive [7,8]. In image processing the image extend is a priori known, this means that a support constraint operator can be used. It can be noted that if the support of $\mathbf{x}$ is $L_m \times L_n$ then, for causal filters, the support of $\hat{\mathbf{x}}_{ll}, \hat{\mathbf{x}}_{lh}, \hat{\mathbf{x}}_{hl}, \hat{\mathbf{x}}_{hh}$ is $(L_m + L_f - 1) \times (L_n + L_f - 1)$, $(L_m + L_f - 1) \times (L_n + L_g - 1)$, $(L_m + L_g - 1) \times (L_n + L_f - 1)$, $(L_m + L_g - 1) \times (L_n + L_g - 1)$, respectively . It can be noted that if $\mathbf{x}$ is positive, then $\hat{\mathbf{x}}_u$ is not necessarily positive and the positivity constraint can not be applied. If the impulse response of the low-pass analysis filter f is positive then the positivity constraint can be applied for the deconvolution of the approximation image . For example the Haar filter [6] is positive.

The vectors $\mathbf{V}_u^{(n)}$ and the relaxation parameters $\mu_u^{(n)}$ are calculated in order to:

a) assume the convergence.

we choose

$$\mathbf{V}_u^{(n)} = \mathbf{H}^T \mathbf{y}_u - \mathbf{A}_u C_u \mathbf{x}_u^{(n-1)} \qquad (21)$$

then

$$\mu_u^{(n)} = \mu_u = constant_u \qquad (22)$$

according to the Bialy theorem a sufficient condition is

$$\mu_u \leq 2 / \left(\lambda_{\mathbf{A}_u}^2\right)_{max} \qquad (23)$$

or, both in the circular approximation and in exact linear convolution via circular convolution we obtain:

$$\mu_u \leq \frac{2}{\max\left(\left|H(k,l)\right|^2 + \alpha_u \left|B_u(k,l)\right|^2\right)} \qquad (24)$$

where $H(k,l)$ and $B_u(k,l)$ denote the 2D-DFT of the impulse response of the degrading system and of the regularization filter, respectively.

b) or to reduce a positive functional of the error [7] at each step.

$$J_u^{(n)} \leq J_u^{(n-1)} \text{ with } J_u^{(n)} = \frac{1}{2} \varepsilon_u^{(n)T} \mathbf{A}_u \varepsilon_u^{(n)}$$

where

$$\boldsymbol{\varepsilon}_u^{(n)} = \mathbf{x}_u - \mathbf{x}_u^{(n)}$$

then

$$\mu_u^{(n)} = \frac{\mathbf{V}_u^{(n)T} \mathbf{E}\mathbf{c}_u^{(n-1)}}{\mathbf{V}_u^{(n)T} \mathbf{A}_u \mathbf{V}_u^{(n)T}} \qquad (25)$$

$\mathbf{E}\mathbf{c}_u^{(n-1)}$ is the observable approximation or detail signal error at step (n-1) defined by

$$\mathbf{E}\mathbf{c}_u^{(n-1)} = \mathbf{H}^T \mathbf{y}_u - \mathbf{A}_u C_u \mathbf{x}_u^{(n-1)} \qquad (26)$$

by taking

$$\mathbf{V}_u^{(n)} = \mathbf{E}\mathbf{c}_u^{(n-1)} \qquad (27)$$

It results an optimized convergence rate [7]

It has been noted that the usual constraint of image positivity is lost when we consider the previous method of deconvolution in a multiresolution scheme except for the approximation in the case of positive impulse response of analysis low-pass filters $f(n)$. To preserve the advantage of using a positivity constraint in the iterative scheme, we propose to reconstruct the signal at each step and then to apply the positivity constraint to the reconstructed signal.

The resulting Iterative Multiresolution Deconvolution with reconstruction at each step (IMD-RS) algorithm is:

$$\hat{\mathbf{x}}_{ll}^{(0)} = 0;, \hat{\mathbf{x}}_{lh}^{(0)} = 0; \hat{\mathbf{x}}_{hl}^{(0)} = 0;, \hat{\mathbf{x}}_{hh}^{(0)} = 0;$$

loop: *for* $u = ll, lh, hl, hh$

Compute: $\mathbf{V}_u^{(n)}, \mu_u^{(n)}$

$$\hat{\mathbf{x}}_u^{(n)} = C_u \hat{\mathbf{x}}_u^{(n-1)} + \mu_u^{(n)} \mathbf{V}_u^{(n)}$$

end for

$$\hat{\mathbf{x}}^{(n)} = C\Big[\left(\tilde{\mathbf{F}} \otimes \tilde{\mathbf{F}}\right) C_{ll} \hat{\mathbf{x}}_{ll}^{(n)} + \left(\tilde{\mathbf{G}} \otimes \tilde{\mathbf{F}}\right) C_{lh} \hat{\mathbf{x}}_{lh}^{(n)} + \left(\tilde{\mathbf{F}} \otimes \tilde{\mathbf{G}}\right) C_{hl} \hat{\mathbf{x}}_{hl}^{(n)} + \left(\tilde{\mathbf{G}} \otimes \tilde{\mathbf{G}}\right) C_{hh} \hat{\mathbf{x}}_{hh}^{(n)}\Big]$$

$$\hat{\mathbf{x}}_{ll}^{(n)} = (\mathbf{F} \otimes \mathbf{F}) \hat{\mathbf{x}}^{(n)}, \hat{\mathbf{x}}_{lh}^{(n)} = (\mathbf{F} \otimes \mathbf{G}) \hat{\mathbf{x}}^{(n)}$$

$$\hat{\mathbf{x}}_{hl}^{(n)} = (\mathbf{G} \otimes \mathbf{F}) \hat{\mathbf{x}}^{(n)}, \hat{\mathbf{x}}_{hh}^{(n)} = (\mathbf{G} \otimes \mathbf{G}) \hat{\mathbf{x}}^{(n)}$$

$$\mathbf{E}^{(n)} = \left\|\mathbf{y} - \mathbf{H}\hat{\mathbf{x}}\right\|^2$$

if $1 - \frac{\mathbf{E}^{(n)}}{\mathbf{E}^{(n-1)}} \leq \varepsilon_0$ *go to loop*

else stop

5. Simulation and Results

In order to prove the effectiveness of our approach we present simulation results of the IMD-RS algorithm. The original image x has bounded support and is positive (Fig.1). The image is convolved with a boxcar type impulse response h(m,n)=1/64 m,n=1,...,8 and h(m,n)=0 elsewhere in order to have a quite singular matrix $\mathbf{H}$. The additive noise is colored high-band. We have a SNR of 20 dB.

To estimate the quality of the reconstruction we compute at each loop the true error

$$\boldsymbol{\varepsilon}^2 = \left\|\mathbf{x} - \hat{\mathbf{x}}\right\|^2$$

The regularization operator B ($B \equiv B_u$) is Laplacian.

We consider three bases for signal decomposition, we used

- -Haar wavelets base : 2-tap filter [4]
- -Daubechies wavelets base: 9-7-tap filters [9].
- -Coiflet wavelets base: 18-tap filter [9]

As it has been observed in [6] for signal deconvolution in a wavelet multiresolution scheme the best results are obtained with aselective filters.

6. Conclusion

In this paper we have presented an algorithm for the constrained deconvolution of images in a wavelet multiresolution scheme. In comparaison with the full-band deconvolution a significant gain in terms of residual error has been observed.