PERFORMANCE INDICATORS OF THE CORRELATION PROCESS FOR NON AMBIGUOUS DOPPLER FREQUENCY ESTIMATION IN MULTIPLE PRF RADARS

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ABSTRACT

This communication investigates the performance of alias-free Doppler frequency estimation in multiple Pulse Repetition Frequency radar systems. Three performance indicators are proposed for Doppler ambiguity resolution algorithms based on the use of a correlation interval of given width : probabilities of correlation, of false correlation and of false measurement. Under the assumption of Gaussian errors on the ambiguous frequencies estimates for each PRF, closed forms (function only of the interval width, the PRF values and the estimation variance on the ambiguous frequencies) are derived for these indicators. Some examples of the expected behavior of MPRF systems obtained with these indicators are presented and discussed.

1. PROBLEM STATEMENT

Moving target signals (hereafter supposed narrowband) from pulsed Doppler surveillance radars operating with a Pulse Repetition Frequency (PRF) F_r consist of a complex sinusoid at the frequency f_D related to the radial velocity of the target v_r by $f_D = 2v_r/\lambda$, where λ is the wavelength of the carrier, and sampled at the frequency F_r, [8]. With a constant PRF, the Doppler frequency f_D can only be estimated modulo F_r. Consequently, fast modern aircrafts velocity surveillance leads to ambiguous Doppler measurements. The ambiguity phenomenon may also be present for distance measurements. By choosing a low PRF, the radar measurements are non ambiguous in distance but highly ambiguous in Doppler frequency. However, the knowledge of the Doppler frequency of the target (i.e. its radial velocity) turns out to be useful to perform some radar tasks such as target tracking and classification, conflict alert, ... Consequently, these ambiguities have to be resolved. The resolution of aliased measurements is performed by the implementation of multiple PRF (MPRF) systems and has been addressed in several papers, [1, 6, 9]. The basic idea is then to search for coincidences (or correlations) between the unfolded ambiguous frequencies estimated on each burst. The frequency for which correlation occurs is estimated to be the "true" Doppler frequency. Note that this correlation process can be performed either coherently or incoherently, [2, 4, 5]. Because of estimation errors on the ambiguous frequencies computed on each burst, the perfect matching between unfolded frequency estimates is never possible and has to be replaced by a correlation (or matching) interval of given width : a coincidence is then declared when all the unfolded values of the Doppler frequency estimates on each burst lie in the same interval. As the noise level on the signal increases, this matching interval has to widen leading to possible false coincidences. Thus, when choosing the matching interval width, one has to come to a compromise between the risk of false coincidences and the risk to miss the right coincidence. Accordingly, the main purpose of this communication is to tackle the problem of false correlations and to quantify the influence of the choice of the matching interval width on the alias-free Doppler frequency estimation quality. This will be achieved through the definition of performance indicators for the correlation procedure. Section 2 is devoted to some details on alias-free Doppler frequency estimation algorithm. In Section 3, the definitions of three performance indicators are given and it is shown that closed form expressions can be derived in the particular case of Gaussian errors on the ambiguous frequency estimates. In Section 4, numerical examples are presented and some conclusions are drawn.

2. ALIAS-FREE DOPPLER ESTIMATION

2.1. Multiple PRF (MPRF) waveform

It is assumed that the MPRF waveform used to perform the alias-free Doppler estimation consists of N_b bursts, each with a different PRF denoted $F_r(k)$ on the k^{th} burst and n_s pulses per burst. The target signal is supposed to be present on N_b bursts. It is then possible to perform classical frequency estimation on every signal block which leads to N_b ambiguous Doppler frequency estimates denoted $\hat{f}_k$ on burst k. These ambiguous estimates $\{\hat{f}_k\}$ are all related modulo the set of PRF $\{F_r(k)\}$. Thus, the unambiguous Doppler frequency can be recovered by searching for coincidences between the unfolded ambiguous frequencies $\{\hat{f}_k + n_k F_r(k)\}$, [2].

2.2. "Searching for coincidences" algorithms

Because of estimation errors on the ambiguous frequency estimates, the exact matching between the unfolded ambiguous frequencies never occurs. Thus, instead of searching for an exact coincidence, the current solution is to search for an approached coincidence that is declared when all the unfolded frequencies lie "sufficiently" close to each other. In this communication, we consider only algorithms based on the use of a "correlation interval" of given width. The common principle of these algorithms is to search for a set of integers $\{n_k\}$ such that the unfolded frequencies $\{\hat{f}_k + n_k F_r(k)\}$ all lie in the same "correlation interval" of width d. Considering only two PRF $F_r(k)$ and $F_r(l)$, the searched integers $n_k^\star$ and $n_l^\star$ have to be such that :

$$|\hat{f}_k + n_k^\star F_r(k) - (\hat{f}_l + n_l^\star F_r(l))| < d$$

Denoting $\delta\hat{f}_k$ the estimation error on the ambiguous estimate $\hat{f}_k$ and for the "good" (i.e. corresponding to $f = f_D$) value of $n_k = n_k^\star$, we get :

$$\hat{f}_k + n_k^\star F_r(k) = f_D + \delta\hat{f}_k$$

Thus, the correlation condition for the "good" values of $n_k = n_k^\star$ and $n_l = n_l^\star$ writes :

$$|\delta\hat{f}_k - \delta\hat{f}_l| < d$$

or, denoting for convenience $(\delta\hat{f}_k - \delta\hat{f}_l)$ by δ_{kl} :

$$|\delta_{kl}| < d$$

Several algorithms are available to perform the task of searching for correlations, [3, 7, 6]. Their main differences become visible when they are used in a more-than-two PRF system, through the way they combine the ambiguous frequency information obtained from each burst. But, whatever algorithm is chosen, the quality of the alias-free estimation remains highly dependent on the parameters of the MPRF system (i.e. PRF values and width of the matching interval). This is the reason why it is interesting to have at one's disposal indicators on the expected behavior of a MPRF system from the only MPRF parameters and independent of a particular correlation algorithm.

3. PERFORMANCE INDICATORS

To study the performance of the alias-free Doppler frequency estimation in MPRF systems, three indicators have been defined :

3.1. Correlation probability

In this section, we want to quantify the probability of correlation P_c for the frequency f_D. First, let us assume that only two PRF ($F_r(k)$ and $F_r(l)$) out of the N_b PRF possible are used to estimate the Doppler frequency f_D. The correlation criterion for the frequency $f = f_D$ is given by :

$$|\delta\hat{f}_k - \delta\hat{f}_l| < d$$

The estimation errors $\delta\hat{f}_k$ on ambiguous frequencies are assumed to be realisations of the independent Gaussian random variables $\Delta\hat{f}_k \sim \mathcal{N}(0, \sigma_{f,k}^2)$. The random variable $\Delta_{kl} = \Delta\hat{f}_k - \Delta\hat{f}_l$ is then Gaussian $\mathcal{N}(0, \sigma_{f,k}^2 + \sigma_{f,l}^2)$:

$$f_{\Delta_{kl}}(\delta_{kl}) = \frac{1}{\sqrt{2\pi(\sigma_{fk}^2 + \sigma_{fk}^2)}} \exp(-\frac{\delta_{kl}^2}{2(\sigma_{f,k}^2 + \sigma_{f,k}^2)})$$

The correlation probability for f_D is defined as the probability that a correlation is found for the "true" value of the non ambiguous Doppler frequency $f = f_D$, which writes :

$$\begin{aligned} P_c &= \text{Prob}[|\delta\hat{f}_k - \delta\hat{f}_l| \leq d] \\ &= \text{Prob}[-d \leq \delta\hat{f}_k - \delta\hat{f}_l \leq d] \\ &= \text{Prob}[-d \leq \delta_{kl} \leq d] \end{aligned}$$

which leads to :

$$P_c = 2\int_0^d f_{\Delta_{kl}}(\delta_{kl}) d\delta_{kl} = \text{erf}\left[\frac{d}{\sqrt{2}\sqrt{\sigma_{f,k}^2 + \sigma_{f,l}^2}}\right]$$

where erf(x) is the classical *error function*. When N_b different PRF are used to find out N correlations for ambiguity resolution, the total probability P_C is upper-bounded by :

$$P_C \geq P_c^N$$

If a "n correlations-out-of-N possible" criterion is adopted to decide for the actual presence of a correlation, the total correlation probability P_C is, [1] :

$$P_C = \sum_{k=0}^{N-n} C_N^k P_c^n (1 - P_c)^{N-n}$$

3.2. False correlation probability

In this section, we investigate the probability that a correlation occurs for a frequency $f \neq f_D$. It can be shown, that for a given position, the distance between the unfolded frequencies (supposed with no estimation

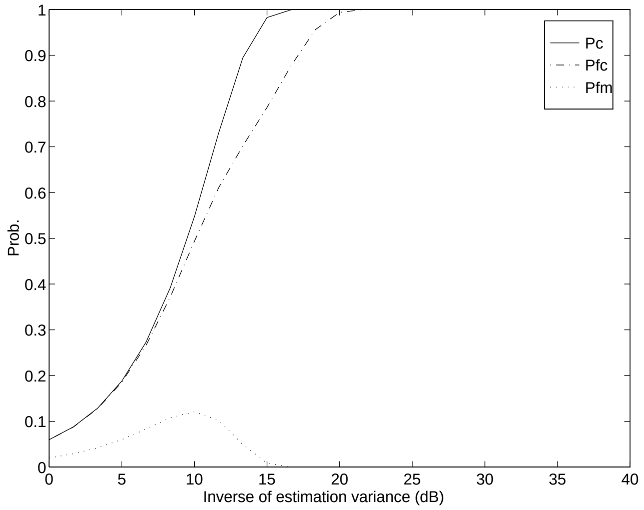

Figure 1: Probabilities P_c, P_{fc} and P_{fm} as a function of the estimation variance on the ambiguous frequencies. $F_r(1) = \frac{20}{19}$, $F_r(2) = \frac{20}{21}$ and $d = 1.5d_{min}$

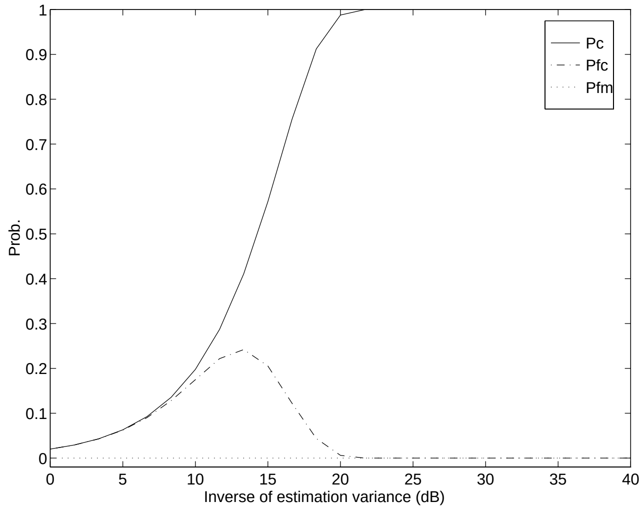

Figure 2: Probabilities P_c, P_{fc} and P_{fm} as a function of the estimation variance on the ambiguous frequencies. $F_r(1) = \frac{20}{19}$, $F_r(2) = \frac{20}{21}$ and $d = 0.5d_{min}$

error) reaches a minimum denoted d_{min} (hereafter, d_{min} is supposed to be positive). Denoting n_k and n_l the two characteristic integers of this position, the difference between the unfolded frequency estimates is :

$$|(\hat{f}_l + n_l F_r(l)) - (\hat{f}_k + n_k F_r(k))| = |\delta_{kl} + d_{min}|$$

The false correlation probability P_{fc} is maximum for this position and is given by :

$$\begin{aligned} P_{fc} &= \text{Prob}[|\delta_{kl} + d_{min}| \leq d] \\ &= \text{Prob}[-d - d_{min} \leq \delta_{kl} \leq d - d_{min}] \end{aligned}$$

Some calculations, [2], leads to :

$$P_{fc} = \frac{1}{2}\,\text{erf}\left[\frac{d + d_{min}}{\sqrt{2}\sigma}\right] + \frac{1}{2}\,\text{erf}\left[\frac{d - d_{min}}{\sqrt{2}\sigma}\right] \quad \text{if } d_{min} < d$$

$$P_{fc} = \frac{1}{2}\,\text{erf}\left[\frac{d + d_{min}}{\sqrt{2}\sigma}\right] - \frac{1}{2}\,\text{erf}\left[\frac{d_{min} - d}{\sqrt{2}\sigma}\right] \quad \text{if } d_{min} > d$$

where $\sigma = \sqrt{\sigma_{f,k}^2 + \sigma_{f,l}^2}$. Rigorously, the computation of P_{fc} should take into account all the possibilities of false coincidence, even for positions other than the one corresponding to d_{min}. However, the contribution of the term corresponding to d_{min} is the major one, and the above expression gives a correct approximation of P_{fc}.

Now, let us examine in details the closest distance value d_{min}. To derive an expression of d_{min}, we consider the case where only two PRF $F_r(1)$ and $F_r(2)$ are used and are such that :

$$\frac{F_r(1)}{F_r(2)} = \frac{m}{n}$$

where $n > m$ and m and n are mutual prime numbers.

Assuming that $f_D = 0$, the ambiguous frequencies for the two PRF $F_r(1)$ and $F_r(2)$ are both equal to 0 and the distance between two given unfolded ambiguous frequencies is :

$$d = n_1 F_r(1) - n_2 F_r(2) = (n_1 m - n_2 n)\frac{F_r(2)}{n}$$

Following Bezout theorem, one can found two integers n_1 and n_2 such that $|n_1 m - n_2 n|$ is minimum and equal to 1. This leads to the following expression of d_{min} :

$$d_{min} = \frac{F_r(2)}{n} = \frac{F_r(1)}{m}$$

The presence of a false correlation for $f \neq f_D$ does not exclude the possibility of a "good" correlation for $f = f_D$. Thus, the P_{fc} is not strictly equal to a false measurement of the non ambiguous Doppler frequency, since one can have both "good" and false correlation at the same time. The false measurement will then depend on the algorithm used to decide between the two candidate correlations. The next section is devoted to the definition of a false measurement probability which corresponds to the conjunction of a false correlation for $f \neq f_D$ and no "good" correlation for $f = f_D$.

3.3. False measurement probability

We assume that $d_{min} > 0$. The false measurement probability is defined as :

$$P_{fm} = \text{Prob}[(|\delta_{kl} + d_{min}| < d) \cap (|\delta_{kl}| > d)]$$

Some mathematical developments, [2], lead to the following expressions of P_{fm} :

$$P_{fm} = \frac{1}{2}\,\text{erf}\left[\frac{d + d_{min}}{\sqrt{2}\sigma}\right] - \frac{1}{2}\,\text{erf}\left[\frac{d}{\sqrt{2}\sigma}\right] \quad \text{if } d_{min} < d$$

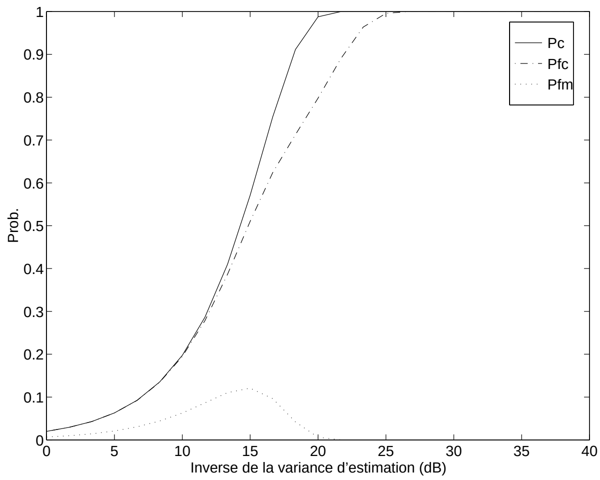

Figure 3: Probabilities P_c, P_{fc} and P_{fm} as a function of the estimation variance on the ambiguous frequencies. $F_r(1) = \frac{60}{59}$, $F_r(2) = \frac{60}{61}$ and $d = 1.5d_{min}$

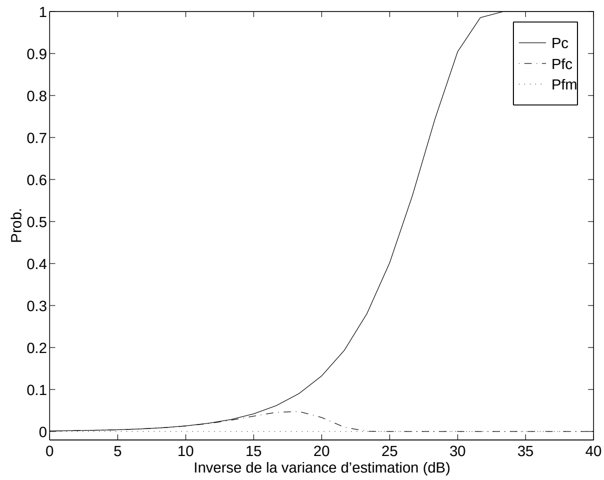

Figure 4: Probabilities P_c, P_{fc} and P_{fm} as a function of the estimation variance on the ambiguous frequencies. $F_r(1) = \frac{60}{59}$, $F_r(2) = \frac{60}{61}$ and $d = 0.5d_{min}$

$$P_{fm} = \frac{1}{2}\,\text{erf}\left[\frac{d_{min}}{\sqrt{2}\sigma}\right] - \frac{1}{2}\,\text{erf}\left[\frac{d}{\sqrt{2}\sigma}\right] \text{ if } 2d \geq d_{min} > d$$

$$P_{fm} \approx 0 \text{ otherwise}$$

It is noteworthy that the expressions of P_c, P_{fc} and P_{fm} are only function of the matching interval width d, the PRF values through d_{min} and the variance σ on the ambiguous frequencies estimates.

4. NUMERICAL SIMULATIONS

Figures (1) to (4) sketch the evolution of P_c, P_{fc} and P_{fm} as a function of the variance on the ambiguous frequencies estimates to be correlated for different PRF values $F_r(1)$ and $F_r(2)$ and different width of the matching interval. These figures illustrate the significant influence of the choice of the correlation interval width and PRF values on the performances of a MPRF alias-free estimation system. It can be seen that a compromise has to be reached between a high P_c and a low P_{fm}. For instance, a narrow correlation interval considerably lowers P_c and P_{fc} but shifts the P_c curve towards lower estimation variance on the ambiguous frequencies. In particular, if d is taken greater than d_{min}, the P_c, but also the P_{fc}, tends towards 1 when the estimation variance decreases, see figures (1) and (3). On the contrary, if d is chosen smaller than d_{min}, the P_{fc} tends towards 0 when the estimation variance decreases, see figures (2) and (4). With regard to the choice of the PRF values, it is to be noticed that selecting closer $F_r(k)$ allows to lower the P_{fc} and the P_{fm} but also shifts the P_c curve to the right.

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