A Morphological Algorithm for Photomosaicking

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ABSTRACT

We define a morphological algorithm to combine two overlapping images into a single one by a process named photomosaicking. By means of a very powerful morphological operation, namely, the watershed transformation, the method described here considers global information of a correlation image to obtain a seam which is connected, irregular and, thus, more realistic than those defined by the existing methods.

1 Introduction

By analyzing remote sensing images we can obtain a large number of informations about our planet resources. Frequently, to extract these informations we need consider the case in which the study of a region extends to other neighboring regions represented by different images. In such a case, it is necessary to join scenes of the neighboring regions possibly acquired from one or more sensors. The process of composing these images is called *photomosaicking*. Photomosaicking can also be used to reconstruct regions which were corrupted by spurious effects such as cloud covering and shadows [2].

The problems related to photomosaicking concern mainly the definition of a seam between two or more overlapping images. This seam indicates where one image ends and the other image begins. These images should be combined into a single one in such a way that the final image does not have spurious artificial edges. These artificial edges are represented by a perceptible difference between the points within the region of overlap [6].

The local aspects of the other algorithms for photomosaicking [5], [6], [7] [9] lead to a regular seam with only one pixel per line (for a horizontal photomosaic) or column (for a vertical photomosaic).

As we will see in the next sections, by means of morphological operations we define a connected, irregular and, hence, more realistic seam whose structure depends basically on global features of an overlap image.

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This work is organized as follows. Section 2 introduces some basic notions on Mathematical Morphology. Section 3 describes an algorithm for photomosaicking and illustrates the method. Final comments and conclusions are drawn in section 4 and 5.

2 Background

We need the following fragment of theory. Let $f(x)$ be a discrete and binary image, i.e, $\{f(x) \in 0{,}1\text{: } x \in \mathcal{Z}^2\}$, where $\mathcal{Z}$ denotes the set of integers. This image can be denoted by a set $\mathbf{X}$ given by $\mathbf{X} = \{x \in \mathcal{Z}^2\text{:}f(x) = 1\}$. The complement $\mathbf{X}^c$ of a set $\mathbf{X}$ is given by $\mathbf{X}^c = \{x \in \mathcal{Z}^2\text{:}f(x) = 0\}$. The symmetric $\check{\mathbf{X}}$ of a set $\mathbf{X}$ is denoted by $\check{\mathbf{X}} = \{\text{-}x\text{: } x \in \mathbf{X}\}$. The translation $\mathbf{X}_u$ of a set $\mathbf{X}$ by vector u is given by $\mathbf{X}_u = \{z\text{: } z{=}x{+}u,\ x \in \mathbf{X}\}$. The set difference $\mathbf{X}\backslash\mathbf{Y}$ of sets $\mathbf{X}$ and $\mathbf{Y}$ is given by $\mathbf{X}\backslash\mathbf{Y} = \mathbf{X} \cap \mathbf{Y}^c$.

Let the structuring element $\mathbf{B}$ be a finite set. The first relation we can define between sets $\mathbf{X}$ and $\mathbf{B}$ is dilation given by $\mathbf{X} \oplus \mathbf{B} = \{u : \mathbf{X} \cap \mathbf{B}_u \neq \emptyset\} = \bigcup_{b \in B} \mathbf{X}_b$

The erosion of a set $\mathbf{X}$ by a structuring element $\mathbf{B}$ is denoted by $\mathbf{X} \ominus \mathbf{B} = \{u : \mathbf{B}_u \subseteq \mathbf{X}\} = \bigcap_{b \in \check{B}} \mathbf{X}_b$, where for a square grid the structuring element $\mathbf{B}$ can be the elementary 4- or 8-connected set.

The above operations can be extended to a gray level image g (see, for example, [3]). For a flat structuring element the erosion and dilation of g by $\mathbf{B}$ can be denoted by

$$(g \ominus B)(x) \;=\; min\{g(u) : u \in \mathbf{B}\} \;\; and \qquad (1)$$

$$(g \oplus B)(x) \;=\; max\{g(u) : u \in \mathbf{B}\} \qquad (2)$$

The digital geodesic dilation [4] of size n, in the $\mathcal{Z}^2$-space, can be obtained from n iterations of a geodesic dilation of size 1, denoted by

$$D_X^1(\mathbf{Y}) = (\mathbf{Y} \oplus \mathbf{B}) \cap \mathbf{X}, \qquad (3)$$

and hence

$$D_X^n(\mathbf{Y}) = \underbrace{D_X^1(D_X^1(...D_X^1(\mathbf{Y})))}_{n\ times} \qquad (4)$$

2.1 Watersheds of a function

The watersheds of a function are geometric features which are very useful in picture segmentation [1]. An intuitive approach of the watershed of an image can be the following [1]. Every minimum of the topographic surface is embedded in a basin. Every time an overflow occurs, a dam can be built on the surface to prevent it. The set of all dams corresponding to the boundary of the basins is called the watersheds of the image. This catchment basins can also be seen as the influence zone of the function minima.

The computation of the watersheds is closely related to another morphological operation called thinning [8]. Let g be a gray level image and $\mathbf{B}_1$ and $\mathbf{B}_2$ be two disjoint subsets of a structuring element $\mathbf{B}$. The function p corresponding to the thinning of the image g by $\mathbf{B}$=($\mathbf{B}_1$, $\mathbf{B}_2$), defined at point x, can be denoted by

$$f(x) = \begin{cases} \max_{y \in B_2}[g(y)] & iff \ \max_{y \in B_2}[g(y)] < \\ & g(x) \leq \min_{y \in B_1}[g(y)] \\ g(y) & if \ not \end{cases} \quad (5)$$

This transformation defines the watershed lines of a function and, in this case, the family of the possible used structuring element $\mathbf{B}$ is the homotopic one [1].

3 The morphological algorithm

The method described here extracts global information of a correlation image between two overlapping regions. It is this global feature that allows the definition of a photomosaic by an irregular and connected seam.

The correlation image, given here by the absolute difference of the pixels in the overlapping regions, defines a new image which can be seen as a topographic surface containing "peaks" and "valleys", i.e., points whose height denotes regions of low or high correlation. Thus, we can use the notion of the watersheds of a function to segment this image, finding connected points of high correlation corresponding to its "valleys".

The morphological algorithm for photomosaicking consists mainly of two steps:

- Definition of the watershed lines (Eq. 5) of the correlation image.
- Definition of a two-phase image indicating the points related to each side of the photomosaic.

Morphologically, the two-phase image can be defined by means of the geodesic transformations mentioned in section 2. Let, for a vertical photomosaicking, $\delta\mathbf{U}$ and $\delta\mathbf{D}$ be the marker sets corresponding to the uppermost and bottommost edges of a binary image $\mathbf{W}$. This image indicates the points of the correlation image belonging to the watershed lines. To these sets we associate labels U and D, respectively.

The geodesic dilation (Eq. 3) of the markers $\delta\mathbf{U}$ and $\delta\mathbf{D}$ in image $\mathbf{W}$ can be denoted by

$$\mathbf{G} = D^{\infty}_{W^c}(\delta\mathbf{U}) \cup D^{\infty}_{W^c}(\delta\mathbf{D}), \quad (6)$$

Informally, D^{∞} means execution of the geodesic dilation until idempotence. A two-phase image p containing labels U and D is such that

$$p(x,y) = \begin{cases} U, if \ [(x,y) \in \mathbf{G}] \subset D^{\infty}_{W^c}(\delta\mathbf{U}) \\ D, if \ [(x,y) \in \mathbf{G}] \subset D^{\infty}_{W^c}(\delta\mathbf{D}) \\ 0, otherwise \end{cases} \quad (7)$$

i.e., the pixels of the image p at position *(x,y)* correspond to the points of the two-phase image having value U (if the points in $\mathbf{G}$ belong to the dilated set $\delta\mathbf{U}$) or D (if the points in $\mathbf{G}$ belong to the dilated set $\delta\mathbf{D}$).

After this step, the points $(\mathbf{G} \oplus \mathbf{B}) \setminus \mathbf{G}$ of the image $\mathbf{W}$, corresponding to the lowest correlation points, are eliminated and we continue executing the operations denoted by Eq. 6 and Eq. 7. We repeat this step until a line separating two regions, labeled U and D, is found. The method can be described by the following steps (for a vertical photomosaic):

1. Define a binary image $\mathbf{W}$ representing the watershed lines of a correlation image.
2. Propagate labels U (from the uppermost edge) and D (from the bottommost edge) until a watershed line is found.
3. Repeat until only one watershed line separating U and D labeled regions is found.
   - Identify the phase adjacent to the lowest correlation watershed points.
   - Eliminate these low correlation points.
   - Continue propagating labels U and D.

Fig. 3 shows the final seam of the photomosaic defined according to images 1 and 2 (the frame indicates the overlapping regions). Notice that the seam follows naturally the white lines present in both images. These lines correspond to very low and narrow "valleys" of the correlation function.

4 Some Comments

In order to point out the difference between the method discussed here and those taking into account only local features of the image, let us consider, for a vertical photomosaic, the image in Fig. 4 which represents the absolute difference of the points in the overlapping images (the polygon indicates a low correlation region).

The algorithm discussed in [6] is a good example to illustrate such a difference. Basically, it creates, for a

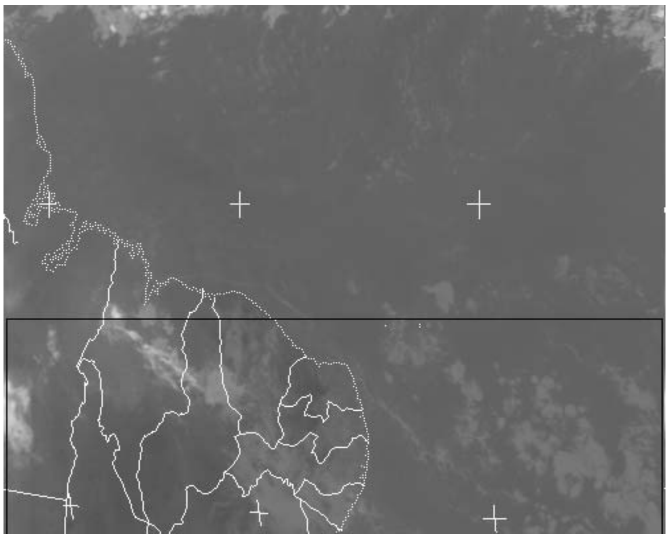

Figure 1: Uppermost side of the photomosaic.

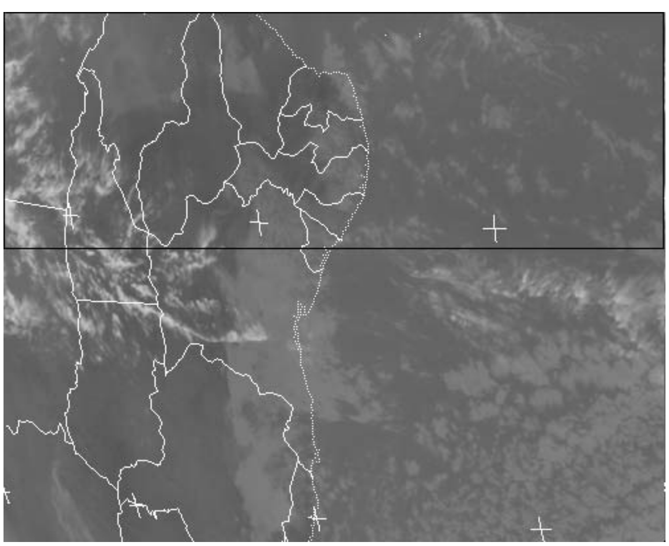

Figure 2: Bottommost side of the photomosaic.

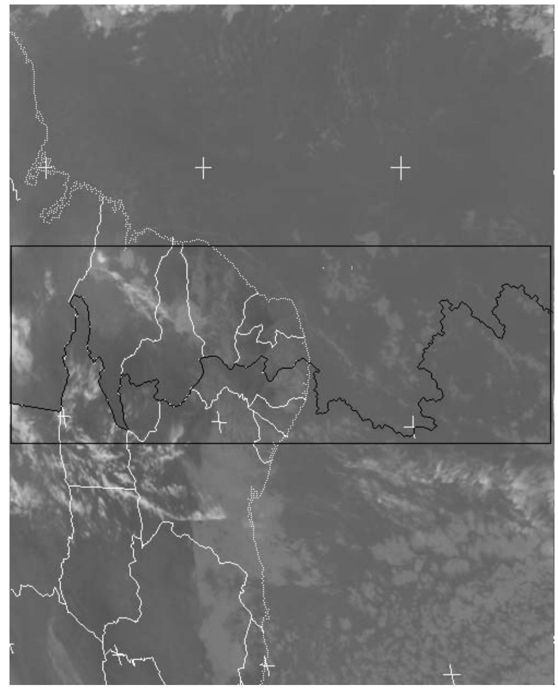

Figure 3: The mosaic and its final seam.

4	8	8	7	7	3	5	6	4	4
0	8	8	7	7	3	5	4	4	4
5	2	8	7	6	5	5	4	4	0
5	2	9	8	4	4	5	4	2	3
7	3	9	9	4	4	6	4	0	5
7	3	9	8	4	4	6	0	5	6
7	3	9	8	5	4	2	3	4	6
7	3	8	8	5	3	5	4	5	7
8	2	8	7	5	3	5	5	5	8
8	5	5	5	5	5	5	5	8	8

Figure 4: A correlation image.

vertical photomosaic, a regular seam (one point per column) given by the lowest value, in each column, of a correlation image **D** defined by

$$D_i = \sum_{j=-\alpha/2+1}^{\alpha/2} \mid U_{i+j} - D_{i+j} \mid, \qquad (8)$$

where U_k and D_k are the gray level of the current column in the uppermost and bottommost side of the two overlap images. Eq 8 computes the sums of differences over α points. To minimize discontinuities in vertical direction we consider that after detection of a point at position (i, j), the next point $(k, j{+}1)$ should be defined in a $w + 1$ neighborhood, where $i - w/2 \leq k \leq i + w/2$.

Fig. 5 shows the final seam obtained over the correlation image of Fig. 4 ($w = 4$). Notice that, in this case, some low correlation points (within small squares) belong to the seam. Obviously, we can avoid this problem by increasing the size w of the neighborhood. Nevertheless, in practice, this increasing implies the introduction of discontinuities in the vertical direction.

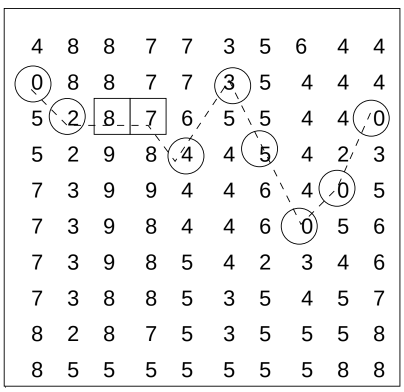

Figure 5: Definition of a seam based on local criterion.

Fig. 6 shows the result of the morphological method. Observe that the line does not cross the low correlation region and it is connected. The connectivity criterion is guaranteed by the watershed algorithm which is a homotopic operation.

5 Conclusion

We have proposed a new algorithm for photomosaicking based on a very powerful morphological operation used in image segmentation. By taking into account global information of an overlap region, the method defines a seam which is connected, irregular and, thus, more realistic than those defined by the existing methods for photomosaicking. Furthermore, due to the parallel nature of the morphological operations, the algorithm reported here can be easily described according to a SIMD programming model. Interesting extensions to this work concern the application of this method to color images.

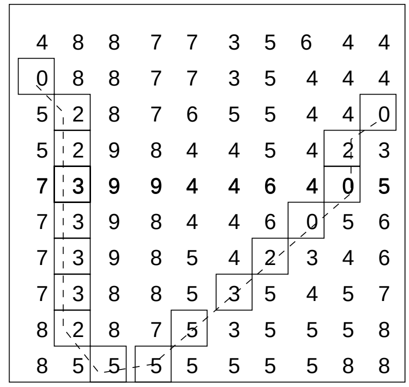

Figure 6: Definition of a seam based on the watersheds of a function.

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