CONSTRAINED DECONVOLUTION: A GAME THEORY APPROACH IN AN H_∞ SETTING

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ABSTRACT

In this paper we solve the constrained deconvolution problem by state space approach in an H_∞ setting. The problem addressed is the design of a nonlinear estimator that guarantees H_∞ performance on infinite horizon for the estimation error by using the Game Theory technic. The method proposed is useful in cases where the statistics of the disturbance and the noise signal are not completely known. We used the technic proposed to estimate heat production rate from the knowledge of the temperature.

1 INTRODUCTION

In the last years, many significant results on the constrained deconvolution have been proposed. A state space formulation and the solution by a Game Theory approach is a new formulation. This deconvolution method can be considered (after adequate modelling) as a state vector observer design. The observer design can be done with the work of [1]. In the case of [1], we must solve two Algebraic Ricatti Equations (ARE) to design the estimator even if there are no model incertainties. In this paper we use technics developped in [4] and [5]. We obtain a solution which requires only one ARE.

2 SYSTEM DESCRIPTION AND MOTIVATION

Consider the following block diagram.

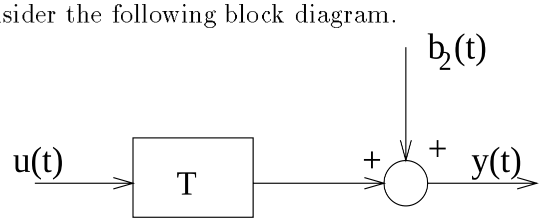

figure 1

where

- u is the unknown signal to be restored.
- (T) is the transfer function of the distorting process.
- b_2 is the additive noise assumed to be L_2.
- y is the measured signal.

(T) is supposed to be causal and u is assumed to be smooth.
The deconvolution problem is to restore u from the knowledge of y and (T). It is well known that deconvolution is a typically ill-posed problem. In this paper we propose to introduce a regularisation based on the positivity of u(t).

3 DECONVOLUTION AND STATE ESTIMATION

If u(t) is a positive smooth signal, it can be considered as the output of a Wiener or a Markov process (P) (see [1]), passing through a nonlinearity insuring positivity for instance (figure 2).

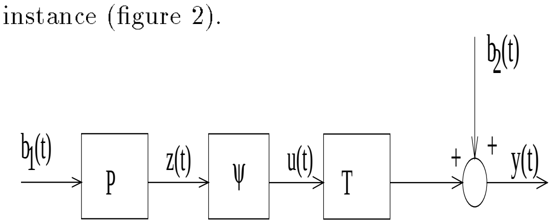

figure 2

Let η_1 and η_2 the state vectors describing respectively the Wiener or the Markov process (P) and the distorting process (T). Then the process of figure 2 can be described by the following state equation.

$$\left\{ \begin{array}{lcl} \dot{x}(t) & = & A\,x(t) \;+\; g(x(t)) \;+\; B\,w \quad x(0)=0 \\ y(t) & = & C\,x(t) \;+\; D\,w \\ z(t) & = & L\,x(t) \\ u(t) & = & \psi(z) \end{array} \right. \tag{1}$$

where

$$x = \begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix}; \quad w = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}; \tag{2}$$

and where $g(.)$ is a nonlinear vector function.

- (i) $g(x) = \begin{pmatrix} 0 \\ 0 \\ . \\ . \\ \psi(z) \end{pmatrix}$

- (ii) $g(0) = 0$
- (iii) There exists some $k > 0$ such that for any x_1 and x_2

$$\|g(x_1) - g(x_2)\| \leq k\|x_1 - x_2\|$$

(In others words g is a Lipschitz function.)

We are concerned with obtaining an estimate z_e of z and therefore an estimation of the input u by using the measurements $Y_t = \{ y\ (\tau);\ 0 \leq \tau \leq t\}$. Let z_e=F $\{Y_t\}$ where F is a nonlinear estimator to be designed. Consider the system given by (1). We propose the following estimator.

$$\begin{cases} \dot{x_e}(t) = (A - KC)x_e(t) + g(x_e(t)) + Ky \\ y_e(t) = Cx_e(t) \\ z_e(t) = Lx_e(t) \\ u_e(t) = \psi(z_e) \end{cases} \quad (3)$$

K is the estimator gain matrix. The error in the state estimate $\overline{x} = x - x_e$ has the following dynamics:

$$\begin{cases} \dot{\overline{x}}(t) = (A - KC)\overline{x}(t) \\ \qquad + (g(x(t) - g(x_e(t)) + (B - KD)w \end{cases} \quad (4)$$

Define the filtering error by $e = (z - z_e)$
Given the system (1) and a prescibed level of noise attenuation $\lambda > 0$, find a filter F such that the estimate error dynamic is globally unifomly asymptotically stable and

$$\left\{\|e\|_2^2 < \lambda^2\|w\|_2^2, \quad 0 \neq w \in L_2\right\} \quad (5)$$

for all g with a Lipshitz constant k. We propose the following theorem.

Theorem
Consider the system (1) satisfaying assumptions stated above. Given a constant $\lambda > 0$, if there exists a real $\epsilon > 0$ such that the following ARE has a symmetric positive definite solution S.

$$AS + SA' - S(C'R^{-1}C - \lambda^{-2}\overline{L}'\overline{L})S + \overline{B}\,\overline{B}' + \epsilon I = 0 \quad (6)$$

where
$\overline{L} = \{L'L + \lambda^2 k^2 I\}^{\frac{1}{2}}$; $\overline{B} = [B \ \ I]$;
$R = DD'$
I is the identity matrix of appropriate dimensions.

then, the estimator gain is $K = SC'R^{-1}$.

4 APPLICATION

We used this method by using experimental data. We apply this deconvolution method to estimate the heat production rate of a chemical reaction from the knowledge transfer between heat production and temperature of calorimeter.
With the following positivity constraints, the results have been concluding.

- (a) $u = z^2$
- (b) $u = |z|$
- (c) $u = \begin{cases} u = 0 & if \quad z \leq 0 \\ u = z & if \quad z > 0 \end{cases}$

For the choosen application:
P$(s) = \frac{1}{s + 1e^{-5}}$

T$(s) = \frac{2.32*1e^{-2}s + 1.236*1e-2}{s^3 + 1.468s^2 + 3.563*1e^{-1}s + 1.075*1e^{-1}}$

The figure 3 is an application of the method described in [6]; the figure 4 is obtained by using [1]; the figure 5 is obtained in applying the results stated above with the positivity constraint (c).

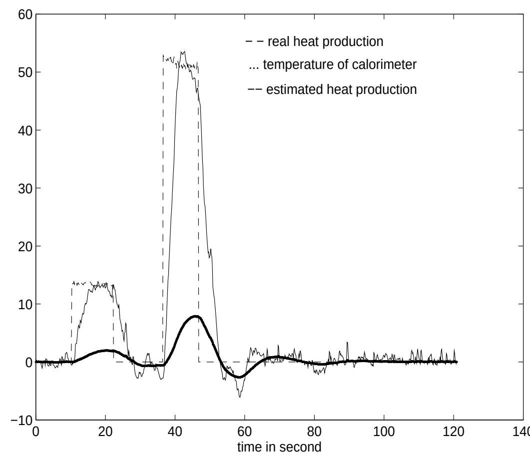

figure 3

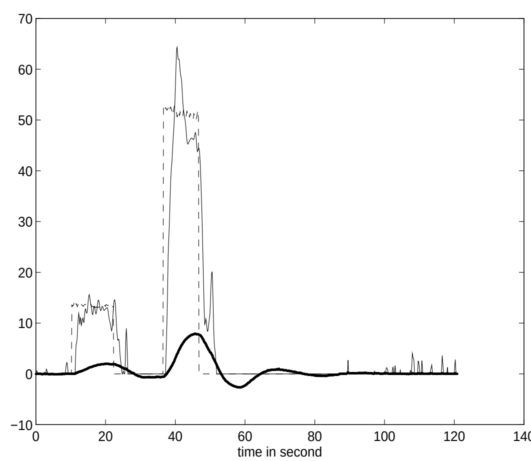

figure 4

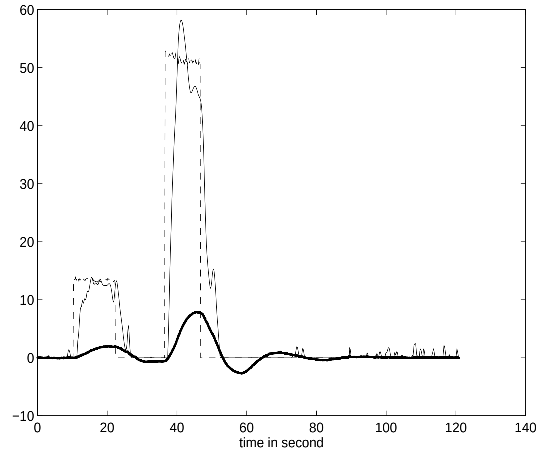

figure 5

The described method through this paper is more easy to use and seems to give best results in restoration terms.

5 CONCLUSION

This paper deals with the $H\infty$ constrained deconvolution problem by using a Game Theory Approach. We use this method to restore positive signals degraded by linear fixe systems. We obtain a simple estimator that can be used on-line. We give a real example to apply the proposed method.

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