LINE SPECTRUM PAIRS IN PATTERN RECOGNITION

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ABSTRACT

The optimal Bayesian Classifier is often difficult to implement because of its complexity. For Gaussian parameters, the Bayes decision rule reduces to a simple centroid distance rule. However, the centroid distance rule fails for non-Gaussian parameters with non-convex probability density functions (p.d.f.). This paper studies some statistical properties of Line Spectrum Pairs (LSP). These statistical properties can be used to study the convexity of LSP point clusters in pattern recognition applications.

1 INTRODUCTION

A major problem in pattern recognition is the determination of the "optimal" classification rule for a given parameter vector. The solution to this problem is given by the Bayes Classifier when the parameter vector statistics are known. However, implementation of the Bayes classifier is often difficult because of its complexity. The Bayes decision rule reduces to a centroid distance rule for Gaussian parameter vectors, leading to a simple classifier. For non-Gaussian parameter vectors with non-convex probability density functions (p.d.f.), surprising results can be obtained with the centroid distance rule [6]. A statistical analysis of the reflection and cepstrum coefficients has shown that their p.d.f. can be non-convex. Thus, the centroid distance rule is not an effective classifier in these cases[6]. This paper studies some of the statistical properties of Line Spectrum Pairs (LSP) coefficients. LSP coefficients have been effectively used for quantization and coding [4],[5]. However, to our knowledge, there are no theoretical studies of these parameters when used for pattern recognition. LSP coefficients are usually computed from the Linear Prediction Coefficients (LPC). There are, at least, three applications for which the LPC parameter vector (AR parameter vector) statistics are assumed to be known[6]:

- **Pattern recognition**-LPC parameters are random, and their statistics (characterizing the intra-class scattering) are usually assumed to be known (generally Gaussian) [1].
- **Estimation theory**-LPC parameters are deterministic and have to be estimated. Most commonly used LPC parameter estimators can be assumed Gaussian (according to the Mann and Wald theorem) when these parameters are estimated for sufficiently large data records [2]. This Gaussian assumption is unrealistic when AR parameters are estimated with the autocorrelation method. It is well-known that the autocorrelation method yields estimators which belong to a bounded stability domain. The Gaussian assumption implies that the distribution of the LPC parameter vector, although being bounded, is very close to the Gaussian distribution.
- **Theory of random coefficient AR models**-many studies have been carried out under the Gaussian assumption [3].

In what follows, the LPC parameter vector of a nth order AR process will be modelled as a Gaussian random variable, a, with mean m_a and covariance matrix Σ_a. The first part of the paper derives a recursive procedure for obtaining the p.d.f. of LSP coefficients from the p.d.f. of the AR parameter vector. The second part of the paper uses this p.d.f. to determine the shape of point clusters in the LSP coefficient representation space.

2 LSP COEFFICIENTS

For a given nth order minimum phase LPC polynomial $A_n(z) = 1 + \sum_{i=1}^{n} a_i z^{-i}$, two LSP polynomials denoted by $P_n(z)$ and $Q_n(z)$ can be constructed by setting the $(n+1)^{th}$ reflection coefficient (parcor coefficient) k_{n+1} to $+1$ or -1:

$$P_n(z) = A_n(z) - z^{-(n+1)} A_n\left(z^{-1}\right) \qquad (1)$$

$$Q_n(z) = A_n(z) + z^{-(n+1)} A_n\left(z^{-1}\right) \qquad (2)$$

The conditions $k_{n+1} = +1$ and $k_{n+1} = -1$ correspond to complete closure and complete opening of the glottis in the acoustic tube model, respectively. Let:

$$P_n(z) = \sum_{i=0}^{n+1} p_i z^{-i}, Q_n(z) = \sum_{i=0}^{n+1} q_i z^{-i} \qquad (3)$$

with $p_0 = 1, p_{n+1} = -1$ and $q_0 = 1, q_{n+1} = +1$. With these conditions, the coefficients of the LSP polynomials $P_n(z)$ and $Q_n(z)$ are linked to the LPC parameters by the following relations:

$$p_i = a_i - a_{n+1-i} \qquad i \in \{1, ..., n\} \qquad (4)$$

$$q_i = a_i + a_{n+1-i} \qquad i \in \{1, ..., n\} \qquad (5)$$

For n even (the cases of n odd and n even only differ in some details), the LSP polynomials can be expressed as:

$$P_n(z) = \left(1 - z^{-1}\right) \prod_{i=2,4,\ldots,n} \left(1 - 2z^{-1}\cos\omega_i + z^{-2}\right) \qquad (6)$$

$$Q_n(z) = \left(1 + z^{-1}\right) \prod_{i=1,3,\ldots,n-1} \left(1 - 2z^{-1}\cos\omega_i + z^{-2}\right) \qquad (7)$$

The parameters $\{\omega_i\}_{i=1,\ldots,n}$ are the LSP parameters. For a stable LPC polynomial $A_n(z)$, the LSP polynomials (6) and (7) have very interesting properties for quantization and coding [4][5]:

- all zeros of $P_n(z)$ and $Q_n(z)$ alternate with each other on the unit circle.
- the LSP parameters $\{\omega_i\}$ satisfy the "ordering property":

$$0 < \omega_1 < \omega_2 < \ldots < \omega_{n-1} < \omega_n < \pi \qquad (8)$$

3 LSP COEFFICIENT P.D.F.

This part presents a recursive method for determining the p.d.f. of LSP coefficients as a function of the p.d.f. of the LPC parameter vector $a = [a_1, ..., a_n]^t$. Define two polynomials $H_{n-2}(z) = \sum_{i=0}^{n-1} h_i z^{-i}$ and $K_{n-2}(z) = \sum_{i=0}^{n-1} k_i z^{-i}$ such that:

$$P_n(z) = H_{n-2}(z)\left(1 - 2z^{-1}\cos\omega_n + z^{-2}\right) \qquad (9)$$

$$Q_n(z) = K_{n-2}(z)\left(1 - 2z^{-1}\cos\omega_{n-1} + z^{-2}\right) \qquad (10)$$

It can then be shown that the polynomial coefficients, for $i = 1, ..., n/2$, satisfy:

$$p_i = h_i - 2h_{i-1}\cos\omega_n + h_{i-2} \qquad (11)$$

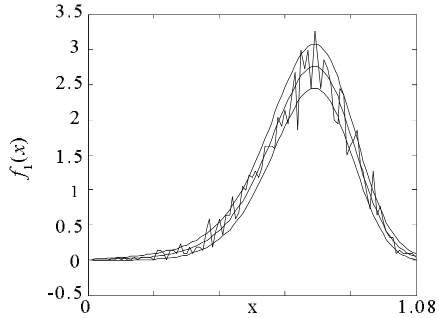
$$q_i = k_i - 2k_{i-1}\cos\omega_{n-1} + k_{i-2} \qquad (12)$$

with $h_0 = k_0 = 1, h_{-1} = k_{-1} = 0, p_i = a_i - a_{n+1-i}$ and $q_i = a_i + a_{n+1-i}$. These relations allow the determination of the p.d.f. of the vector $V_{n-2}^T = [a_1, ..., a_{n-2}, \omega_{n-1}, \omega_n]$ as a function of the p.d.f. of the LPC parameter vector $a = [a_1, ..., a_n]^t = V_n$. In a similar way, the p.d.f. of the vector $V_{n-4} = [a_1, ..., a_{n-4}, \omega_{n-3}, \omega_{n-2}, \omega_{n-1}, \omega_n]^t$ can be determined as a function of the p.d.f. of the vector V_{n-2}. With $p/2$ iterations (assuming the LPC parameter vector p.d.f. is known), the LSP vector p.d.f. can be computed (see Appendix for the example of an order 2 AR process). For simplicity, the study is restricted to Gaussian LPC parameters. However, other cases could be studied similarly. Consider the case of a 2nd order Gaussian AR parameter vector, with two conjugate poles $p_1 = \rho e^{j\varphi}$ and $p_2 = \rho e^{-j\varphi}$. The mean and covariance matrix of this random vector are :

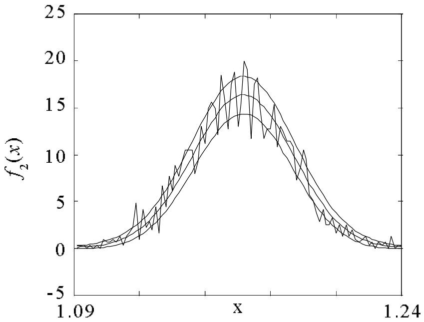
$$m_a = \begin{pmatrix} -2\rho\cos\varphi \\ \rho^2 \end{pmatrix} \qquad (13)$$

$$\Sigma_a = \sigma^2 M \qquad (14)$$

M is a unit norm matrix and σ^2 characterizes the intra-class scattering. Simulations are performed with $\rho = 0.8, \varphi = \frac{\pi}{4}$ and $\Sigma_a = 10^{-2}\begin{pmatrix} 1 & 0.9 \\ 0.9 & 1 \end{pmatrix}$. Fig. 1. shows a comparison between the theoretical and estimated LSP parameter p.d.f. with 95% confidence intervals. The theoretical p.d.f. and the histograms of the LSP parameters are in good agreement.



(a) coefficient ω_1.



(b) coefficient ω_2.

Fig. 1. Theoretical and estimated LSP parameter p.d.f. with 95% confidence intervals.

4 LSP POINT CLUSTERS

The last part of the paper studies the convexity of the point clusters in the LSP representation space. When classifying using the centroid distance rule, it is well-known that surprising results can be obtained with parameters whose p.d.f. is not convex. In Fig. 2, all points belonging to the first class (highlighted with a star) will be mis-classified with the centroid distance rule. The convexity or non-convexity of the LSP parameter p.d.f. can be studied using the results of the previous section. Figs. 3 (a) and (b) show the 3D p.d.f. of the previous order 2 LSP vector and the corresponding level lines. The LSP parameter vector p.d.f. is convex, yielding a case for which the centroid distance rule can be used. A theoretical proof of this convexity property cannot be easily given for any order. However, numerous simulations have been performed which always confirm this property.

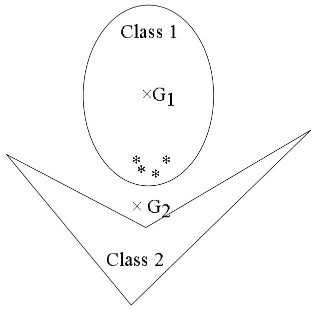
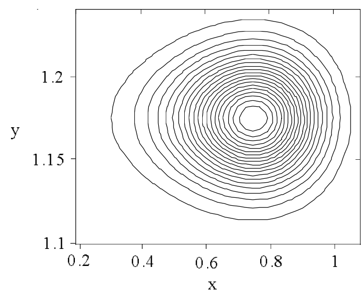
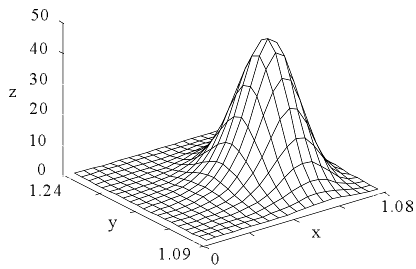


Figure 1: Fig. 2. Classification Model



(a) Level Lines



(b) $3D$ p.d.f.

Fig 3. Level lines and $3D$ plot of the LSP coefficient p.d.f.

5 CONCLUSION

The first part of the paper presented a recursive method for determining the LSP coefficient vector p.d.f. from the LPC vector p.d.f. The second part of the paper showed that the LSP coefficient p.d.f. is shown to be convex, yielding a case for which the centroid distance classifier can be implemented. This study shows that LSP coefficients are well suited for classification, contrary to reflection and cepstrum coefficients [6]. It also allows us to explain some good results obtained with these coefficients in vector quantization.

6 Appendix A : LSP coefficient p.d.f. for order 2

This appendix gives the example of the computation of the LSP coefficient p.d.f. as a function of the p.d.f. of an order 2 LPC parameter vector. The two LSP coefficients are linked to LPC parameters by the following relations:

$$\cos\omega_2 = \frac{1}{2}\left(a_2 - a_1 - 1\right) \tag{15}$$

$$\cos\omega_1 = \frac{1}{2}\left(1 - a_1 - a_2\right) \tag{16}$$

which lead to:

$$a_1 = -\cos\omega_1 - \cos\omega_2 = g_1\left(\omega\right) \tag{17}$$

$$a_2 = 1 - \cos\omega_1 + \cos\omega_2 = g_2\left(\omega\right) \tag{18}$$

with $\omega = (\omega_1, \omega_2)^t$. Eq. (15) and (16) show that the LSP vector ω is real if the vector a belongs to the stability domain D_a of an order 2 AR process. The jacobian matrix corresponding to the transformation from $a = (a_1, a_2)^t$ to ω is:

$$J = \sin\omega_1 \sin\omega_2 \tag{19}$$

The p.d.f. of a Gaussian vector with mean m and covariance matrix Σ is:

$$f\left(x\right) = \frac{1}{2\pi\sqrt{\det\Sigma}} \exp Q\left(x; m\right) \qquad x \in \mathbb{R}^2 \tag{20}$$

where $Q\left(x; m\right)$ is the quadratic form:

$$Q\left(x; m\right) = -\frac{1}{2}\left(x - m\right)^t \Sigma^{-1}\left(x - m\right) \tag{21}$$

To insure the stability of the LPC parameter vector a, it will be assumed that the distribution of a is a truncated Gaussian distribution:

$$g\left(a\right) = \frac{\alpha I_D(a)}{2\pi\sqrt{\det\Sigma}} \exp Q\left(a; m_a\right) \qquad a \in \mathbb{R}^2 \tag{22}$$

with:

$$\begin{array}{lll} I_D(a) = 1 & \text{if} & a \in D_a \\ I_D\left(a\right) = 0 & \text{if} & a \notin D_a \end{array}$$

and:

$$\frac{1}{\alpha} = \iint_{D_a} f\left(x\right) dx$$

The p.d.f. of the LSP coefficient vector $\omega = (\omega_1, \omega_2)^t$ can then be determined:

$$f\left(\omega\right) = \frac{\sin\omega_1 \sin\omega_2}{2\pi\sqrt{\det\Sigma}} \exp Q\left(g\left(\omega\right)\right) I_\Delta\left(\omega\right) \qquad \omega \in \mathbb{R}^2 \tag{23}$$

with $g\left(\omega\right) = \left[g_1\left(\omega\right), g_2\left(\omega\right)\right]^t$ and:

$$\begin{array}{lllll} I_\Delta\left(\omega\right) & = & 1 & \text{if} & 0 \le \omega_1 \le \omega_2 \le \pi \\ I_\Delta\left(\omega\right) & = & 0 & \text{else} & \end{array}$$

References

[1] R. O. Duda and P.E. Hart, Pattern Classification and Scene Analysis, Wiley, New York, 1973.

[2] S. Kay and J. Makhoul, "On the Statistics of the Estimated Reflection Coefficients of an Autoregressive Process," IEEE Trans. Acoust. Speech and Sig. Proc., Vol. ASSP-31, No. 6, Dec. 1983, pp. 1447-1455.

[3] D. F. Nicholls and B. G. Quinn, "Random Coefficient Autoregressive Models : an Introduction," Lecture notes in Statistics, Vol. 11, Springer, New York, 1982.

[4] S. Saoudi, and J. M. Boucher, "A new efficient algorithm to compute the LSP parameters for speech coding," Signal Processing, Vol. 28, No 2, Aug. 1992.

[5] N. Sugamura, and N. Farvardin, "Quantizer Design in LSP speech analysis and synthesis", IEEE journal on selected areas in communications, Vol. 6, No. 2, Feb. 1988.

[6] J.Y. Tourneret and B. Lacaze, "On the Statistics of Estimated Cepstrum and Reflection Coefficients of an Autoregressive Process", Signal processing, Vol. 43, No 3, May 1995.