AN ADAPTIVE BLIND EQUALISER WITH AUTOMATICALLY CONTROLLED STEP-SIZE

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ABSTRACT: *Although research into blind equalisation has been on-going for more than a decade, the existing blind equalisation algorithms are often inefficient in combating the impairments introduced by mobile communication channels. New and efficient algorithms are hence needed. The performance characteristics of a recently proposed blind clustering technique in the presence of frequency selective fading and Doppler-effects in a mobile communication environment is first studied. A new, fast convergence algorithm is then introduced based on modification of the Super-Exponential (SE) algorithm, followed by a clustering technique with an automatically controlled step-size. Simulations show that the new algorithm converges very fast and can get rid of the constellation rotation problem encountered when applying the SE method to time-varying channels.*

I. INTRODUCTION

An adaptive blind equaliser is used to recover a signal that is corrupted by intersymbol interference and noise without the help of a training signal. There are basically two categories of blind equalisers, namely the Bussgang and Higher-order statistics based algorithms. The Bussgang equalisers [1-3] use different LMS-type techniques to solve this problem. Since the second-order statistic is phase blind, statistics of higher order are therefore considered in the realisation of blind equalisation of nonminimum phase (NMP) channels.

Higher-order statistics based equalisers utilise the higher order statistics, moments or cumulants of the data at the output of the communication channel. Researchers like Giannakis [4], Swami and Mendel [5], Zheng and McLaughlin [6] and many others are working on some close-form relations between unknown parameters and the observed signal's cumulants of various order or their respective polycepstra. Once a channel impulse response is obtained, it can then be used to design an equaliser. These blind equalisers, although very general and powerful, need extra steps to recover the transmitted signal and therefore require extensive computation. More recently, Shalvi and Weinstein presented a class of iterative algorithms (SE) that converge monotonically at a very fast rate to the desired response regardless of initialisation [7]. They proved that maximising the Kurtosis of a complex equaliser output under certain power constraints, leads to a global maximum. However, the final MSE value obtained in some cases is unacceptable as this method aims to reduce the ISI only. Recently, Ueng and Su [8] focused on this problem of the SE algorithm and suggested two classes of adaptive blind algorithms for PAM data, which possess the properties of fast convergence rate with small steady-state ISI and MSE.

II. SOFT CLUSTERING TECHNIQUE

Karaoguz and Ardalan [9] have presented a soft decision-directed blind algorithm which was based on maximising the a-posterior (MAP) probability density function of the transversal equaliser output with respect to the equaliser weights. This algorithm is most effective for reconstructing BPSK and QPSK signals and is always referred to as the blind clustering algorithm.

The present study makes use of the blind clustering algorithm to improve the SE algorithm to speed up the convergence rate and to solve the phase rotation problem encountered when dealing with both static and time-varying multipath channels.

For a given QPSK constellation, each symbol travels via energy spread over a random number of unequal length paths and all the multipath signals are combined at the receiver. By the ‘Central limit theorem’ the distribution of the summation of random variables approaches a Gaussian distribution as the number of paths increase. Hence, the output of the equaliser around each transmitted data symbol is Gaussian distributed with a priori known mean and variance. Figure 1 illustrates the resulting configuration for the QPSK constellation, where the circles denote the contours of the surface of the bell-shaped Gaussian distribution.

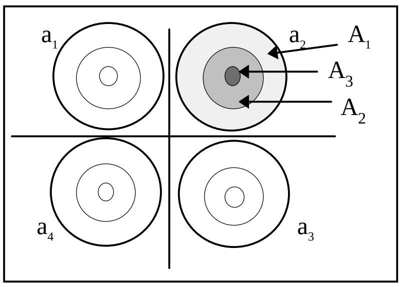

Figure 1 Modelling with Gaussian distributions

III. ALGORITHM WITH CLUSTERING TECHNIQUE

The SE method [7] causes the combined impulse response to converge quickly to the desired response by maximising the

Kurtosis of a complex equaliser output under certain power constraints. The tap weight vector is updated by,

$$\mathbf{C}'(i) = \mathbf{C}'(i-1) + \frac{\beta}{\delta}\mathbf{Q}_i \mathbf{y}_i^* z(i)\left(|z(i)|^2 - \frac{E\{|a_i|^4\}}{E\{|a_i|^2\}} \right) \quad (1)$$

where β is the forgetting factor,

$$\mathbf{y}_i = [y(i), y(i-1), \ldots, y(i-L+1)]^T \quad (1a)$$

$$\delta = C_4[a_i] / E\left[|a_i|^2\right] \quad (1b)$$

y_i is the received complex imput vector stored in the equaliser taps $\mathbf{C}'$ at time i, δ is defined by the fourth order cumulant C_4 and the expected values of the input symbols $\{a_i\}$ in (1b) and $\mathbf{Q}_i$ is an inverse of the correlation matrix $\mathbf{R}_L$ which is defined as,

$$\mathbf{R}_L = E\left[\mathbf{y}_i \mathbf{y}_i^{\mathbf{T}}\right] \quad (1c)$$

The blind equaliser output is

$$z_i = z_R + jz_I = \mathbf{C}'^{\mathbf{T}} \mathbf{y}_i \quad (1d)$$

$\mathbf{Q}_i$ is updated by a recursive formula which must be initialised by batch processing the data segment first.

Consider the output of the blind equaliser as a data point, and apply a generalised decision directed algorithm (GDD) at the output of the equaliser. The GDD algorithm is based on a decision procedure that is analogous to the Bayes decision rule. This considers the fact that any given equaliser output can be from any one of the clusters in the constellation. The algorithm maximises the generalised objective function (2) by moving the data point toward the centre of the nearest cluster, taking into account the probability that the observed equaliser output might well be one of the transmitted symbols (a_1, a_2, a_3, a_4, refer to Figure 1).

$$\Psi(z_R, z_I) = -\frac{1}{2\sigma}\exp\left[\left(z_R - \mu_R\right)^2 + \left(z_I - \mu_I\right)^2\right] \quad (2)$$

where μ_R and μ_I are the means of the real and imaginary parts of the clusters with values of ± 1. The update equation can be written as,

$$\mathbf{C}'_R(i) = \mathbf{C}'_R(i) + \alpha \frac{\partial \Psi(\mathbf{C}'_R, \mathbf{C}'_I)}{\partial \mathbf{C}'_R}$$
$$\mathbf{C}'_I(i) = \mathbf{C}'_I(i) + \alpha \frac{\partial \Psi(\mathbf{C}'_R, \mathbf{C}'_I)}{\partial \mathbf{C}'_I} \quad (3)$$

where $\mathbf{C}'_R$, $\mathbf{C}'_I$ are the real and imaginary parts of the complex FIR weight vector and α is the learning rate. The probability that an equaliser output can be from a cluster further away from the given equaliser output is less than the probability of a closer cluster updating the filter weights.

The update equations in (3) continuously adjust the complex weight vector so that the output of the blind equaliser forms four clusters where the clusters have real, imaginary means (μ_m, μ_n; m, n = 1, 2) and variance values specified by σ^2. The partial derivatives can be written as,

$$\frac{\partial \Psi(\mathbf{C}'_R, \mathbf{C}'_I)}{\partial \mathbf{C}'_R} = -\frac{1}{\sigma^2}\sum_{m=1}^{2}\sum_{n=1}^{2} \mathbf{\Omega}_{mn}(z_R, z_I)\left[(z_R - \mu_m)x_R + (z_R - \mu_n)x_I\right] \quad (4)$$

$$\frac{\partial \Psi(\mathbf{C}'_R, \mathbf{C}'_I)}{\partial \mathbf{C}'_I} = -\frac{1}{\sigma^2}\sum_{m=1}^{2}\sum_{n=1}^{2} \mathbf{\Omega}_{mn}(z_R, z_I)\left[(z_I - \mu_n)x_R + (z_R - \mu_m)x_I\right] \quad (5)$$

and

$$\mathbf{\Omega}_{mn}(z_R, z_I) = \frac{1}{\left(1 + \exp\left(\frac{-2\mu_m z_R}{\sigma^2}\right)\right)\left(1 + \exp\left(\frac{-2\mu_n z_I}{\sigma^2}\right)\right)} \quad (6)$$

where $\mu_k = (-1)^k$ and

$$\sum_{m=1}^{2}\sum_{n=1}^{2} \mathbf{\Omega}_{mn}(z_R, z_I) = 1$$

The update equations are the sum of 4 decision-directed updates corresponding to 4 clusters in the QPSK constellation. The probability that an output belongs to a certain cluster is determined by the joint sigmoid function in (6) which can take values between 0 and 1.

IV. VARIABLE STEP SIZE

A simple way to increase the convergence speed of a blind algorithm is to apply a sequence of gradually varying adaptation step sizes which better fit the current state of the convergence process. It is known that the choice of the step size reflects a trade-off between misadjustment and the speed of adaptation [10]. The step size should be chosen to ensure stable operation at the beginning of the convergence process and it should be selected to minimise the residual excess error in the steady state. So far, there is no general theory on the selection of the step size for an existing blind equaliser algorithm. Researchers usually resort to simulation experiments with a much smaller step size than that derived by Ungerboeck [11]:

$$\alpha = 1 / \left(L \cdot E\left[|x_i|^2\right]\right) \quad (7)$$

where L is the number of equaliser taps.

In this paper, we make use of a simple automatic control of the equaliser's step size [10] to further improve the performance of the modified SE method. The equations of the system are,

$$y_n = \sum_{i=1}^{L} C_n^i \cdot x_{n-i}$$
$$e_n = y_n - d_n, \quad (8)$$

Let d_n denote the desired symbol, where y_n is the recovered symbol, C_n^i are the equaliser's coefficients at time n, and

x_{n-i} is the input signal contained in the tapped delay line of the equaliser on the i-th position.

The magnitude of the output error e_n can be used as a measure of how ‘close’ the received signal is to the transmitted signal where different areas in the constellation plane can be classified according to the distance of the equaliser output from the centre of each cluster (Figure 1). If the equaliser output signals y_n fall into these areas, we can expect that a rough equalisation has already been achieved. For a QPSK signal, the i-th constellation point can be defined as belonging to different areas A_i,

$$y_n \in A_i$$
if and only if (9)
$$\left| y_n - d_n \right| \le const$$

Thus, we adopted a system whereby a different value of the step-size is used when the received signal falls into different regions. Because the Euclidean distance of a data symbol to the nearest threshold is equal to one obviously $const \le 1$.

The use of the higher or lower step size is dependent on the event $y_n \in A_i$. However, a single event $y_n \in A_i$ can switch the adaptation of the larger step size on, even though the equaliser can be far away from a rough equalisation of the channel. To prevent this, the algorithm can monitor a few recent values of the output error and base its choice on a series of y_n values rather than a single one. A flag is set ($f_n = 1$) if such a received signal exists that $y_n \in A_i$, ($f_n = 0$ otherwise). Selecting a threshold as *TH*, then

$$\text{if} \quad \sum_{i=1}^{L} f_{n-i} > TH \quad \text{then} \quad \alpha = \alpha_i \tag{10a}$$

$$\text{otherwise} \quad \alpha = \alpha_{min} \tag{10b}$$

The switching between adaptation of higher or lower step size is then performed more smoothly.

V. SIMULATIONS AND DISCUSSION

The performance of the proposed algorithm with variable step size was studied using a computer simulation. The influence of the automatic step size control on the convergence speed of the equaliser was investigated.

The channel reported by Shalvi and Weinstein {H = [0.4 1 -0.7 0.6 0.3 -0.4 0.1]} was applied in the simulations. Transmission using QPSK modulation where {Re(y), Im(y)} ∈ {±1} was simulated. The additive white, i.i.d. Gaussian noise was added to the output of the channel. The equaliser's length was equal to 16. The forgetting factor was chosen to be 5.10^{-3}. The optimum initial step size for the 16-tap equaliser was chosen to be 1.10^{-2}. When condition (10a) is fulfilled the step sizes are chosen to be 2.10^{-2}, 4.10^{-2} and 8.10^{-2}. The algorithm with the above mentioned step sizes with ‘smoothed’ switching (L = 16, TH = 8, $const$ = 0.5, 0.75, 0.9 respectively) was simulated.

The results of simulation are shown in Figure 2. By using the automatic step size control a saving of about 500-600 iterations is possible compared to the algorithm with a constant step size in the case of SNR of 20 dB.

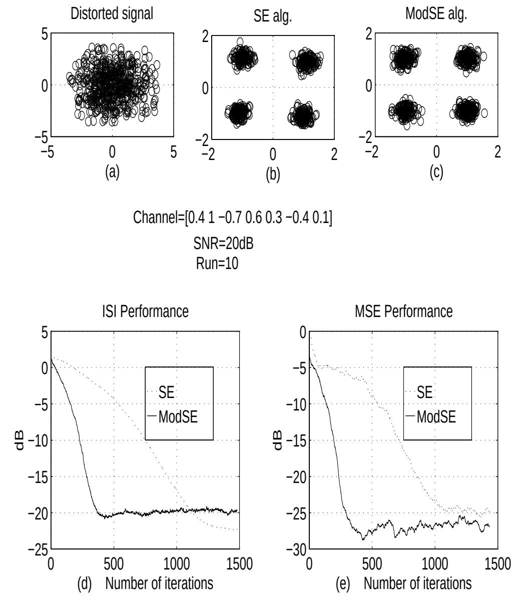

Figure 2. Simulation results for the case of a 7-tap channel with SNR chosen to be 20 dB.

The performance of the new algorithm was also studied with application to a mobile radio channel. The data generator outputs symbols at a rate of 500 kHz. This results in a symbol duration for the quadrature and inphase signal of 20μs. The simulations were carried out for vehicle speeds of 37 mi/hr and 74 mi/hr which gave Doppler frequencies of 50 Hz and 100 Hz respectively.

Mobile communication channels suffer from rapidly changing channel characteristics due to the fading and Doppler phenomena. The baseband multipath Rayleigh fading channel model used in our simulations comprised of four propagation paths with time delays of 0, 20, 40 and 60 μsec with relative power in each multipath of 0.63, 0.98, 0.23 and 0.4 respectively. In each path the received signal is the summation of signals with the same propagation delay but statistically independent phases. This scattering phenomenon will cause fading. We make use of the commonly used ‘Jakes’ model [12]. The maximum Doppler shift (Fd) is related to the velocity of movement v_m, and the transmit frequency f_c, by

$$\text{Fd} = \frac{v_m * f_c}{c} \tag{11}$$

where c is the speed of light.

The time-varying characteristics of the real and imaginary components of the first coefficient of the fading channel are illustrated in Figure 3 together with the performance of the two algorithms when applied to a 4 path fading channel with Doppler frequency of 50 Hz and 100 Hz respectively.

In Figure 3, the reconstructed eye-constellations by using the modified algorithm show no phase rotation in both cases and the results show the superior performance of the new algorithm.

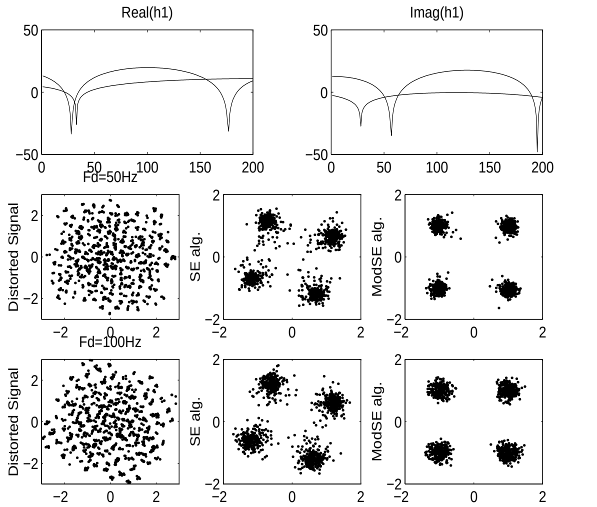

Figure 3. Simulation result for the case of 4-path fading channel with SNR chosen to be 30 dB, where h(1) is the coefficient of the first tap of the multipath fading channel for both cases with Fd of 50 and 100 Hz.

The ISI performance of the existing SE and the improved method is illustrated in Figure 4. The new algorithm results in a saving of about 500-600 iterations for the case where the Doppler frequency is 50 Hz and a saving of 400-500 iterations where the Doppler frequency goes up to as high as 100 Hz.

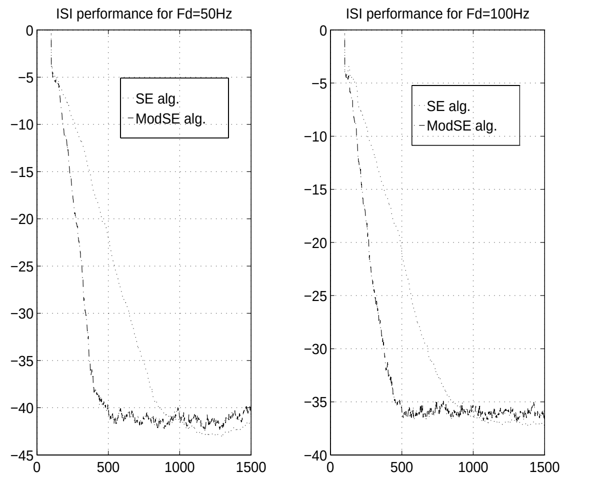

Figure 4. ISI performance for the case of a fading channel with Doppler frequency of 50 and 100 Hz and with SNR of 30 dB

In Figure 5, a significant improvement in the steady state MSE value was obtained after convergence by using the new algorithm. This is due to the problem of phase-rotation encountered with the application of the SE algorithm to a fading channel. In both cases where the channels are static and time-varying, with SNR of 20 dB and 30 dB respectively, simulation results show that the new algorithm gave fast convergence rates and obtained very low steady-state MSE and ISI.

VI. CONCLUSIONS

We have proposed an algorithm for blind equalisation by using the existing SE and blind clustering algorithm with automatically controlled step size. This is based on the idea of adjusting the equaliser tap gains to maximise the likelihood that the received i.i.d. signal comes from a mixture of the four Gaussian clusters with known means. The proposed procedure increases the computational complexity of the algorithm only slightly and thus can be easily implemented.

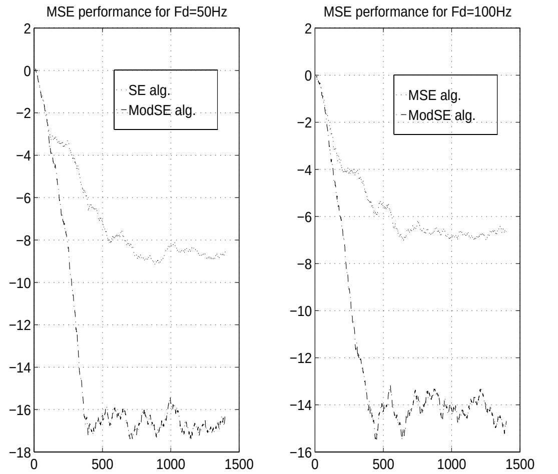

Figure 5. MSE performance for the case of a fading channel with Doppler frequency of 50 and 100 Hz with SNR of 30 dB**.**

References

[1] Y. Sato, "A method of self-recovering equalisation for multilevel amplitude-modulation system," IEEE Trans. Comm., vol. COM-23, pp. 672-682, June 1976.

[2] A. Benveniste and M. Goursat, "Blind equalisation," IEEE Trans. Comm., vol. COM-32, pp 871-883, Aug. 1984.

[3] D. N. Godard, "Self-recovering equalisation and carrier tracking in two dimensional data communication system," IEEE Trans. Comm., vol. COM-28, pp. 1867-1875, Nov. 1980.

[4] G. B. Giannakis and J. M. Mendel, "Identification of nonminimum phase systems using higher order statistics," IEEE Trans. Acoust., Speech, Signal Processing, vol. 37, pp. 360-377, Mar. 1989.

[5] A. Swami and J. M. Mendel, "Closed-form recursive estimation of MA coefficients using autocorrelations and third order cumulants," IEEE Trans., Acoust., Signal Processing , vol. 37, pp. 1794-1795, Nov. 1989.

[6] F. C. Zheng, S. McLaughlin and B. Mulgrew, "A second and fourth order cumulant based blind equalisation algorithm for nonminimum phase channels." Proceeding Int. Signal Processing workshop on Higher order statistics, Chamrous, France, pp. 129-132, July 1991.

[7] O. Shalvi and E. Weinstein, "Super-exponential methods for blind deconvolution" IEEE Trans. on information theory vol. 39 No. 2 pp. 504-519 March 1993.

[8] F. B. Ueng and Y. T. Su, "Adaptive Blind Equalisation Using Second and Higher Order Statistics," IEEE Journal on Selected areas in Communications, pp. 132-140, vol. 13 No. 1 Jan. 1995

[9] J. Karaoguz and S. H. Ardalan, "A soft decision-directed blind equalisation algorithm applied to equalisation of mobile communication channels," in Proc. ICC'92 (Chicago), 1992, vol. 3, pp. 343.4.1-343.4.5

[10] K. Wesolowski, "Adaptive blind equalisers with automatically controlled parameters." IEEE Trans. on Comm., pp. 170-172, vol. 43. No. 2/3/4 Feb./March/April 1995.

[11] G. Ungerboeck, "Theory on the speed of convergence in adaptive equalisers for digital communication," IBM J. Res. Develop., vol. 11, pp. 546-555, 1972

[12] W. C. Jakes, *Microwave mobile communications* New York : Wiley 1974.