A MODIFIED FILTERBANK FOR TRACKING MULTIPLE SINUSOIDAL SIGNALS

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ABSTRACT

A modified adaptive filterbank structure is presented to track multiple sinusoids which is based on the resonator-in-a-loop filterbank structure [1]. The advantages of this configuration over the previous resonator-in-a-loop filterbank structure are two-fold: its better enhanced transfer function characteristics, and the easier and more accurate determination of its signal-to-noise enhancement ratio. The simulation results using both the filterbank structures are presented and confirm the improved performance of the new filterbank.

1 INTRODUCTION

In recent years, the resonator-in-a-loop filterbank has been used in several signal processing areas, such as adaptive signal processing [1], [2], subband coding and decoding [1], trigonometric transforms [3] and filter design [4]. In the area of the adaptive signal processing, that structure has been employed to track, enhance and notch multiple sinusoidal signals [1], and to analyze short-time data [2]. Moreover, a typical simulation result for a general harmonic signal was also presented in [1]. Because of its simple parallel and loop structure, that filterbank is very easily realized in custom VLSI circuitry and also via software downloaded to a DSP board to carry out real-time signal processing. In addition, the unique orthogonal property of that structure results in a simple gradient Gaussian-Newton algorithm [1], [5] for adaptive signal filtering, which also promotes efficient realization of the entire system.

Based on this resonator-in-a-loop filterbank, this paper presents a new adaptive filterbank structure. The advantage of this configuration over the previous resonator-in-a-loop filterbank structure given by [1] is that each enhanced transfer function arising out of our proposed structure has much better stopband characteristics than those of the original resonator-in-a-loop filterbank. So this filterbank can provide much better enhancement (isolating) signals than obtainable with the original resonator-in-a-loop filterbank [1]. In addition, we have found that it is easier and more accurate for the modified filterbank to approximately determine its signal-to-noise enhancement than that of the original resonator-in-a-loop filterbank. The main task of this modified structure, as well as the former filterbank, is to track and estimate multiple sinusoidal signals. The simulation results using both this modified structure and the original resonator-in-a-loop filterbank are presented and used to confirm the improved performance of this new filterbank.

2 THE MODIFIED FILTERBANK STRUCTURE FOR MULTIPLE SIGNAL TRACKING

Consider the situation for tracking a composite signal made up of N sinusoids. A new structure shown in Figure 1 is introduced and employed to isolate this composite signal into individual component sine waves. The structure within the dotted line is the resonator-in-a-loop filterbank proposed by Martin and his colleagues [1] for tracking and estimating N multiple sinusoidal or N harmonic signals. The rest of the figure utilizes our proposed additional parts to construct a new structure for carrying out N sinusoidal composite signal tracking and parameter estimation.

The resonator transfer function from resonator input to its output ($\hat{y}_{fb,i}(k)$) is given as [1]:

$$\hat{H}_{bp,i}(z) = G\frac{\hat{Y}_{fb,i}(z)}{\hat{E}(z)} = \frac{2a_i z^{-1} - 2z^{-2}}{1 - 2a_i z^{-1} + z^{-2}} \qquad i=1, 2, 3, \ldots, N \qquad (1)$$

The new modified transfer function from the resonator input to the modified resonator output ($y_{fb,i}(k)$) can be easily derived as:

$$H_{bp,i}(z) = G\frac{Y_{fb,i}(z)}{E(z)} = \frac{1 - z^{-2}}{1 - 2a_i z^{-1} + z^{-2}} \qquad i=1, 2, 3, \ldots, N \qquad (2)$$

We select the transfer function of the pseudosensitivity signal from resonator input to its output as [1], [2]:

$$H_{ps,i}(z) = G\frac{Y_{ps,i}(z)}{\hat{E}(z)} = \frac{2\sin(2\pi\nu_i)z^{-1}}{1 - 2a_i z^{-1} + z^{-2}} \qquad (3)$$

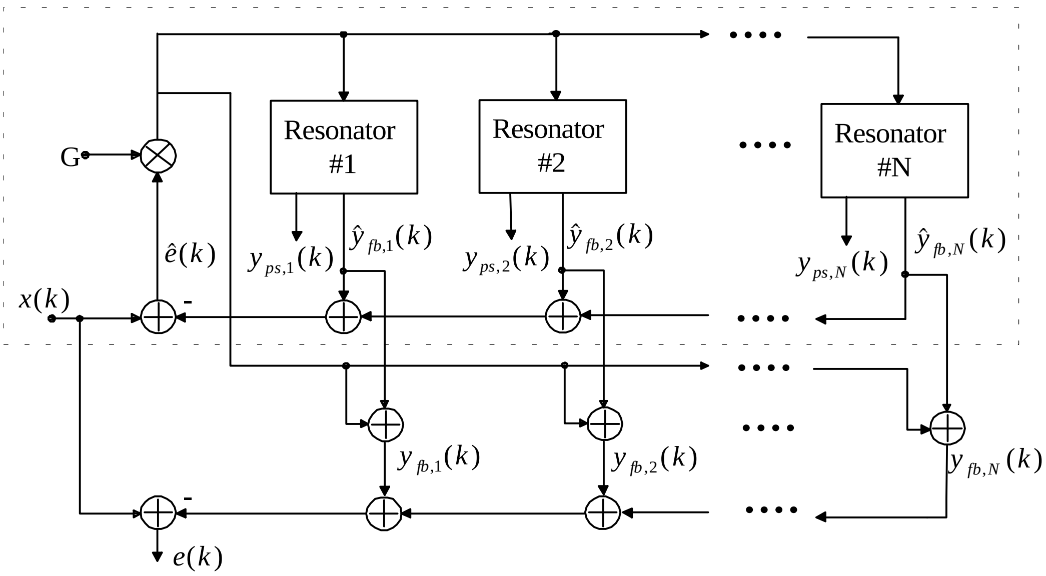


Figure 1 The new filterbank structure based on the resonator-in-a-loop filterbank [1] (shown within dotted line block)

We can then derive the global enhanced transfer functions for the resonator-in-a-loop filterbank structure and our modified new structure at node (*i*) as:

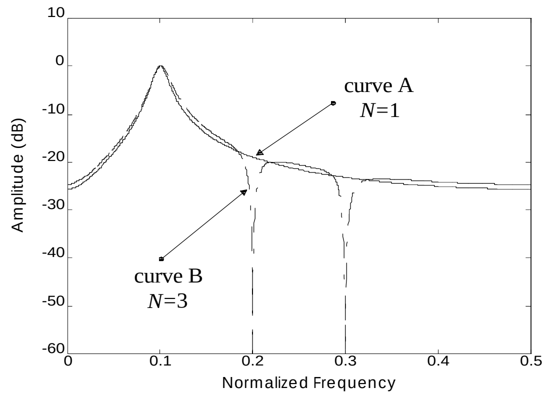
$$\hat{H}_{enh,i}(z) = \frac{G\hat{H}_{bp,i}(z)}{1+G\sum_{j=1}^{N}\hat{H}_{bp,j}(z)} \tag{4}$$

$$H_{enh,i}(z) = \frac{GH_{bp,i}(z)}{1+G\sum_{j=1}^{N}H_{bp,j}(z)} \qquad i=1,\, 2,\, 3,\, \dots,\, N \tag{5}$$

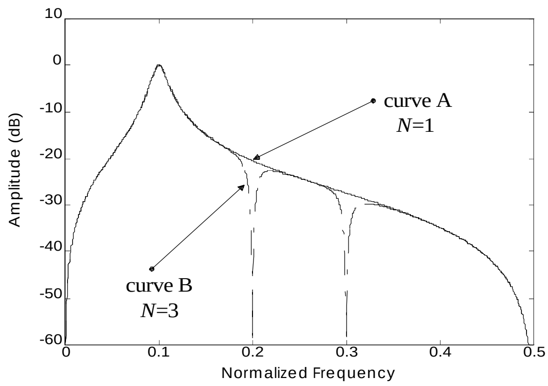
The global pseudosensitivity transfer function, from the filterbank input to the pseudosensitivity output, is chosen as [8]:

$$H_{ps\_enh,i}(z) = \frac{GH_{ps,i}(z)}{1+G\sum_{j=1}^{N}\hat{H}_{bp,j}(z)} \tag{6}$$

Consider the enhanced transfer functions of the modified filterbank and the former resonator-in-a-loop filterbank. Figure 2 shows typical enhanced transfer function amplitude curves for the resonator-in-a-loop filterbank (shown in Figure 2(a)) and for our new modified filterbank (in Figure 2(b)), where the enhanced transfer functions are referred to the input signal *x(k)* and to the bandpass output signals $y_{fb,i}(k)$ and $\hat{y}_{fb,i}(k)$, respectively. The lengths for both filterbanks are assumed to be N=1 and N=3. The normalized frequencies are set to be 0.1 for the single length filterbank and 0.1, 0.2 and 0.3 for the filterbank with N=3. Here curve A in Figure 2(a) and (b) shows the single length filterbank amplitude transfer function, whereas curve B denotes the filterbank with length 3.



(a) The original filterbank



(b) The modified filterbank

Figure 2 The enhanced amplitude transfer functions at $\nu = 0.1$ for both filterbanks with N=1 and N=3; curve A: single length filterbank, curve B: filterbank with length 3

According to curve B in Figure 2(a) and (b), it is obvious that the modified filter has almost the same enhanced unity gain, at the enhanced point, and better stopband, at dc and Nyquist frequencies, in comparison to the resonator-in-a-loop filterbank. Therefore, this modified filterbank can achieve better enhancement performance than that of the original resonator-in-a-loop filterbank. The extra cost is negligible, only amounting to the incorporation of adders.

Meanwhile, comparing curve A and curve B in both Figures 2(a) and 2(b), it is interesting to note that two amplitude transfer curves in Figure 2(b) for our proposed modified structure are much closer to each other (except at $\nu = 0.2$ and 0.3) than those in Figure 2(a). It has been proved that the single second-order IIR enhanced filter has exactly the same mathematical signal-to-noise enhancement ratio expression [5]. So, we may adopt the view that approximately the signal-to-noise enhancement ratio at node (i) for our proposed new filterbank is almost equal to the signal-to-noise enhanced ratio of the single-length modified filterbank. Obviously, this approach for the modified filterbank is more accurate than that for the former resonator-in-a-loop filterbank.

We also choose the adaptive algorithm [1], [6]-[8] corresponding to Equation (7) to update the weight coefficients as:

$$w_i(k+1) = w_i(k) - \mu \frac{y_{ps,i}(k)e(k)}{\left\| y_{ps,i}(k) \right\|^2 + P_{\min}} \tag{7}$$

where $y_{ps,i}(k)$ is the pseudosensitivity output, $e(k)$ is the error or notch output, $P_{\min}$ is used to prevent the step size, μ, being divided by a very small value and $w_i(k)$ is the updated weight coefficient related to a_i by $a_i = 1 - w_i^2(k)/2$. In addition $\left\| y_{ps,i}(k) \right\|^2$ is the estimated power of the pseudosensitivity signal which is calculated as:

$$\left\| y_{ps,i}(k) \right\|^2 = (y_{bp,i}(k)^2 + y_{ps,i}(k)^2)/2 \tag{8}$$

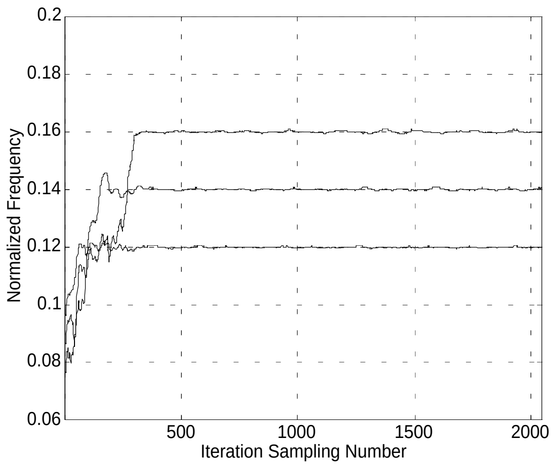
Based on the updated weight coefficient, $w_i(k)$, in (7), we can estimate the multiple sinusoidal signal frequencies ν_i related to it as [6]:

$$\nu_i(k) = \arcsin(w_i(k)/2)/\pi \tag{9}$$

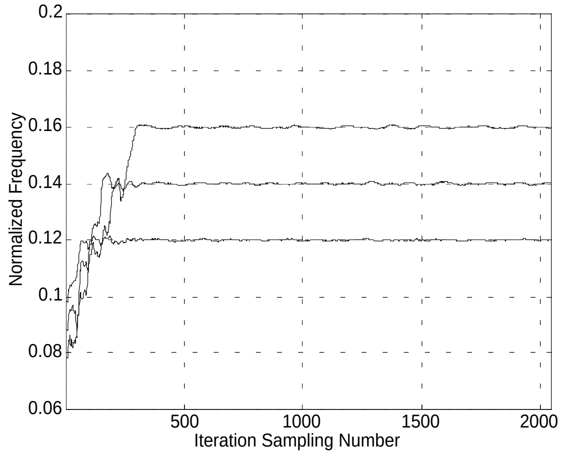
3 SIMULATION RESULTS

A typical example using these two filterbank structures for tracking three composite sinusoids in the presence of additive white Gaussian noise has been carried out to compare the performance of these two structures. Here the input signal contained sinusoids of normalized frequencies 0.12, 0.14 and 0.16, the SNR was 10 dB. The approximate Gauss-Newton Gradient algorithm [1], [5]-[8] in Equation (7) was used to update the weight coefficients of each adaptive filter. The other initial parameters for both the filterbanks were as follows: the lengths of both the filterbanks were set to N=3, the damping factor G was equal to 1/36, the step size, μ, values for the adaptive filters #1, #2 and #3 were 0.001, 0.0009, 0.0008 and the initial frequencies for the resonators #1, #2 and #3 were set to 0.08, 0.09 and 0.1, respectively. The parameter for preventing division by zero, $P_{\min}$, was 0.002.

Figure 3 shows the tracking of the three normalized frequencies with using these two structures. The enhanced signal amplitude spectra are illustrated in Figure 4, employing a 400-sample DFT from iteration 1501 to 1900.



(a) Using the original filterbank



(b) Using the modified filterbank

Figure 3 The tracking frequencies for three composite sinusoidal signals embedded in white noise

From the simulation results obtained in Figure 3, it is easy to see that the convergence speed of the modified filterbank is very similar to the original resonator-in-a-loop filterbank. So the new modified filterbank does not affect the fast convergence speed of the original resonator-in-loop filterbank.

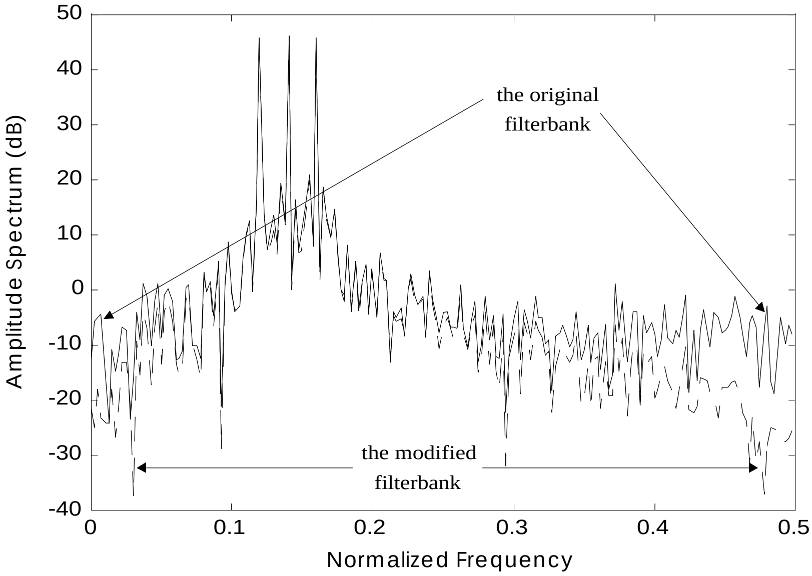


Figure 4 The enhanced signal spectra for three composite sinusoidal signals embedded in white noise with using both filterbanks (the signal spectra obtained as 400-point DFTs from iteration 1501 to 1900)

From Figure 4, we can see that the enhanced signal spectra around the passband are consistent. The more important point in Figure 4 is that the enhanced or extracted signal using the modified filterbank has lower level noise spectrum at the stopband than that with the former resonator-in-a-loop filterbank, especially at around dc and Nyquist frequencies. So, this result is identical to that discussed for Figure 2 and proves that the modified filterbank can achieve better signal-to-noise enhancement ratio than that of the original filterbank.

4 CONCLUSIONS

A new modified filterbank structure for tracking multiple sinusoidal signals has been established. Comparisons to the original resonator-in-a-loop filterbank confirm that the enhancing behaviour of this new configuration is much better than that of the original filterbank. Also, it has been found that it is much easier with this new configuration to determine the approximate signal-to-noise enhancement ratio than with the original one. Simulations were provided to reinforce the theoretical analysis of this modified filterbank. It is expected that this new modified filterbank can also be used to substitute for the former resonator-in-a-loop filterbank in various applications, and that consistent performance improvement will result.

5 REFERENCES

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