REGULARIZED IMAGE DECONVOLUTION IN A WAVELET SCHEME.

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Abstract

This paper addresses the problem of deconvolution in a multiresolution scheme. It results a deconvolution problem at each level of resolution. The Miller regularized approach is used and the normal equations are solved using a constrained iterative algorithm. Simulations show the advantages of this approach.

1. Introduction

We consider the general problem of image deconvolution. The image forming mathematical model is

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \qquad (1)$$

where the vectors $\mathbf{y}$, $\mathbf{x}$, $\mathbf{n}$ represent the row-ordered distorded image, the original image and the additive noise, respectively. The degradation is space invariant, it follows that the matrix $\mathbf{H}$ is a block Toeplitz matrix constructed with the point spread function (psf). The degrading operator $\mathbf{H}$ is perfectly known. Most of the previously reported works in the literature consider a white noise. This paper addresses the image deconvolution when the noise $\mathbf{n}$ is colored.

Following Miller [1] we will consider the regularized mean square solution of (1). This solution minimizes the functional:

$$J\{\mathbf{x},\alpha\} = \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 + \alpha\|\mathbf{B}\mathbf{x}\|^2 \qquad (2)$$

where $\mathbf{B}$ is the regularization operator and α the regularization parameter.

The result requires the resolution of the following normal equations:

$$\left(\mathbf{H}^T\mathbf{H} + \alpha\mathbf{B}^T\mathbf{B}\right)\hat{\mathbf{x}} = \mathbf{H}^T\mathbf{y} \qquad (3)$$

The role of $\mathbf{B}$ is to move the small eigenvalues of $\mathbf{H}$ away from zero and to incorporate prior knowledge about the regularity of the solution. The parameter α ensures a compromise between the available data and the prior knowledge. The solution of (3) is not generaly positive. To overcome this difficulty an iterative constrained algorithm is often used to obtain a positive solution close to $\hat{\mathbf{x}}$.

In order to adapt both the regularization parameter and the constraint into frequency bands taking into account the colored noise and the frequency distribution of the image energy, we consider the image deconvolution in a wavelet multiresolution scheme.

2. Modeling the Deconvolution in a multiresolution scheme.

Deconvolution in a wavelet based multiresolution scheme has been a recent subject of study. Although it has been adressed in many differents ways [2-3] the proposed approach is new.

Wavelet decomposition of a discrete image into an approximation at a coarser resolution 2^{-1} and vertical, horizontal and diagonal images details requires firstly filtering the image rows by both a low-pass 1-D filter f and a high-pass 1-D filter g and decimating the outputs and secondly proceed in a similar maner for the columns of two resulting sub-images [4]. The approximation-image corresponds to both the rows and columns low-frequency (ll). The vertical (resp. horizontal) detail-image corresponds to both the rows high frequencies (resp. low) and the columns low frequencies (resp. high) (hl resp. lh). The diagonal detail-image corresponds to both the rows and columns high frequencies (hh). Filters f and g are derived from a scaling function $\phi(t)$ and from $\phi(t)$ and a wavelet $\psi(t)$, respectively. It can be noted that the decimation process is used for the non-expansion of the data which is of interest in image compression schemes. In sharp contrast with data compression the decimation is without technical interest in the deconvolution problem. Then in our approach the output of the filters are not decimated. A four-band decomposition is illustrated in Fig.1 while the reconstruction scheme is depicted in Fig.2.

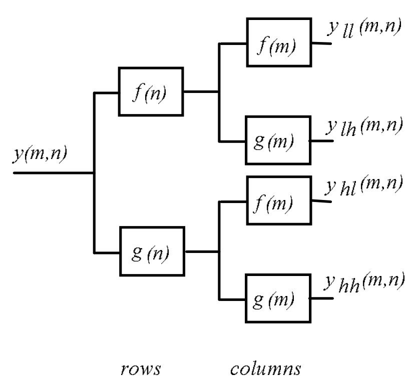

Fig. 1 Four-band image decomposition