CFAR DETECTOR FOR BACKGROUND NOISE WITH TWO-PARAMETER DISTRIBUTION

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ABSTRACT

We present a double-parameter CFAR with very reasonable losses and low computational complexity. Its basic architecture has been conceived from tail extrapolation theory. The detector uses a detection threshold, set from the measured PFA which is obtained with an auxiliary threshold (pseudothreshold), lower than the final detection threshold.

Starting from the basic scheme, a CFAR detector for Weibull clutter has been designed; both, the pseudothreshold control mechanism and the correction of the basic extrapolation equation are described.

1 INTRODUCTION

In multiple detection problems (images, SAR, radar, sonar,...) we are faced with the problem of extracting targets (objects, planes,...) immersed in a perturbation background which can be approximated by a statistical, two-parameter (shape and scale) model.

The design of CFAR detectors for such a class of problems has traditionally relied [1][2] on estimating the model parameters from samples in a reference window centered on the cell under test. The detection threshold can be then computed from the model and the estimated parameters so that the system exhibits the CFAR property.

There are two problems that have hindered practical implementations of the above mentioned detection systems: i) The great number of reference cells required to guarantee acceptable SNR losses (defined as the increase in received power which is needed to yield the same PD as the ideal detector, where the model parameters are assumed to be known). ii) The excessive computational load associated to the estimation of both parameters.

In this contribution, we will study the synthesis process of a two-parameter CFAR with reasonable losses and acceptable computational load for a real-time system. The detector has been studied for Weibull background noise, although the same system is CFAR for a wide family of two-parameter distributions (K, Gamma, Exponential).

The contribution describes the synthesis method for a two-parameter CFAR detector, based on tail extrapolation techniques [4]. The proposed scheme is shown in figure 1. The detector is based on two reference windows, one of which is included inside the other (M<N). The first reference window is used to estimate the mean value of the background noise (amplitude normalization). It consists of a one-parameter CFAR [3] with a relatively small window (M approximately some tens) and perfectly assumable computatyinal load.

The second parameter (shape) is obtained through the processing of a N-cell window, where N>>M. The estimation of the shape parameter is related to that of the variance, and is thus very expensive from a computational point of view (besides, it requires a great number of samples, N). In order to reduce that load, we have restored to a tail extrapolation technique [4]. It is based on the assumption that our background noise distribution belongs to the generalised exponential family (two-parameter model), with normalised mean. The shape parameter of such a distribution is estimated by counting the alarms at the output of an auxiliary threshold, much lower than the detection threshold. This scheme exhibits a much reduced computational cost than direct estimation of variance, though N must be greater for equal performance. The CFAR threshold is then determined from the shape parameter, the desired PFA and an extrapolation formula.

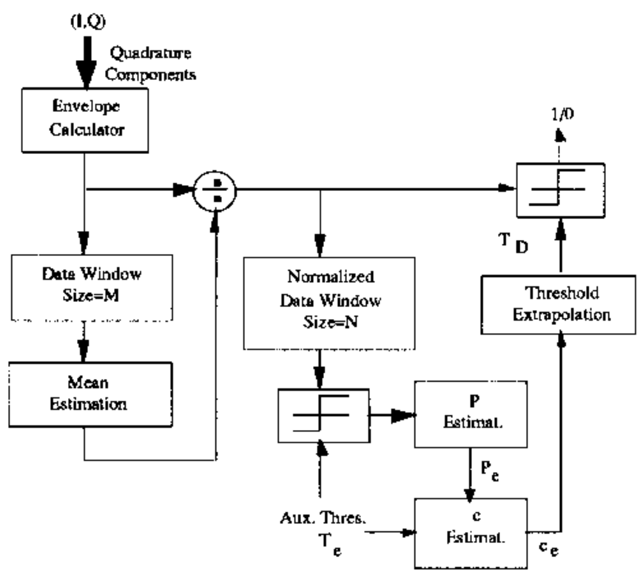

Fig. 1: Basic diagram of proposed CFAR detector.

A last point is that the samples in the reference window used to estimate the shape parameter (size N) must be homogeneous. Our system is supposed to operate on regions from a previously segmented image. Given the fact that segmentation does noy yield perfectly homogeneous regions, an energy normalisation is still required to dampen such variations.

The paper begins with a brief explanation of the proposed detector, studied assuming the ideal amplitude normalization [5]. We expose that synthesis of a CFAR detector as the one proposed is feasible applying the technique of tail extrapolation. The extension of the initial synthesis method to the whole detector is then considered. In this phase, the effect of the amplitude normalization is included. We arrive at the conclusion that, even in the case of small normalization references, both the detector and the method of design are valid.

2 SYNTHESIS OF THE DOUBLE PARAMETER CFAR

First of all, we have to model the clutter using a statistical family characterized by a two parameter probability density function. In our case, the Weibull distribution has been chosen. As was stated before, prior to comparison with the detection threshold, the signal is processed by an amplitude normalizer (we assume a linear detector). Although, in practice, the normalization operation may change the distribution of the input signal, when the number of cells used for estimation is moderately large, it can be assumed that the distribution warping is negligible for most purposes. Thus, we will assume that the distribution at the detector input is Weibull with normalized mean. In section 4 we will generalize the design method for the case of non-ideal normalization. The expressions for the density and complementary distribution functions of the normalized Weibull random variable are [5]:

$$f(x) = \Gamma^c\left(\frac{1}{c}\right)\cdot\left(\frac{x}{c}\right)^{c-1}\cdot\exp\left\{-\left(\frac{x}{c}\cdot\Gamma\left(\frac{1}{c}\right)\right)^c\right\} \quad ; \quad x \geq 0 \qquad (1)$$

$$F(t) = 1 - \exp\left\{-\left(\frac{t}{c}\cdot\Gamma\left(\frac{1}{c}\right)\right)^c\right\} \quad ; \quad t \geq 0 \qquad (2)$$

where c is the shape parameter (related to the impulsivity of the clutter) which will be assumed to be in the range [0.5 - 2.0] (between highly impulsive clutter and Gaussian noise). Most clutter types can be characterized by a shape parameter within that interval [7].

For the Weibull distribution (1), the tail extrapolation equation (2) can be writen [5]:

$$\mathrm{Ln}\{-\mathrm{Ln}(P)\} \cong c\cdot\mathrm{Ln}\left(\frac{t}{c}\cdot\Gamma\left(\frac{1}{c}\right)\right) \qquad (3)$$

The estimation equation (the one used to determine the shape parameter) is obtained taking in (3) c as the independent unknown, P (estimated probability with the pseudothreshold, PFA_e) as the dependent variable and t (pseudothreshold T_e) as a parameter. In the extrapolation equation, however, t (final detection threshold) is taken as the independent unknown, the shape parameter c (estimated through the estimation equation) as the dependent variable and the final PFA (P in (3)) as a parameter.

Secondly, we will have to obtain the performance in PFA in order to check that the detector is effectively CFAR with respect to both parameters. The mean PFA at the output detector could be obtained by averaging the complementary distribution function (2) with the density function of the threshold extrapolated by (3). Unfortunately, it is difficult to explicitly obtain the probability density function of the detection threshold t. However, it is reasonable to accept that the PFAs with the pseudothreshold and with the detection threshold are independent. This is due, in the first place, to the fact that the extrapolation is usually made from a burst to the next one. Under this independence assumption, the expression for the mean of the final PFA of the detector can be shown to be (taking into account (3) and (2)):

$$E\{PFA_F\} = \sum_{i=0}^{N}\exp\left\{-\left(\frac{g(i/N)}{c}\cdot\Gamma\left(\frac{1}{c}\right)\right)^c\right\}\cdot p(i/N) \qquad (4)$$

where: i represents the number of cells where amplitude has exceeded the pseudothreshold, N is the number of cells in which the estimation has been carried out, p(.) is the binomial distribution and g(.) corresponds to the extrapolation function (3). The expression (4) should be calculated numerically.

2.1 Estimation Equation

Before designing the parameters of the detector, we will carry out a study of the solutions of equation (3). The range of the possible roots of this equation sets some constraints on the pseudothreshold space to achieve the desired performance: PFA approximately constant for values of the shape parameter in the range (0.5 - 2.0). Analyzing the equation (3) for c we can draw the conclusion that, for a given estimation threshold (T_e) and an estimated PFA (PFA_e), this equation can have two, one or no solutions for c. $\mathrm{Ln}(PFA_e)$ in (3) is a single maximum function of c, which has no solutions for values of PFA_e over such value. At the maximum, it has one solution and under it, there are two (as illustrated in Fig. 2).

In figure 2, the estimation equation (3) is graphically presented (continuous lines), with the pseudothreshold (T_e) as a parameter. The valid solutions to the estimation equation belong to the region within the dashed lines. The region is horizontally limited by the specified range of variation of the shape parameter. The lower limit is given by the curve of constant PFA which can be estimated with a given precision with the number of available cells N. This PFA represents an absolute limit imposed by the number of cells. Moreover, it is the same for all shape parameters. The upper limit is specified by the curve of the maxima of $\mathrm{Log}(PFA_e)$ in equation (3) (with T_e as a parameter). The PFA_e of the detector must be lower than its value, to guarantee one solution for c.

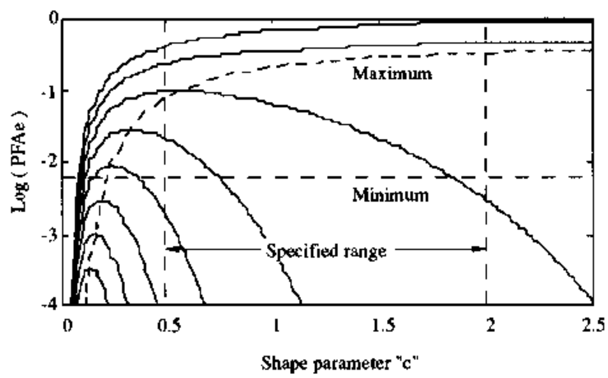

Fig. 2: Solutions to the tail extrapolation equation (T_e parameter).

From figure 2 it can be deduced that with a constant pseudothreshold it is not possible to fall within the region of valid solutions for the whole range of shape parameters. Hence in order for the detector to work properly, some algorithm for pseudothreshold control must be incorporated. The selected approach tries to keep the PFA constant at the output of the pseudothreshold. The following pseudothreshold updating algorithm is used (being K an adaptation constant):

$$\mathrm{Ln}(T_e(i+1)) = \mathrm{Ln}(T_e(i)) - K\cdot\{\mathrm{Ln}(-\mathrm{Ln}(P)) - \mathrm{Ln}(-\mathrm{Ln}(PFA_e))\} \qquad (5)$$

This algorithm originates from the linear relationship between the logarithm of the threshold and the double logarithm of the PFA, expressed by (3). This property guarantes an uniform convergence.

2.2. Extrapolation Function

The final PFA (equation (4)) is represented in figure 3 for an illustrative case: $PFA_D=10^{-5}$, N=1000 (estimation cells). It shows the final PFA as a function of the estimation PFA.

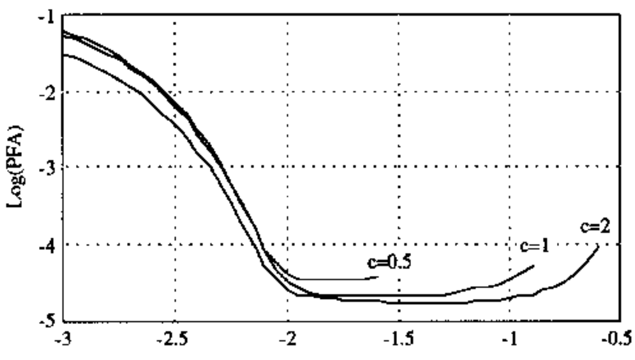

Fig. 3: Mean PFA for some values of the shape parameter (N=1000, $PFA_D=10^{-5}$).

The main characteristic of this curves is that the final mean PFA does not match the one of design, 10^{-5}. It can be observed that there is a small bias, practically constant, in the range which will be the proper one for our detector to work within. Furthermore, the bias depends on the shape parameter. This means that the detector based on the extrapolation of the threshold by (3) is not CFAR with respect to the shape parameter. Hence, provided we want the detector to have this property we will need to distort the extrapolation equation (3). With that purpose, we will substitute in (3) the term PFAD (equivalent to P in the extrapolation step) by a function of the auxiliary threshold:

$$PFA_D=g(T_e) \quad (6)$$

Equations (6) are calculated by an iterative numerical procedure. The mean PFA is obtained with (4) using (3) without distortion (for every shape parameter). The PFA_D is corrected in the extrapolation equation (derived from (3)) as a function of the resulting bias in each of the shape parameters (for the T_e corresponding to that parameter). This method converges after a few iterations (it has been halted when the maximum bias was lower than twice the desired PFA). By correcting the curve of extrapolation we have reached the CFAR property with respect to the two parameters of the distribution.

2.3 Performance Evaluation

We will evaluate the performance in probability of detection (PD). The model of target fluctuation will be taken Gaussian or Swerling-1, 2 [7]. The PD will be measured as a function of the clutter shape parameter and the signal to clutter power ratio (S/C). The losses in S/C will be obtained. They are defined as the increase of S/C over that necessary for the optimum detector (shape parameter assumed known) to get a certain PD (usually 0.5 or 0.9).

We show, in figure 4, the results for the case: $PFA_D=10^{-5}$, N=1000. The continuous line corresponds to the ideal detector and the dashed line to the proposed CFAR. The losses for shape parameters greater than 0.8 are under 0.9 dB (PD=0.5).

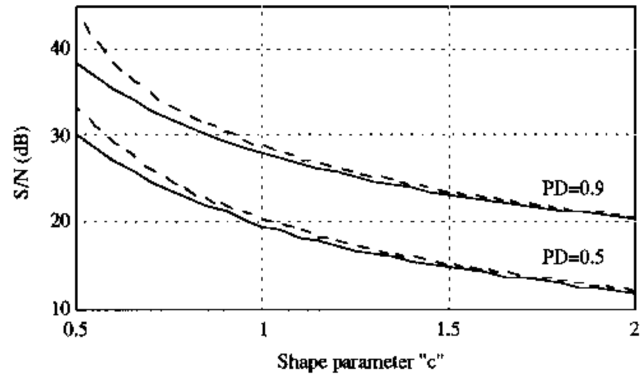

Fig. 4: SNR for a given PD as a function of the shape parameter.

3. EFFECT OF THE FINITE SIZE OF THE MEAN ESTIMATION WINDOW

An aspect to be discussed is the influence of the finite size of the normalization window. In the previous section we have systematically accepted thus far that the normalization (mean estimation, Fig.1) performed on the received data was ideal. This allowed us to assume that the clutter had an ideally normalized Weibull distribution and to proceed with the design process.

Figure 5 show (for N=1000) the final true PFA when the threshold is calculated (extrapolated assuming an ideally normalized Weibull) from the true data (data normalized with a sample of a given finite size M). The cases of clutter shape factors of c=1.0 and c=2.0 have been selected because, for them, (approximate) closed expressions are available for the probability of false alarm at the output of the CFAR [3], [6].

It can be concluded that, for M larger than a few tens, the difference between the actual PFA and the PFA (10^{-5}) predicted for our detector using the ideal normalization assumption is not practically noticeable. In the next section we generalize the design process to deal with the effect of the non-ideal normalization.

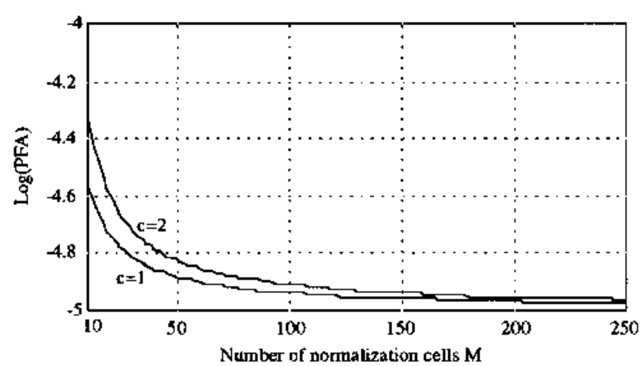

Fig. 5: Effect of the finite size of the CFAR reference on the expected PFA ($PFA_D=10^{-5}$, N=1000)

4 CORRECTION OF DESIGN PROCESS

In the former section we have checked that, for a small number of cells in the normalization window (M), the detector that assumes an ideal normalization suffers an increase in the PFA, which is, in addition, variable with the shape parameter.

We commented in the introduction that, even though the detector has been designed for Weibull clutter, the technique is valid for a wide range of possibilities. In this section we will adapt the method of design of the proposed CFAR detector to the distribution at the output of the normalizer. To that end, we simply have to substitute in equation (6) the complementary distribution function of the Weibull random variable by that of the output of the amplitude normalizer. The threshold is to be determined according to the tail extrapolation technique (equation (5)). Since the normalizer effect is to slightly increase the tail of the distribution, the estimated shape parameter will also be slightly larger, and will thus compensate the increase in PFA.

The distribution function at the output of the normalizer is difficult to obtain in a closed-form expression (except in the case c=1). Therefore, it has been obtained by means of a semianalytical technique.

Since the expression of the distribution function of the Weibull random variable is known, in order to obtain that output it is only necessary to generate the threshold (T*amplitude average) and to average the corresponding values of the complementary distribution function. The number of trials that are necessary to reach an accurate convergence has been determined by the method in reference [2]. The resulting distribution, tabulated as a function of c and T, is entered in (6). The design process follows identical steps as those carried out in section 2.

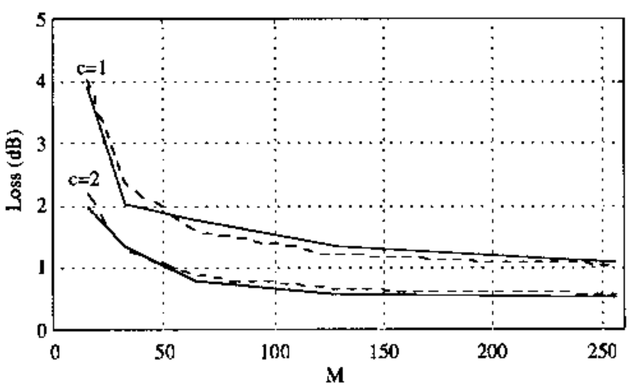

Fig. 6: Detection losses as a function of M (PD=0.5, PFA=10^{-5}, N=1000)

Finally, a number of experiments have been carried out for several values of M, in which the tracking filter of the pseudothreshold has been included, for a Swerling-1 target. The PFA remains close to the value of design. The detection performance has been determined through Monte Carlo simulation. Results have been plot in figure 6 for two values of the shape parameter 1 and 2, PD=0.5, PFA=10^{-5} and N=1000. In this curve the detection losses are represented for several values of M: 16, 32, 64, 128 and 256.

Along with the loss of the whole detector (continuous line), in figure 6 we have also represented the curve (dashed line) corresponding to the additive estimation of these losses. That is, by considering that the losses of the two-parameter CFAR are equal to the sum of those pertaining to the scheme of mean normalization (c known) and those from the one proposed in section 2 (mean known). The conclusion is that this approach is accurate enough, which supports the application of this method to the comparison to other two-parameter CFAR detectors that was used in [5].

5 CONCLUSIONS

In this paper we have demonstrated that the two parameter detector proposed in [5], which assumes a quasi-ideal amplitude normalization, is valid when the number of reference cells in the normalization window is small.

We have also shown that the losses of the two parameter CFAR detector are approximately equal to the sum of those of a CFAR with amplitude normalization (known c) and those of a CFAR with ideal normalization which estimates c. The assumption now justified was used to compare the proposed detector to other classical two parameter CFAR systems. Specifically, data in figure 6 have been chosen according to the test in reference [2]. The CFAR with maximum likelihood estimators for a reference size of 32 cells shows losses of 3.2 and 6.8 dB, for c=1 and c=2 respectively. As can be proved, the loss in resolution implied by the use of a detector such as the one proposed is widely compensated by the reduction both in losses and in computational load.

6 REFERENCES

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