SOURCE SEPARATION USING SECOND ORDER STATISTICS

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ABSTRACT

It is often assumed that blind separation of dynamically mixed sources can not be accomplished with second order statistics. In this paper it is shown that separation of dynamically mixed sources indeed can be performed using second order statistics only. Two approaches to achieve this separation are presented. The first approach is to use a new criterion, based on second order statistics. The criterion is used in order to derive a gradient based separation algorithm as well as a modified Newton separation algorithm. The uniqueness of the solution representing separation is also investigated. The other approach is to use System Identification. In this context system identifiability results are presented. Simulations using both the criterion based approach and a Recursive Prediction Error Method are also presented.

1 INTRODUCTION

Source separation algorithms has become a well established research area in the signal processing community. Several papers concerned with this topic have been published, cf [3, 2, 9]. However, there are few papers concerned with source separation for dynamic channels using second order statistics cf. [9, 8, 10]. In the present paper two different methods for source separation using second order statistics are described. The first algorithm is based on a criterion, which makes it possible to derive an algorithm in an analytic manner. The second method solves the source separation problem by considering it as a system identification problem which can be solved with, for example, a Recursive Prediction Error Method (RPEM).

2 PROBLEM FORMULATION

In figure 1 a cross-mixture scenario is depicted. Two white signals $\xi_1(n)$ and $\xi_2(n)$ are used as source generating signals. These signals are convolved with two linear filters and the outputs are $x_1(n)$ and $x_2(n)$, referred to as the source signals. The following assumptions are introduced

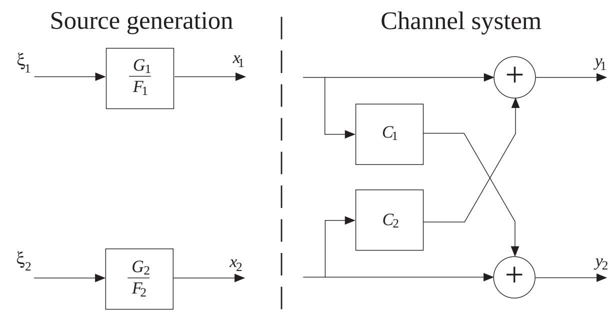

Figure 1: *Source generation and channel system.*

C1: $\xi_1(n)$ and $\xi_2(n)$ are realizations of mutually uncorrelated identically distributed sequences with non zero variance and zero mean.

C2: The filters $G_1(q^{-1})/F_1(q^{-1})$ and $G_2(q^{-1})/F_2(q^{-1})$ are asymptotically stable.

The source signals are unmeasurable and inputs to a system, referred to as the channel system. The channel system produces two outputs $y_1(n)$ and $y_2(n)$ which are measurable and referred to as the observables. The objective is to extract the sources from the observables.

3 CRITERION BASED APPROACH

The algorithm to be presented in this section is based on the assumption that the channel system consists of finite impulse response (FIR) filters. The model for one possible separation structure is depicted in figure 2. This structure is referred to as the feed-forward separation structure. The inputs to the feed-forward separation structure are the observable signals. The output from the separation structure, $s_1(n)$ and $s_2(n)$, depend on the two adaptive FIR-filters, D_1 and D_2, and can be written as

$$\begin{aligned}\begin{bmatrix} s_1(n,\theta) \\ s_2(n,\theta) \end{bmatrix} &= \begin{bmatrix} 1 & -D_2 \\ -D_1 & 1 \end{bmatrix} \begin{bmatrix} y_1(n) \\ y_2(n) \end{bmatrix} \\ &= \begin{bmatrix} 1 - C_1 D_2 & C_2 - D_2 \\ C_1 - D_1 & 1 - C_2 D_1 \end{bmatrix} \begin{bmatrix} x_1(n) \\ x_2(n) \end{bmatrix}. \quad (1)\end{aligned}$$

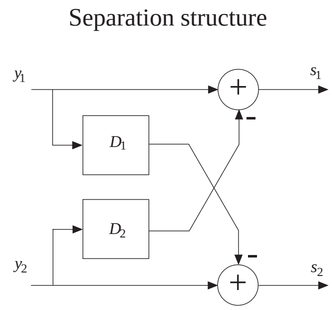

Figure 2: *Source generation, channel system and separation structure.*

where $\theta = \begin{bmatrix} d_{1,0} & \dots & d_{1,U-1} & d_{2,0} & \dots & d_{2,W-1} \end{bmatrix}^T = \begin{bmatrix} \mathbf{d}_1^T & \mathbf{d}_2^T \end{bmatrix}^T$. In order to extract the sources from the observables the parameter vector θ must equal the true parameter vector $\theta_0 = [c_{1,0} \dots c_{1,P-1} c_{2,0} \dots c_{2,L-1}]^T = \begin{bmatrix} \mathbf{c}_1^T & \mathbf{c}_2^T \end{bmatrix}^T$. This is only possible if the natural assumption

C3: $P \leq U$ and $L \leq W$

is introduced. The assumption C3 is reassuring that the channel system can be modeled by the feed-forward separation structure.

Several algorithms found in the literature are not based on any criterion minimization which leads to a combinatorial explosion of possible algorithms. A detailed treatment of the combinatorial problem can be found in [4]. The combinatorial explosion can be eliminated by using the following criterion

$$V(\theta) = \sum_{l=-M}^{M} \left(R_{s_1 s_2}(l, \theta)\right)^2, \tag{2}$$

where $R_{s_1 s_2}$ is the cross-correlation of the signals s_1 and s_2. If the true parameter vector, θ_0 is inserted in the criterion function then $V(\theta_0) = 0$. The cross-correlation $R_{s_1 s_2}(i, \theta)$ is, under assumption C1 and C2, given by

$$\begin{aligned} R_{s_1 s_2}(l) \quad = \quad & R_{y_1 y_2}(l) - \mathbf{d}_2^T \mathbf{r}_{y_2 y_2}(l) - \mathbf{d}_1^T \mathbf{r}_{y_1 y_1}(l) \\ & + \mathbf{d}_2^T \mathbf{R}_{y_2 y_1}(l) \mathbf{d}_1, \end{aligned} \tag{3}$$

where

$$\mathbf{r}_{y_1 y_1}(l) = [R_{y_1 y_1}(l) \dots R_{y_1 y_1}(l + W - 1)]^T, \tag{4}$$

$$\mathbf{r}_{y_2 y_2}(l) = [R_{y_2 y_2}(l) \dots R_{y_2 y_2}(l - U + 1)]^T, \tag{5}$$

$$\mathbf{R}_{y_2 y_1}(l) = [\mathbf{r}_{y_2 y_1}(l) \dots \mathbf{r}_{y_2 y_1}(l + W - 1)], \tag{6}$$

$$\mathbf{r}_{y_2 y_1}(l) = [R_{y_2 y_1}(l) \dots R_{y_2 y_1}(l - U + 1)]^T. \tag{7}$$

The gradient, $\nabla V = \partial V / \partial \theta$, can now be calculated as

$$\nabla V(\theta) = \sum_{l=-M}^{M} 2 \frac{\partial R_{s_1 s_2}(l, \theta)}{\partial \theta} \left(R_{s_1 s_2}(l, \theta)\right). \tag{8}$$

This gradient in conjunction with a stochastic gradient type of algorithm exhibits slow convergence when the source signals are similar. To increase the speed of convergence a Newton algorithm can be derived. The Newton algorithm uses the Hessian matrix of V in order to recalculate the gradient used in equation (8). However, the Hessian matrix is in general not positive definite. If the Hessian matrix is indefinite then the search in some directions can be the opposite to the expected search direction. To eliminate this uncertainty a modified Hessian matrix is used, defined by

$$\tilde{\mathbf{H}} = 2 \sum_{l=-M}^{M} \mathbf{P}(l), \tag{9}$$

where

$$\mathbf{P}(l) = \begin{bmatrix} \frac{\partial R_{s_1 s_2}(l)}{\partial \mathbf{d}_1} \left(\frac{\partial R_{s_1 s_2}(l)}{\partial \mathbf{d}_1} \right)^T & \frac{\partial R_{s_1 s_2}(l)}{\partial \mathbf{d}_1} \left(\frac{\partial R_{s_1 s_2}(l)}{\partial \mathbf{d}_2} \right)^T \\ \frac{\partial R_{s_1 s_2}(l)}{\partial \mathbf{d}_2} \left(\frac{\partial R_{s_1 s_2}(l)}{\partial \mathbf{d}_1} \right)^T & \frac{\partial R_{s_1 s_2}(l)}{\partial \mathbf{d}_2} \left(\frac{\partial R_{s_1 s_2}(l)}{\partial \mathbf{d}_2} \right)^T \end{bmatrix}.$$

The modified damped Newton algorithm is

$$\theta(n) = \theta(n-1) - \mu(n) \tilde{\mathbf{H}}^{-1} \psi(n, \theta(n-1)). \tag{10}$$

3.1 Parameter identifiability and convergence

The issue of parameter identifiability (PI) has been investigated, cf [5]. In brief, for causal channels, the problem is parameter identifiable using second order statistics if the channel system contains at least one root in each channel and $1 - C_1 C_2$ is minimum phase. However, it might be possible to relax the conditions on the algorithm in [5].

Investigation of the convergence properties of the algorithm given in equation (10) has been conducted. For source signals with $F_1 = F_2 = 1$ it turns out that a necessary and sufficient condition for convergence is that it is not simultaneously true that $1 - C_1 C_2$ is linear phase and that the sources have identical color, cf [6].

4 SYSTEM IDENTIFICATION APPROACH

The system identification problem can be stated as the problem of estimating all polynomials in figure 1. Note that the source separation problem requires only the identification of the channel so that the source signals x_1 and x_2 can be recovered.The channel filters are assumed to be ARMA filters with

$$C_1 = \frac{B_1}{A_1}, \; C_2 = \frac{B_2}{A_2}. \tag{11}$$

The resulting equations can be put in matrix form as

$$\begin{aligned} \mathbf{y} \quad & = \quad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{B_2}{A_2} \\ \frac{B_1}{A_1} & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \\ & = \quad \begin{bmatrix} \frac{G_1}{F_1} & \frac{G_2 B_2}{F_2 A_2} \\ \frac{G_1 B_1}{F_1 A_1} & \frac{G_2}{F_2} \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \end{bmatrix}, \end{aligned} \tag{12}$$

where G_1, G_2 etc. are polynomials in the unit delay operator z, and the signals y_1, y_2 etc. are functions of the discrete time variable $t = 1, 2, ...$ (To simplify notation we omit the dependence on z and t whenever possible). Without any restriction A_1, A_2, F_1 and F_2 are assumed to be monic and G_1 and G_2 are assumed to be minimum phase.

4.1 System Identifiability

This section deals with the parameter identifiability of the system presented in equation (12), using second-order statistics only. The following assumption is introduced.

C4: No cancelations occur either in the elements of the spectral matrix $\mathbf{W} = E\left[\mathbf{y}\mathbf{y}^H\right]$ or in its determinant.

Proposition 1 *Under assumption C4, the system given by equation (12) is PI if the filters of both the channels have more poles than zeros.*

Proposition 2 *Under assumption C4, the system given by equation (12) is PI if the sources are purely autoregressive, both filters of the channel have at least one zero, and the filter $A_1A_2 - B_1B_2$ is minimum phase and of degree larger than those of B_1 and B_2.*

Proposition 3 *The system given by equation (12), with a static channel, is locally parameter identifiable (LPI) with exactly two solutions if the sources are colored differently, i.e. G_1/F_1 is not proportional to G_2/F_2 and the product of the channel gains differs from unity.*

Proposition 4 *In the case of G_1/F_1 proportional to G_2/F_2 and static channels the system given by equation (12) is not even locally identifiable.*

Proofs Can be found in [1]. □

A previous article on PI for the source separation problem has been published in [7]. In that paper LPI is shown using second order statistics and accepting a channel flop, but only for static channels. Those results are covered by Proposition 3 of the current paper.

5 SIMULATIONS

5.1 The Criterion based algorithm

According to assumptions C1 and C2 the source signals $x_1(n)$ and $x_2(n)$ can be generated by filtering two mutually uncorrelated sequences through two autoregressive filters. One filter has a complex pole pair at radius 0.8 and angle $\pi/4$. The other filter has a radius of 0.8 and variable angle from 0 to π. The channel system consists of two filters $D_1 = 0.3 + 0.1q^{-1}$ and $D_2 = 0.1 + 0.7q^{-1}$. The auto and cross correlations of y_1 and y_2 have been estimated prior to minimization of the criterion. These

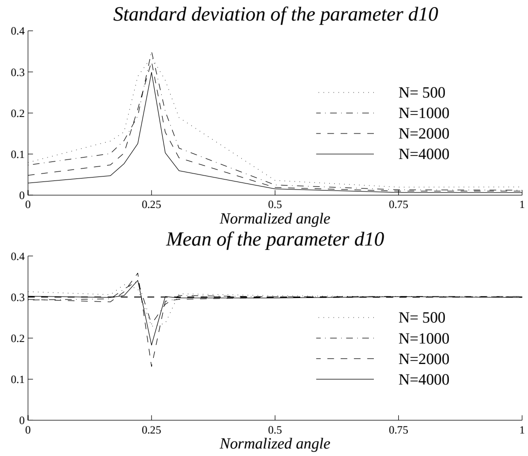

Figure 3: *The standard deviation and mean value as function of the relative angle between the poles of the source filters.*

estimates are based on a realization size of $N = 500$, 1000, 2000, and 4000 samples. For each source filter pair and realization size, N, a standard deviation and a mean value have been calculated based on 100 realizations. The criterion minimizes only the cross-correlation lags from -2 to 2. From figure 3 it can be seen that the variance is reduced with increasing number of samples. The simulation also reveals the narrow band where separation is "hard". This band corresponds to sources of similar, or even identical color. Note that only the parameter $d_{1,0}$ is presented in figure 3. However, the other parameters have a similar behavior.

The second simulation illustrates the convergence behavior. Three cases are used, these are summarized in table 1. In all three cases G_1 and G_2 are unity. For

Case	F_1	F_2	$roots(1 - C_1C_2)$
1.	$0.8\angle \pm \pi/4$	$0.8\angle \pm \pi/4$	0.5 and 3
2.	$0.8\angle \pm \pi/4$	$0.8\angle \pm \pi/4$	0.5 and 2
3.	$0.8\angle \pm \pi/4$	$0.8\angle \pm 3\pi/4$	0.5 and 2

Table 1: *The settings for simulation two*

each case 100 realizations, of size $N = 16000$, have been run and the resulting parameter standard deviation have been summarized in table 2. Note that case one corre-

Prop.	Case 1	Case 2	Case 3
c_{10}	0.187	0.041	0.006
c_{11}	0.683	2.187	0.006
d_{10}	0.175	0.041	0.005
d_{11}	0.698	2.157	0.005
cond($\tilde{\mathbf{H}}$)	700	10^{16}	80

Table 2: *The standard deviation of the estimated parameters and the condition number of the modified Hessian.*

sponds to $1 - C_1C_2$ being mix-phase whereas case two and three corresponds to linear-phase. Note that the condition number for case 2 is very high. The interesting thing is that the convergence property, in section 3.1, holds even if $F_1 = F_2 \neq 1$. However, for the general case this remains to be proven.

5.2 Simulations using RPEM

In this section we present simulation results to illustrate the theory of section 4. From the class of available parameter estimation methods we choose the recursive prediction error method (RPEM). A description of how this method can be applied to the source separation problem can be found in [1]. Recursive signal separation with ARMA-channel filters based on a criterion employing only second order statistics is believed to be novel. Figure 4 presents both estimated parameters and root mean square (rms) values of the difference between the true and the estimated source signals.

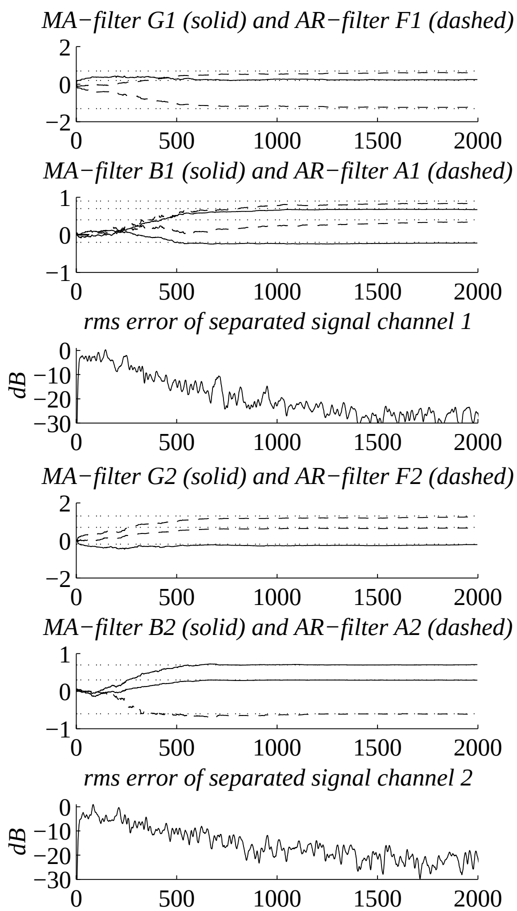

Figure 4: *Parameter estimates and normalized rms values. The true parameter values dotted.*

The system used in this section is PI according to Proposition 1. The filter B_1/A_1 contains more poles than zeros while B_2/A_2 has equally many in this example.

In the RPEM algorithm both the source filters and the channel were modeled by using the true number of parameters.

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