# COMBINED BLOCK CODE AND DIVERSITY OVER A RAYLEIGH FADING CHANNEL WITH SOFT DECISION DECODING 

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#### Abstract

Three new Block Code Diversity combing schemes for digital transmission over a Rayleigh fading channel have been proposed. It has been shown that improved performance in term of probability of error versus signal to noise ratio has been obtained using soft-decision Viterbi decoding together with diversity techniques. Simulation results have been included to verify the performance of the proposed schemes.


## 1. INTRODUCTION

Mobile communication channels encounter fading. This is due to natural and man-made obstacles, in that there is often no direct line of sight path between transmitter and receiver. Therefore the received signal fluctuates or fades. Diversity combining techniques have been used to combat multipath fading.

In all diversity schemes the same message is transmitted over $\mathrm{M} \geq 2$ different effective channels. The signals received from the $M$ channels are combined in the receiver to create a more reliable copy of the transmitted message.

Block coding improves the performance of power limited and/or band-limited channels. The decoding process can be realised with Viterbi decoding, which has the advantage that soft decision can be incorporated very easily.

In this paper we introduce three novel combined block code and diversity schemes namely; Combined Block Code-Code Diversity (CBC-CD), Combined Block Code-Average Diversity (CBC-AD) and Combined Block Code Selection Diversity (CBC-SD). All three schemes have been decoded using soft decision Viterbi decoding. In CBC-CD soft decisions on diversity channels and decoding are carried out
simultaneously by using a trellis and Viterbi algorithm [1, 2]. In CBC-AD the $M$ diversity signals are averaged on a symbol by symbol basis to provide a single more reliable signal of the same dimensions as the original transmitted signal. All of the additional signal processing required by diversity combining operation is performed prior to decoding. In CBC-SD the selection operation for diversity branch with strongest signal level is performed prior to decoding.

It is the purpose of this work to evaluate and compare the performance and complexity of these three schemes.

## 2. SYSTEM DESCRIPTION

Block coded digital communication system with Mdiversity reception is shown in Figure.1. The digital message to be transmitted is coded by a block encoder, $(n, k)$, in which the codeword vector is represented by $\left\{C_{j}\right\}, j=1,2, \ldots \ldots . n$. The $n$-bit blocks out of the encoder are mapped and block interleaved to break up burst errors caused by fading, then modulated, producing the transmitted signals $\left\{S_{j}\right\}, j=1,2, \ldots \ldots . n$. The system is modeled as M independent multipath Rayleigh channels corrupted by additive White Gaussian noise $n(t)$. The receivers are assumed to employ matched filters to get the maximum signal to noise ratio at sampling times. The receiver values $\left\{r_{i j}\right\}, i=1,2, \ldots \ldots . M ; j=1,2, \ldots . . n$ are the unquantized samples of the outputs. After deinterleaving, these samples are quantised by Q-level quantisation and then accumulated in a $(M \times n)$ buffer matrix as $\left\{y_{i j}\right\}$ values, which can be used in any diversity combination scheme before sending them to the Viterbi decoder.

## 3. SOFT-DECISION DECODING OVER RAYLEIGH FADING CHANNEL

Considering ( $n, k$ ) block code, with Binary-PSK modulation. After interleaving to adequate depth, binary symbols are sent through the channel and received with energy $a_{n}^{2} E_{s}$. Where $a_{n}^{2}$ is the channel fading effect attached to the $n t h$ symbol in the channel time index sequence, and $E_{s}$ is the average received energy per symbol.

Performance analysis for ML decoding on Rayleigh fading channel follows a union bound procedure, for which we need the two-codeword probability of error, averaged over the fading distribution. Considering two codewords $\boldsymbol{X}_{\boldsymbol{0}}$ and $\boldsymbol{X}_{\boldsymbol{i}}$ that differ in $w$ positions within the block. With perfect side information and antipodal signaling, the two codeword error probability, conditioned upon a certain fading sequence $\boldsymbol{a}$, is [3] .

$$
\begin{equation*}
P\left[\boldsymbol{X}_{0} \rightarrow \boldsymbol{X}_{i} \mid \boldsymbol{a}\right]=Q\left(d_{E} / 2 N_{0}\right)^{1 / 2} \tag{1}
\end{equation*}
$$

Where $d_{E}^{2}=\left(a_{1}^{2}+----+a_{w}^{2}\right)\left(4 E_{s}\right)$ is the
Euclidean Distance between code sequences, modified by the channel gain in a given position. Only $w$ positions contribute to the total Euclidean Distance. Substituting this distance expression into (1) and using an exponential bound on the Q -function gives

$$
\begin{equation*}
P\left[\boldsymbol{X}_{\boldsymbol{0}} \rightarrow \boldsymbol{X}_{\boldsymbol{i}} \mid \boldsymbol{a}\right] \leq \frac{1}{2} \prod_{i=1}^{w} e^{-a_{i}^{2} 2 E_{S} / N_{0}} \tag{2}
\end{equation*}
$$

To remove conditioning the fading amplitude, we assume the fading variables are independent Rayleigh variables. Averaging of (2) then leaves the upper bound.

$$
\begin{equation*}
P\left[\boldsymbol{X}_{0} \rightarrow \boldsymbol{X}_{i} \mid \boldsymbol{a}\right] \leq \frac{1}{2} \cdot \frac{1}{\left(1+2 E_{s} / N_{0}\right)^{w}} \tag{3}
\end{equation*}
$$

Showing that the probability of confusing two sequence having distance $w$ is inversely proportional to the wth power of SNR, also meaning that effectively we have achieved wth-order diversity when sequences differ in this many positions. The final upper bound on codeword error probability $\left(P_{E}\right)$ then uses the weight spectrum of the code in a union bound.
$P_{E} \leq \frac{1}{2} \sum_{w=d_{\mathrm{nin}}}^{n} \frac{1}{\left(1+\frac{2 E_{s}}{N_{0}}\right)^{w}} \approx \frac{1}{2} \cdot \frac{1}{\left(R \frac{2 E_{b}}{N_{0}}\right)^{d_{\mathrm{in}}}}$
assuming dominance at high SNR by the minimum shortest error events. At high SNR $P_{E}$ for soft-decision decoding diminishes as $\left(E_{b} / N o\right)^{d m i n}$, there by the effective diversity order of the block coding strategy is equivalent to the minimum distance of the code, $L=$ dmin.

## 4. DIVERSITY

The concept of higher time diversity plays a crucial role in the performance of communication over fading channels. Traditionally, $L$-fold time diversity is obtained by means of repeating a symbol in $L$ different channels, which experience independent fading. In this paper combined block code diversity schemes.

### 4.1. Combined Block-Code-Diversity

In CBC-CD scheme ( $n, k$ ) block coded symbols are transmitted through $M$ independent fading channels, in order to achieve higher time diversity than obtainable separately. In CBC-CD the signals received from the $M \geq 2$ channels are combined in the receiver to create a more reliable copy of the transmitted message. We will assume that the fading is independent at the different receiving antennas. If the number of branches of space diversity is $M$, then with ideal interleaving and coherent reception, the error event probability is now given by
$P_{E} \approx \frac{1}{2} \cdot \frac{1}{\left(R \frac{2 E_{b}}{N_{0}}\right)^{d_{\min }+M}}$

Effective diversity $L=d_{\text {min }}+M$. For M-branch diversity, the received signal on the $i t h$ branch
$r_{i}=a_{i} z+n_{i}$

Where $z$ is the transmitted signal, $n_{i}$ is $i$ th additive white Gaussian noise in the $i$ th branch, and $a_{i}$ is the fading sample in the $i t h$ branch. The fading samples
are assumed to be statistically uncorrelated from branch to branch and Rician or Rayleigh distributed.

In the BPSK CBC-CD scheme, offers a time diversity of $L=5$ by implementing $(7,4)$ linear block encoder with $d_{\text {min }}=3$ in combination with space diversity with $M=2$. The $M$ received CBC-CD diversity signals are regarded as an M-dimensional signal. The corresponding decoding trellis is then modified accordingly. Each trellis transition corresponds to an M-dimensional signal with $M$ identical coordinates. In B-CD the $M$ received symbols attached to the trellis transition are identical. The decoding metric (DM) is
$D M_{C B C-C D}=\sum_{i=1}^{M}\left\|r_{i}-a_{i} z\right\|^{2}$

### 4.2. Combined Block Code Average Diversity Combining

In CBC-AD, the M diversity signals are averaged on a symbol by symbol basis to provide a single, more reliable signal of the same dimensions as the originally transmitted signal. All of the additional signal processing required by the combining operation is thus performed prior to decoding. There is thus no need to modify the decoder itself. The decoding metric is defined as follows:
$D M_{C B C-A D}=\left\|\frac{1}{M} \sum_{i=1}^{M}\left(r_{i}-a_{i} z\right)\right\|^{2}$

### 4.3. Combined Block Code Selection Diversity

CBC-SD method is more suitable for mobile communication system, because of its simple implementation. In this method the diversity branch having the highest signal level is selected. The selection process is performed prior to decoding. Thus there is no need to modify the Viterbi decoder. In this investigation the poor performance of CBC-SD resulted in being considered as unsutable.

## 5. TRELLIS CONSTRUCTION

Mceliece [4] recently introduced a method in which the encoder generator matrix total span is reduced by means of columns and rows permutation. The generator matrix with smallest span will have smaller
trellis. Thus the process of Viterbi decoder becomes simpler and faster. we consider generator matrix of a (7, 4) linear block code, from which we generate minimum span generator matrix, and drawing it's trellis diagram. The performance of this linear block code is investigated over Rayleigh fading channel.
$\mathbf{G}=\left[\begin{array}{lllllll}\underline{1} & \underline{1} & \underline{0} & \underline{1} & \underline{0} & \underline{0} & \underline{1} \\ 0 & \underline{1} & \underline{0} & \underline{1} & \underline{0} & \underline{1} & 0 \\ \underline{1} & \underline{0} & \underline{0} & \underline{1} & \underline{1} & 0 & 0 \\ \underline{1} & \underline{1} & \underline{1} & 0 & 0 & 0 & 0\end{array}\right] \cdots \cdots \cdots \begin{gathered}\text { span } \\ 7 \\ 5 \\ 5\end{gathered}$
Span of each row starts with 1 and ends with 1.
Total Span of ( 20 ) is obtained by the permutation of rows 1, 3, 4 of the above $\mathbf{G}$ matrix.
$\mathbf{G}=\left[\begin{array}{lllllll}0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0\end{array}\right] \cdots \cdots \cdots \begin{array}{r}5 \\ 5\end{array}$
Total Span of ( 17 ) is obtained by the permutation of rows 2 , 3 , of the above $\mathbf{G}$ matrix
$\mathbf{G}=\left[\begin{array}{lllllll}0 & 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0\end{array}\right] \cdots \cdots \cdots \begin{array}{r}5 \\ 4 \\ 4 \\ 3\end{array}$
Total Span of ( 16 ) is obtained by the permutation of rows 1,2 , of the above $\mathbf{G}$ matrix
$\mathbf{G}=\left[\begin{array}{lllllll}0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0\end{array}\right] \ldots \cdots \cdots \begin{array}{r}4 \\ 4 \\ 4 \\ 3\end{array}$

Total Span of ( 15 ) is obtained by the permutation of rows 1,2 , of the above $\mathbf{G}$ matrix. Drawing its trellis as shown in Figure.2.

## 6. SIMULATION AND RESULTS

To evaluate the three proposed schemes. The random data have been encoded by a $(7,4)$ Hamming encoder. The coded symbols are block interleaved before being transmitted in the form of Binary PSK signaling. For the purpose of soft-decision decoding, the analogue samples are quantised uniformly into 8 levels. The
decoding trellis is implemented using minimum span generator matrix. Two channel diversity has been investigated. The simulation results are given in Figures (3-4), which show that CBC-CD , CBC-AD and CBC-SD at probability error of $\left(10^{-5}\right)$ offer coding gain of approximately $22 \mathrm{~dB}, 21 \mathrm{~dB}$ and 13 dB respectively over uncoded BPSK.

## 7. CONCLUSIONS

To reduce the fading effects of mobile communication three combined Block Code Diversity schemes have been introduced, and their performance and complexity has been assessed. Although CBC-CD scheme offers higher coding gain compared to CBCAD and CBC -SD schemes. In CBC-AD scheme the additional complexity is limited to a pre-decoding averaging circuit. While in CBC-CD a significant increase in decoding complexity is encountered as diversity order increases. For higher order of diversity the performance improvement is heavily off-set by the corresponding decoding complexity increase. Thus CBC-AD scheme is the preferred approach for practical systems in comparison to the other two schems.

## REFERENCES:

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Figure.1.Block Coded Diversity Modem Over Rayleigh Channel


Figure.2. Trellis for $(7,4)$ block encoder


Figure.3. Performance of (7, 4 ) Block Coder over Rayleigh fading channel


Figure.4. Performance of (7, 4 ) Block Coder over Rayleigh fading channel

