MANEUVERING TARGET MOTION ANALYSIS USING BSPLINE REPRESENTATION*

Laurent DERUAZ

Thomson-Sintra ASM, 525 route des dolines, BP 157
06903 Sophia-Antipolis cedex, FRANCE

ABSTRACT

Target motion analysis (TMA) for a rectilinear source movement (RSM) has been intensively studied in the last ten years. But difficulties still exist, especially when source heading or speed changes are within the same time as the conventional TMA convergence time. This paper is concerned with a new method of batch TMA for maneuvering sources using a non-linear least-squares fit between the whole set of measurements and a BSpline trajectory representation. It provides a good way to globally estimate both the instants of maneuvers and their number with an experimentally robust model order selection method. This work includes tests on actual data from at-sea recordings.

1 Introduction

Target motion analysis (TMA) for a rectilinear source movement (RSM) has been intensively studied in the last ten years [2]. But difficulties still exist, especially when source heading or speed changes are within the same time as the conventional TMA convergence time. This is typically the case of evading or attacking targets.

Many approaches to cope with such maneuvering target motion analysis (MTMA) have been proposed:

- extended Kalman filters which is known to exhibit divergences even for RSM(and specially in the bearing-only tracking case) [1]
- conventional "Batch" TMA with RSM reinitialization on maneuver detection based on predicted residuals; here, the performance is strictly dependent on the minimum time for TMA convergence.

But only global methods taking into account the whole measurement sequence to rebuilt the complete trajectory are really able to deal with such evading/attacking sources. Besides hidden Markov models [3], this paper is concerned with a new method of batch TMA for maneuvering sources using a non-linear least-squares fit between the whole set of measurements and a BSpline trajectory representation [4]. It provides a good way to globally estimate both the instants of maneuvers and their numbers with a robust model order selection method.

The crucial point stands in the initialization procedure. Usual technics are measurement linearization related to position, or direct linear regression on position estimation when possible. These ones will be very easy to extend from RSM model if the target coordinates are linked to trajectory parameter with a linear function. The first idea in linearly modeling trajectory is to extend the RSM model to polynomial one using 0 to $P-1$ order position vector time-derivative. However this method is not able to modelize usual maneuvering trajectories leaving unnegligible bias. Relying on the fact that trajectories often look like piecewise RSM, it is natural to use the linear representation of piecewise polynomials: the BSpline representation. Our work includes the following points:

- Maneuver instants optimization with constrained Polytope method [5]
- Model bias and Cramer-Rao bounds studied with simulated data.
- Use of a model selection criterion experimentally robust to measurement bias.
- Tests on actual data from at-sea recordings.

In this paper we will limiting to TMA in the (O, x, y) plane, assuming the z coordinate is known.

2 Basis of "Batch" TMA methods

2.1 Principles

Given at each snapshot a noisy vector of independent measurements $\underline{m}$ so that

$$\underline{m}(t) = \underline{M}\left(\underline{X}(t)\right) + \underline{b}(t), \tag{1}$$

with $\underline{M}$ the measurement model, $\underline{b}_m$ the additive noise and $\underline{X}(t)$ the source state vector. More generally $\underline{M}$ can also depend on first derivative of $\underline{X}(t)$ if we take Doppler into account. To simplify, we will not mention it hereafter in this paper.

*This work was partly supported by DRET, Paris (France)

The basic principle of "batch" methods consist in minimizing the quadratic criterion

$$C = \sum_t^T \sum_i^N \frac{\left\| m_i(t) - M_i\left(\underline{X}(t), \underline{\dot{X}}(t)\right)\right\|^2}{\sigma_i^2}, \qquad (2)$$

with Diag($\underline{\sigma}$) the noise covariance, T snapshot number and N the number of different measurement per instant. This is the MLE solution for gaussian zero mean measurement noise and independent between different measurements and instants. In order to be able to solve this problem, it is necessary to reduce the unknown number in modeling the state vector $\underline{X}$ with P component parameter vector $\underline{A}$. In this way, it exists a Φ function so that

$$\underline{X}(t) = \underline{\Phi}(t, \underline{A}), \qquad (3)$$

that allows to write (1) as

$$\underline{m}(t) = \underline{M}\,(t, \underline{A}) + \underline{b}(t). \qquad (4)$$

If the Φ function is continuously derivable respectively to $\underline{A}$, the criteria (2) can be minimized with a specific descent algorithm like [6].

2.2 Initialization

The crucial point of these methods stand in the initialization procedure. Usual technics are measurements linearization toward state vector $\underline{X}(t)$, or linear regression on direct state vector estimation when possible. This procedure will be very easy to extend from RSM one if Φ is a linear function of $\underline{A}$. Indeed if we can write $\underline{X}(t) = \Phi(t)\underline{A}$, $\Phi(t)$ being now a matrix:

- in the first case there exist a function $\underline{f}$ of $\underline{m}(t)$, a matrix $C(t)$ and a new noise $\underline{v}_m$so that

$$\begin{aligned}\underline{f}\,(\underline{m}(t)) &= C(t)\underline{X}(t) + \underline{v}(t)\\ &= C(t)\Phi(t)\underline{A} + \underline{v}(t),\end{aligned} \qquad (5)$$

- in the second we assume we can invert the measurements model (1) by

$$\begin{aligned}\underline{M}^{-1}\,(\underline{m}(t)) &= \underline{X}(t) + \underline{w}(t)\\ &= \Phi(t)\underline{A} + \underline{w}(t),\end{aligned} \qquad (6)$$

  with $\underline{w}$ a new noise that is not generally gaussian with zero mean.

In the two cases, if we considerate all the measurements instants, we have a linear system in $\underline{A}$ that can be solve directly by mean square by the pseudo-inverse of respectively $C(.)\Phi(.)$ and $\Phi(.)$. These results are only an approximation because $\underline{v}(t)$ and $\underline{w}(t)$ are not a gaussian with zero mean noise and we cannot strictly say that the solution of MV is minimization of mean square of (5) or (6).

3 The polynomial Model

The first idea in linearly modeling trajectory is to extend the RSM model to polynomial one using 0 to $P-1$ order position vector derivative as parameter vector $\underline{A}$:

Given t_r the reference instant $\Theta = \left(\underline{r}^{(m)}(t_r)\right)_{m=1..M}$ with $\underline{r}^{(m)}(t_m) = \begin{pmatrix} x^{(m)}(t_r) \\ y^{(m)}(t_r)\end{pmatrix}$ and $\underline{T}(t) = \left((t-t_t)^m/i!\right)_{m=1..M}$, we have $\underline{r}(t) = \Theta\underline{T}^\top(t)$. Therefore, if $\underline{X}(t) = \underline{r}(t)$, we have for the parameter vector $\underline{A} = \mathrm{Vect}(\Theta^\top)$ and so, from (3) we have

$$\Phi(t, \underline{A}) = \begin{pmatrix} \underline{T}_{K,\underline{\tau}}(t) & 0 \\ 0 & \underline{T}_{K,\underline{\tau}}(t)\end{pmatrix}\underline{A}$$

that is a linear expression making usable the descent algorithm initializations described in section 2.2.

However this method is not able to modelize usual maneuvering trajectories implying significant bias.

4 The BSpline Model

4.1 The basic idea

In fact it is obvious to remark that trajectories often look like piecewise RSM.

The direct expression of a piecewise RSM $p(t)$ is given by

$$\begin{aligned}p(t) &= \sum_{m=1}^{M-1} \left[p(t_{m+1})(t-t_m) + p(t_m)(t_{m+1}-t)\right] I_{[t_m,t_{m+1}[}(t)\\ &= \sum_{m=1}^{M} p(t_m)B_{m,2,\underline{t}}(t)\end{aligned}$$

where $I_{[t_m,t_{m+1}[}(t) = 1$ for $t \in [t_m, t_{m+1}[$ and 0 elsewhere.

The functions $\left(B_{m,2,\underline{t}}\right)_{m=1..M}$ form a basis of piecewise polynomials of degree 1 for $\underline{t} = (t_m)_{m=1..M}^\top$ breaking point sequence vector (we have of $M-1$ segment and $M-2$ maneuvers): these functions are called the BSpline representation of piecewise polynomial of degree 1 [4].

More generally, each $K-1$ degrees piecewise polynomial of breaking point sequence $\underline{t}$ is decomposable in a BSpline basis of order K noted $\left(B_{n,K,\underline{\tau}}\right)_{n=1..N}$, where $\underline{\tau}$ is called the knot vector sequence. $\underline{\tau}$ vector contains vector $\underline{t}$ elements that can be replicate: if the m^{th} breaking point of the piecewise polynomial has continuity of his derivative until $K-n_m-1$ order, t_m will be repeated n_m times in $\underline{\tau}$. In these conditions we have $N = K + \sum_{m=2}^{M-1} n_m$ as shown in [4, p 113].

4.2 Application to TMA

The trajectory will be assumed as an M point piecewise polynomial (including so $M-1$ pieces) of order $K-1 = 1$ or 2 for each coordinate x and y. It will be represented by a single BSpline basis (see basis examples on figures 1 and 2) put in vector $\underline{B}_{K,\underline{\tau}}(t) = \left(B_{n,K,\underline{\tau}}(t)\right)_{n=1..N}$. Given the sequence $\underline{t} = (t_m)_{m=1..M}^\top$

there exist a unique coefficient set $\Theta = \begin{pmatrix} \alpha_n \\ \beta_n \end{pmatrix}_{n=1..N}$ so that $\underline{r}(t) = \Theta \underline{B}_{K,\underline{\tau}}^{\top}(t)$.

The knot sequence $\underline{\tau}$ will be simply chosen: multiplicity of t_1 and t_m will be K (2 for rectilinear segments model and 3 for paraboloid one) in order to give no constraint on initial and final derivative value of $\underline{r}(t)$ from 0 order included, and the multiplicity of other breaking instant will be 1 to impose continuity of derivative from 0 order to $K-2$ order (continuity for rectilinear segments model and first derivative paraboloid one). In this case we will have $N = M + K - 2 = P + K$ if P is the maneuvers number.

In the particular case of $K = 2$ we have $N = M$ and Θ is corresponding to the source position sequence $(\underline{r}(t_m))_{m=1..M}$ with $\underline{r}(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$ the position vector of the source at time t.

If now $\underline{X}(t) = \underline{r}(t)$ and if the target maneuver instants are known we have, from (3) with a parameter vector $\underline{A} = \mathrm{Vect}(\Theta^{\top})$:

$$\Phi(t, \underline{A}) = \begin{pmatrix} \underline{B}_{K,\underline{\tau}}(t) & 0 \\ 0 & \underline{B}_{K,\underline{\tau}}(t) \end{pmatrix} \underline{A}$$

that is a linear expression making usable the descent algorithm initializations described in section 2.2.

However in practise the maneuver instants are not known: they will be initiate arbitrarily. Moreover, if we take $K = 2$ (piecewise RSM), the first derivative of BSpline toward maneuver instants is not continuous and we cannot use a full descent algorithm.

4.3 Maneuver instants optimization with polytope algorithm.

To avoid a grid method that would lead to a large computational time on more than two maneuvers, we propose to use the polytope method [5] that is iterative and cheaper. This method is also known under "simplex" method (or "complex" method if there is also some constraints on the variables to estimate).

If P is the variable number on which one want to minimize an C function, the initialization is done with a N_P point of the space of the variable to estimate, forming in this way a "Polytope" in a P dimension space. These points are generally drawn at random or for an homogeneous way so that the polytope could contain the maximum amount of space.

At each iteration, points are sorted toward there C value and the algorithm modifies the polytope by moving the worst point toward the centroid of the others. Details on the method are given for example in [5]. As a matter of fact in the unconstrained case one choose $N_p = P + 1$ and $2P$ in the constrained case.

Here we have a constrained problem because we have the natural relation between the $P = M-2$ maneuvering instants $t_0 < t_1 < ... < t_P < t_{P+1}$ (t_0 and t_{P+1} being fixed) so we will have $2P$ points polytopes.

5 Model order selection criteria

5.1 Quadratic criteria properties

Given P_0 the good model order selection associate to state vector $\underline{A}_0$. For $P > P_0$ then we can define the good state vector as $\underline{A}_P$ which is $\underline{A}_0$ completed with $P - P_0$ values that leaves unchanged the $\underline{M}$ function. It is known than in these conditions that the criterium C_P is a centered χ^2_{NT-P} law.

5.2 Differential criterium

Beside Akaïke and Rissanen criteria that lead to minimization of following:

$$\begin{aligned} \text{‘}AIC\text{’} &= C_P + 2P \\ \text{‘}RIC\text{’} &= C_P + P \log N \end{aligned}$$

and that are experimentally not robust to the measurement bias, let us define an intuitive one that we will call the "differential criterium":

$$\text{‘}DC\text{’} = C_P - C_{P+1}$$

As shown above, for $P = P_0$, ‘DC’ is the difference between a $\chi^2_{NT-P_0}$ and a $\chi^2_{NT-P_0-1}$. As the P_0+1 model includes the good P_0 one, the criterium will generally stand for a centered χ^2_1. On the other hand, for $P < P_0$ models are too different from the good one to deal with measurements and so the criterium is large.

So the good order will be the one for which the differential criterion relatively falls down to values near from some unities, standing for a centered χ^2_1. Moreover, as the criterion is differential, it is perhaps less dominated by measurement bias than the others.

6 Simulation and Real data

Several simulations and application to real data have been tested, and this with one or two simultaneous bearing measurements per snapshot. BSpline method gives always better results than RSM one that is sometimes totaly out of the source trajectory. One can see figures 3,4 an abstract of results obtained from simulation and from real data. The total time is about one hour and angles measurements are 15 second spaced. The essential difference between simulation and real data is measurement bias existence: ‘AIC’ and ‘RIC’ never gives a minimum on figure 4 but differential criterion does a break at $P = 4$ for a 3 maneuver detection.

7 Conclusion

This work has shown that BSplines are good tools to match real at sea targets trajectories and differential criterion seems to deal better with real data than classical Akaïke or Rissanen ones. Even though only piecewise rectilinear segments have been shown here, piecewise parabolic segment are also good candidate to modelize real trajectories and they might allows the use of full classical descent algorithm.

References

[1] V.J. Aidala, "Kalman filter behavior in bearings-only tracking applications," *IEEE Trans. Aerosp. Electron. Syst.*, Vol. AES-15, pp 29-39, Jan. 1979

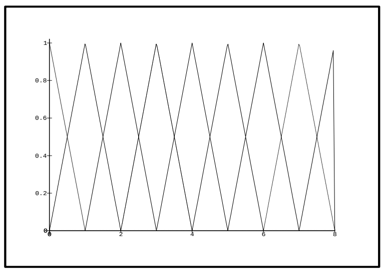
[2] J.M. Passerieux, D. Pillon, D. Van-Cappel, P. Blanc-Benon, "Bearing-Only Tracking in Passive Sonar: from Operator graphical plots to efficient Computer aided techniques", *in Proc. of Underwater Defence & Technology Conf., Paris (France)* 1991, pp. 685-692.

[3] O. Tremois, J.P. Le Cadre, "Maneuvering Target Motion Analysis Using Hidden Markov Model", *IEEE in proceedings of ICASSP*, vol.II, April 1994, p.317-320.

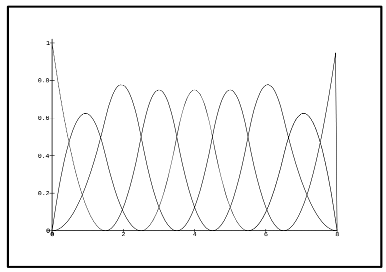
[4] C. De Boor, "A Practical Guide to Splines", *Springer-Verlag, New York*, 1978.

[5] P.E. Gill, W. Murray, and M. Wright, "Practical Optimization", *Academic Press, New York*, 1981.

[6] J.E. Dennis, D.M. Gay, R.E. Welsch, "NL2SOL, An Adaptative Least-Squares Algorithm", *Comm of ACM*, Algorithm 573, vol 7, n°3, september 1981, pp 1-14.

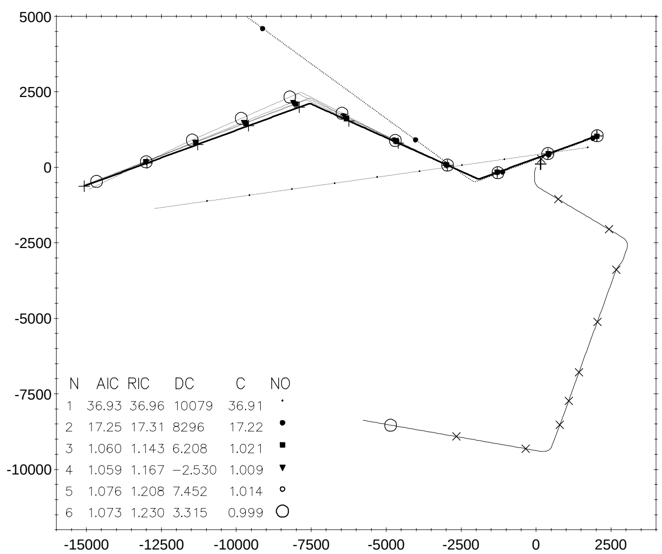


BSplines 2

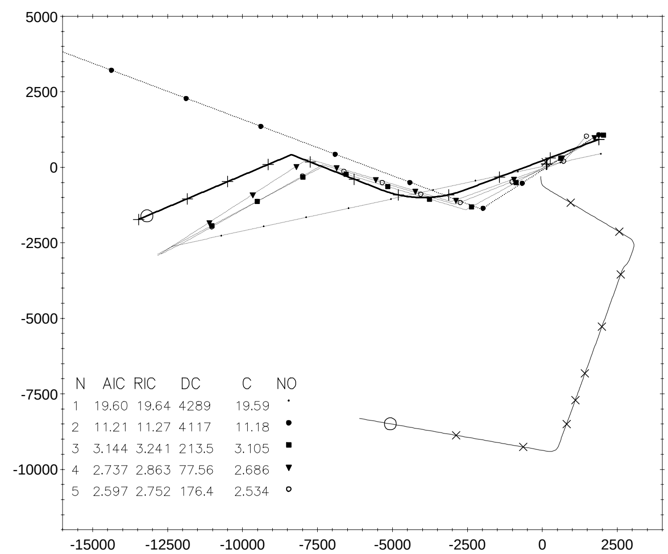


BSplines 3

Figures 1,2: An example of BSpline basis elements of order 2 and 3 with 9 elements.



simulation



real data

Figures 3,4: An example of TMA with BSpline with two simultaneous angle measurement with two antenna 400m spaced. Akaïke, Rissanen and "differential" criteria values are given at the right of each figure from a RSM model to a five maneuvers BSpline model.