TIME DELAY ESTIMATION IN A MULTIPATH CONTEXT

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ABSTRACT

A second-order blind deconvolution algorithm is utilized to improve on interception and classification procedures. It consists of applying the subspace decompostion algorithm described in [10] to several portions of the observation received on a single sensor, and then of estimating the source signal cleaned from its interferences caused by the multipath propagation. Asymptotic performances are lastly analyzed in terms of mean and variance of the estimated filters.

1 Introduction

It is assumed that a useful unknown spread-spectrum signal $s(t)$ is received on a single sensor, corrupted by multipaths and noise. The observation can be modeled as:

$$r(t) = \sum_{p=1}^{K} \alpha_p s(t - \tau_p) + v(t). \tag{1}$$

$v(t)$ denotes the noise. Equation (1) is the result of multiple reflections of the transmitted signal in the propagation channel. The goal is to retrieve the useful signal $s(t)$ from observation $r(t)$, on one hand, and to estimate time delays τ_p and attenuations α_p on the other. This problem is typical in sonar, radar and communication systems.

The case of multipath propagation has been considered by several authors, but with differents hypotheses. In [9], the statistcs of the signal are known. In [2] and [1], the source signal is unknown, but the second sensor receives only the signal plus the noise. In [11], the author considers two sensors, and uses the location of the correlation function's peaks in order to initialize an adaptative Kalman-type algorithm. We can finally mention [4], [5] based on the maximum likelihood. The latter works suppose the source statistics are known. In the present paper, the signal and its statistics are unknown, but its support splits into at least two disjoint portions.

The paper is organized as follows. Section 2 shows how to move from the single sensor case to a multichannel observation. The identification method is described in section 3, and two solutions are considered to obtain the source signal in section 4. In section 5, the statistical performances are derived, followed by simulations (section 6).

2 Problem formulation

The first idea would be to apply the technique proposed in [10] directly to the array of processors. Unfortunately, in interception problems, the sensors are too few and too close for this method to be applicable (it would be ill conditioned). The idea is thus to take single sensor (or beam) and try to follow the same kind of approach.

In sonar interception for instance, the orders of magnitude of various parameters (delays, wavelengths, celerity) are such that the paths gather in clusters; each cluster corresponds to a given number of reflections on the bottom. In practice, the two first clusters are sufficient for the problem to be solved in presence of one source. The first cluster contains paths reflected 0 or 1 time on the bottom, and 0, 1, or 2 times on the surface; the second cluster contains paths reflected twice on the bottom, and 1, 2, or 3 times on the surface.

By isolating the two above mentioned clusters, that are clearly separated, the following (artificial) observations can be built [3]:

$$r_1(t) = \sum_{p=1}^{N_1} \alpha_p s(t - \tau_p) + v_1(t), \tag{2}$$

$$r_2(t) = \sum_{p=N_1+1}^{N_2} \alpha_p s(t - \tau_p) + v_2(t). \tag{3}$$

Note that delays are of course different, and must be numbererd accordingly. Denote $h_1(t)$ and $h_2(t)$ the respective filters applied to $s(t)$, giving $r_1(t) = h_1 * s(t)$ and $r_2(t) = h_2 * s(t)$.

3 Identification method

3.1 Principle

Because the useful signal $s(t)$ has a spread spectrum, we do not want to process $r_i(t)$ in narrow bands. Denote $H_i(z)$ the $z-$transform of $h_i(t)$. Since h_i represent very different paths, polynomials $H_i(z)$ are prime to each other, and the procedure described in [10] applies. In other words, $s(z)$ can be viewed as the GCD of $r_1(z)$ and $r_2(z)$. However, it cannot be computed this way because polynomials $r_i(z)$ have a very high degree. It is more convenient to use the covariance matrix of vector $[r_1(t)\ r_2(t)]$.

In a second stage, time delays can be estimated from $h_i(t)$ by finding the location of its maxima. For this purpose, because delays are not integer multiples of the sampling period, the search must be combined with an interpolation procedure.

3.2 Method used

There are a lot of blind identification methods, but the one described in [10] includes the case of a wide-band source, and does not resorts to high-order statistics. It is assumed that the largest length of the two filters, M, is known. With the assumed notation, we have

$$r_i(t) = \sum_{m=0}^{M} h_i(t-m)s(m) + v_i(t). \qquad (4)$$

Considering N samples of r_i, we build vectors:

$$R_i(n) = [r_i(n), ..., r_i(n-N+1)]^T,$$

so that equation (4) becomes:

$$R_i(n) = \mathcal{H}_i(N)S(n) + V_i(n),$$

where:

$$\mathcal{H}_i(N) = \begin{pmatrix} h_i(0) & \cdots & h_i(M) & 0 & \cdots & \cdots & 0 \\ 0 & h_i(0) & \cdots & h_i(M) & 0 & \cdots & 0 \\ \vdots & & & & & & \vdots \\ 0 & \cdots & \cdots & 0 & h_i(0) & \cdots & h_i(M) \end{pmatrix},$$

$$\text{and} \quad S(n) = [s(n), ..., s(n-N-M+1)]^T.$$

If we take into account two values $i = 1, 2$, then:

$$\begin{pmatrix} R_1(n) \\ R_2(n) \end{pmatrix} = \begin{pmatrix} \mathcal{H}_1(N) \\ \mathcal{H}_2(N) \end{pmatrix} S(n) + \begin{pmatrix} V_1(n) \\ V_2(n) \end{pmatrix},$$

or in compact notation:

$$R(n) = \mathcal{H}(N)S(n) + V(n). \qquad (5)$$

Write $\mathcal{R}_f$ the covariance matrix of a signal f. Then:

$$\mathcal{R}_r = \mathcal{H}(N)\mathcal{R}_s\mathcal{H}(N)^T + \mathcal{R}_v,$$

The method used requires the fact that the matrix $\mathcal{H}(N)$ is full rank, yielding the building of two filters $H_1(z)$ and $H_2(z)$ with no common zeros [10].

The method is based on the subspace decomposition. The $M+N$ eigenvectors corresponding to the largest eigenvalues of $\mathcal{R}_r$ span the signal subspace and the remaining $M-N$ the noise subspace. Since the signal subspace is also spanned by the columns of $\mathcal{H}(N)$, one obtains:

$$G_i^T\mathcal{H}(N) = 0 \quad \text{for} \quad 1 \le i \le N-M, \qquad (6)$$

with $G_i = [g_{1,i}(0), \cdots, g_{1,i}(N-1), g_{2,i}(0), \cdots, g_{2,i}(N-1)]^T$. This relation defines the matrix $\mathcal{H}(N)$ up to a multiplicative constant [10].

In order to estimate $H = [h_1(0), \cdots, h_1(M), h_2(0), \cdots, h_2(M)]^T$, system (6) is solved in the mean-squares sense. Yet, the search for H corresponds to the minimisation of the following quadratic system:

$$q(H) = \sum_{i=1}^{N-M} |G_i^T\mathcal{H}(N)|^2,$$

which is the equivalent to the minimisation of:

$$q(H) = \sum_{i=1}^{N-M} |H^T\mathcal{G}_i(N)|^2,$$

where $\mathcal{G}_i$ is a matrix $2(M+1)\times(M+N)$ defined by:

$$\mathcal{G}_i = \begin{pmatrix} G_{1,i} & 0 & \cdots & 0 \\ 0 & G_{1,i} & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & 0 & G_{1,i} \\ G_{2,i} & 0 & \cdots & 0 \\ 0 & G_{2,i} & \cdots & 0 \\ \vdots & & & \vdots \\ 0 & \cdots & 0 & G_{2,i} \end{pmatrix},$$

with $G_i = [G_{1,i}, G_{2,i}]^T$. Writing $\mathcal{Q} = \sum_{i=0}^{N-M-1} \mathcal{G}_i\mathcal{G}_i^T$, we have thus to minimze the following expression:

$$q(H) = H^T\mathcal{Q}H. \qquad (7)$$

In order to avoid the null solution, a constraint such as $||H|| = 1$ or $H^Tc = 1$ must be imposed. In the former case, the solution H is then the eigenvector corresponding to the smallest eigenvalue of $\mathcal{Q}$.

4 Source signal estimation

4.1 Classical solution

The source signal reconstruction can be performed by using only one estimated filter (h_1 for example), and inverse it to obtain $s(t)$:

$$s(n) = \frac{r_1(n)}{h_1(0)} - \frac{\sum_{p=1}^{M} h_1(p)s(n-p)}{h_1(0)}. \qquad (8)$$

This solution is not robust, because nothing ensures that h_1 admits a stable inverse. If there are zeros outside the unit circle, one could replace their modules by their inverse. The system becomes minimum phase, and the spectrum remains unchanged. However, this procedure does not give entire satisfaction.

4.2 Robust solution

It is better to utilize the two estimated filters h_1 and h_2, and find two polynomial $u_1(z)$ and $u_2(z)$ satisfying the Bezout identity [3]:

$$H_1(z)u_1(z) + H_2(z)u_2(z) = 1. \tag{9}$$

These two polynomials are then used to build the source signal according to the Gauss relation:

$$r_1(z)u_1(z) + r_2(z)u_2(z) = s(z). \tag{10}$$

The Bezout identity (9) tells how to compute polynomials u_1 and u_2 but the degree of the choosen polynomials may be too high. So it is relevant to construct a mean-squares solution. Denote $H_{T,i}$ the following matrix:

$$H_{T,i} = \begin{pmatrix} h_i(0) & 0 & \dots & 0 \\ h_i(0) & h_i(1) & \dots & 0 \\ \vdots & & & \vdots \\ h_i(0) & \dots & h_i(M-1) & h_i(M) \end{pmatrix};$$

equation (9) then becomes:

$$(H_{T,1}, H_{T,2}) \begin{pmatrix} u_1(0) \\ .. \\ u_1(M) \\ u_2(0) \\ \dots \\ u_2(M) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ \dots \\ 0 \end{pmatrix}. \tag{11}$$

If we change the vector $[1,0,...,0]^T$ into the vector $[1,\epsilon]^T$ where ϵ is close to zero, and note $a^T = [h_1(0), 0, \cdots, 0, h_2(0), 0, ..., 0]$ and $u^T = [u_1(0), .., u_1(M),\ u_2(M), ..., u_2(M)]$, we obtain the following system, with same left-hand side as (11):

$$\begin{pmatrix} a^T \\ A \end{pmatrix} u = \begin{pmatrix} 1 \\ \epsilon \end{pmatrix}. \tag{12}$$

The minimisation of the norm of Au subject to $a^T u = 1$ yields:

$$u = \frac{A^{-1}a^T}{a^T A^{-1} a}.$$

The quantity $(a^T A^{-1} a)^{-1}$ can also be used as a test variable to detect the optimal degree of $u_i(z)$.

5 Statistical performances

In this section, the statistical performances of the estimated filter $\hat{H}$ are investigaed when the covariance $\mathcal{R}_r$ is estimated from a limited (but asymptotically large) number of independent snapshots, P.

5.1 Moments of $\hat{\mathcal{G}}_i$

In [7], first and second moments of eigenvectors $\hat{G}_i$ of the estimated covariance matrix are given. These results can be used to compute the first and second moments of the estimated eigenvectors matrix, $\hat{\mathcal{G}}_i$:

$$E[\hat{\mathcal{G}}_i] = \mathcal{G}_i \left(1 - \frac{1}{2P} \sum_{\substack{j \neq i \\ j=1}}^{2N} \frac{\lambda_i \lambda_j}{(\lambda_i - \lambda_j)^2} \right), \tag{13}$$

$$\begin{aligned} E[\hat{\mathcal{G}}_i \hat{\mathcal{G}}_i^T] = & \frac{1}{P} \sum_{\substack{j \neq i \\ j=1}}^{2N} \frac{\lambda_i \lambda_j}{(\lambda_i - \lambda_j)^2} \mathcal{G}_j \mathcal{G}_j^T + \\ & \left(1 - \frac{1}{P} \sum_{\substack{j \neq i \\ j=1}}^{2N} \frac{\lambda_i \lambda_j}{(\lambda_i - \lambda_j)^2} \right) \mathcal{G}_i \mathcal{G}_i^T, \end{aligned} \tag{14}$$

where G_i is the eigenvector of $\mathcal{R}_r$ associated with eigenvalue λ_i.

5.2 Moments of $\hat{H}$

The minimisation of $H^T \mathcal{Q} H$ can be carried out under two different constraints: $H^T c = 1$ or $||H||^2 = 1$.

5.2.1 Constraint $H^T c = 1$

The solution of the minimisation is defined up to a multiplicative constant, μ, that will be ignored in the performance evaluation:

$$H = \mu \mathcal{Q}^{-1} c. \tag{15}$$

Now let us compute the moments of $\hat{H}$. Using equation (15), one can write:

$$E[\hat{H}] = \mu E[\hat{\mathcal{Q}}^{-1}] c.$$

To obtain $E[\hat{\mathcal{Q}}^{-1}]$, we expand up to first order the inverse of $\hat{\mathcal{Q}} = \mathcal{Q}(I + \Delta)$, where Δ is a perturbation that deduces from (13):

$$\begin{aligned} E[\hat{H}] = & \left(I - \frac{1}{P} \sum_{i=1}^{L} \sum_{\substack{j \neq i \\ j=1}}^{2N} \frac{\lambda_i \lambda_j}{(\lambda_i - \lambda_j)^2} \right. \\ & \left. \cdot (\mathcal{Q}^{-1} \mathcal{G}_j \mathcal{G}_j^T - \mathcal{Q}^{-1} \mathcal{G}_i \mathcal{G}_i^T) \right) H. \end{aligned} \tag{16}$$

In order to obtain the second moment of $\hat{H}$, we write: $\hat{\mathcal{Q}}^{-1} = \mathcal{Q}^{-1}(I - \Delta) + o(\Delta)$, and get:

$$E[\hat{H}\hat{H}^T] \approx \mathcal{Q}^{-1} c c^T \mathcal{Q}^{-1} - \mathcal{Q}^{-1} \{ E[\Delta] c c^T + c c^T E[\Delta] \} \mathcal{Q}^{-1}, \tag{17}$$

where the terms in Δ^2 are neglected. Using equation (16) the term $E[\Delta]$ could be calculated explicitly; we do not report here these easy calculations. As a conclusion, the second moment (and the variance) of $\hat{H}$ can be computed.

5.2.2 Constraint $H^T H = 1$

Under this constraint, the solution $\hat{H}$ is the eigenvector of $\hat{\mathcal{Q}}$ associated with the smallest eigenvalue. So the procedure is in principle similar to the one used in section 5.1. But the difficulty is that now, the number K of degrees of freedom is unknown. So we need to estimate K in a first step.

Since $\hat{\mathcal{G}}_i$ are asymptotically Gaussian, matrix $\hat{\mathcal{Q}}$ is asymptotically Wishart, of size $q \times q$ with $q = 2(M+1)$, and with K degrees of freedom. In fact, matrix $\hat{\mathcal{Q}}$ can be decomposed in the sum of K independent vectors, each of covariance Σ: $\hat{\mathcal{Q}} = \sum_{i=1}^{K} x_i x_i^T$, where Σ and K are unknown. Nevertheless, it is possible to estimate them using the standard expressions of the moments of Wishart matrices [6] [8]:

$$E[\hat{\mathcal{Q}}\Sigma^{-1}] = K I_q, \tag{18}$$

$$E[\hat{\mathcal{Q}}^{-1}\Sigma] = \frac{1}{K-q-1} I_q. \tag{19}$$

Multiplying the results of equations (18) and (19) yields the relation: $K = (K-q-1)\, E[\hat{\mathcal{Q}}]\, E[\hat{\mathcal{Q}}^{-1}]$.

Thus, we are able to access K as soon as we have an estimate of $E[\hat{\mathcal{Q}}]$ and $E[\hat{\mathcal{Q}}^{-1}]$. The same kind of result as before can be used, but a first order expansion of $E[\hat{\mathcal{Q}}^{-1}]$ is now not sufficient, and it is necessary to go up to second order. Again, the explicit expressions are not reported here for reasons of space; besides, their calculation does not present any difficulty.

Once K is known, the mean and variance of $\hat{H}$ can be computed as in section 5.1. If we denote H_i the eigenvectors of Q with $H = H_1$ and ν_i the eigenvalues, we obtain:

$$\begin{aligned} E[\hat{H}] &= H\Big(1 - \frac{1}{2K}\sum_{j=2}^{2(M+1)} \frac{\nu_1\nu_j}{(\nu_1-\nu_j)^2}\Big), \\ E[\hat{H}\hat{H}^T] &= \frac{1}{K}\sum_{j=2}^{2(M+1)} \frac{\nu_1\nu_j}{(\nu_1-\nu_j)^2} H_j H_j^T + \\ &\quad \cdot \left(1 - \frac{1}{K}\sum_{j=2}^{2(M+1)} \frac{\nu_1\nu_j}{(\nu_1-\nu_j)^2}\right) H_j H_j^T. \end{aligned}$$

6 Simulations

The source signal is a windowed linear FM code defined as follows:

$$s(t) = \begin{cases} p(t) sin(2\pi(at^2 + bt)) & 0 < t < L-1, \\ 0 & \text{otherwise}, \end{cases} \tag{20}$$

with:

$$p(t) = \begin{cases} 0.5(1 - cos(\pi \frac{t}{N_w})) & 0 < t < N_w, \\ 1.0 & N_w < t < L - N_w, \\ 0.5(1 - cos(\pi \frac{t-L}{N_w})) & L - N_w < t < L-1, \end{cases}$$

$L = 750, N_w = L/10 = 75$, $a = (f_2 - f_1)/2L, b = f_1, f_1 = 0.1$ and $f_2 = 0.18$; and realistic delay values:

$$\begin{aligned} r(t) &= s(t) + .8s(t-5.14) + .7s(t-10.2) + .7s(t-885) \\ &+ .8s(t-893.15) + .9s(t-899.1) - .7s(t-904). \end{aligned}$$

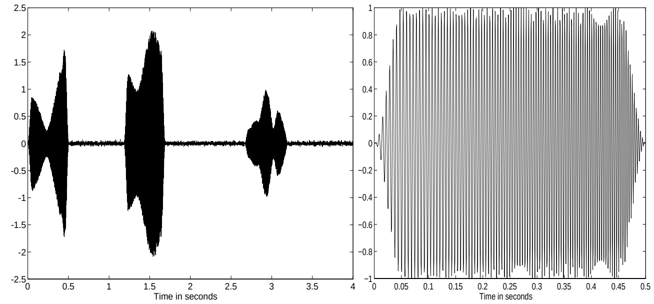

Figure 1: Received and estimated signals.

7 Conclusion

The first contribution of this paper was to show how the technique proposed in [10] could be used in interception problems where a single sensor measurement can be utilized (the multiple sensor approach is ill-conditioned). The second contribution is a performance analysis, based of the asymptotic normality of eigenvectors $\hat{G}_i$, yielding an asymptotically Wishart matrix $\hat{\mathcal{Q}}$.

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