ANALYSIS OF AN ADAPTIVE IIR FILTER FOR MULTIPATH TIME DELAY ESTIMATION

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ABSTRACT

A novel adaptive recursive algorithm is proposed for estimating the interpath delay of a radiated signal in a multipath environment. Using LMS-type adaptation, the estimator is computationally efficient and it provides direct measurements of multipath gain and delay on a sample-by-sample basis. The convergence dynamics and variances of system parameters are derived. It is shown that the optimal performance of the estimator approaches the Cramér-Rao lower bound (CRLB) for high signal-to-noise ratio (SNR) conditions. Computer simulations have validated the capability of the method to track time-varying delays accurately.

1 INTRODUCTION

In radar and sonar data processing, a radiated signal often arrives at a receiver through more than one propagation path. By making use of the passive time delay measurements between the multipath arrivals, useful source localization information can be provided [1]. Other applications of multipath time delay estimation include wireless communication systems, seismology and biomedical engineering. In this paper, a simple multipath delay estimator, called adaptive multipath canceller (AMC), is proposed that provides direct and continuous measurements of multipath gain and delay. It is assumed that the transmitted signal arrived at the sensor goes through two paths in the same plane with the receiver and source. The received waveform can be written as

$$r(k) = s(k) + \alpha s(k-\Delta) + n(k) \tag{1}$$

where the unknown source signal $s(k)$ and the corrupting noise $n(k)$ are jointly stationary and mutually uncorrelated with each other. Without loss of generality, it is assumed that the signal and noise spectra are bandlimited between $-\pi$ and π while the sampling period is unity. The multipath transmission is characterized by the gain factor α as well as the interpath delay Δ. Note that the multipath gain must lie between 0 and 1 while the multipath delay should be larger than zero. Given the received signal $r(k)$, the goal is to estimate Δ, and in many applications, it is desired to derive the multipath gain as well.

Since the multipath gain and delay are nonstationary due to relative motion between the signal source and the receiver, adaptive techniques are necessary for its estimation and tracking. Basically, the AMC is an adaptive recursive filter that eliminates the multipath component in the received signal and it is computationally efficient. In Section II, the structure and algorithm of the AMC are derived and the convergence dynamics of the system parameters are given. Section III shows that the mean square error of the AMC can attain the CRLB for high SNRs. Numerical examples are presented in Section IV to corroborate the analytical derivations and to evaluate the performance of the estimator.

2 THE AMC

To simplify the analysis, we first assume that noise is absent in the received signal. Taking the Z transform of (1) gives [2]

$$R(z) = \left(1 + \alpha \sum_{n=-\infty}^{\infty} sinc(n-\Delta) z^{-n}\right) S(z) \tag{2}$$

where $R(z)$ and $S(z)$ denote the Z transform of $r(k)$ and $s(k)$ respectively and $sinc(v) = sin(\pi v)/(\pi v)$. The idea of the AMC is to remove the multipath component $\alpha s(k-\Delta)$ from $r(k)$ by passing it through an adaptive IIR filter whose transfer function is given by

$$A(z) = \frac{1}{1 + \hat{\alpha} \sum_{i=1}^{M} sinc(i-\hat{\Delta}) z^{-i}} \tag{3}$$

where $\hat{\alpha}$ and $\hat{\Delta}$ represent the estimates of the multipath gain and delay respectively. The filter length, M, is chosen to be larger than the maximum allowable delay. Although a noncausal IIR filter of transfer function $1/(1+\alpha\sum_{i=-M}^{M} sinc(i-\Delta)z^{-i})$ will provide a better model for multipath cancellation, it is, however, an unstable system and its realization is not practically implementable. As a result, $A(z)$, which has been shown to be stable (See Appendix I), becomes a realizable structure of the AMC. It is noted that when the delay is an integral multiple of the sampling interval, exact multipath elimination can be achieved. The system block diagram of the AMC is depicted in Figure 1 where $r'(k)$ represents the filtered output of $r(k)$. If we minimize the mean square value of $r'(k)$, it is expected that $\hat{\alpha}(k) \to \alpha$ and $\hat{\Delta}(k) \to \Delta$ when the multipath is highly resolvable, that is, $\Delta >> 1$. In this case, $A(z)$ is a very accurate inverse modeling filter and $r'(k) \approx s(k)$.

Similar to Widrow's LMS method, stochastic gradient estimates are used in the adaptive process and the iterative AMC algorithm is given by

$$\hat{\Delta}(k+1) = \hat{\Delta}(k) - 2\mu_\Delta r'(k) \sum_{i=1}^{M} f(i - \hat{\Delta}(k)) r'(k-i) \quad (4)$$

and

$$\hat{\alpha}(k+1) = \hat{\alpha}(k) + 2\mu_\alpha r'(k) \sum_{i=1}^{M} sinc(i - \hat{\Delta}(k)) r'(k-i) \quad (5)$$

where

$$r'(k) = r(k) - \hat{\alpha}(k) \sum_{i=1}^{M} sinc(i - \hat{\Delta}(k)) r'(k-i) \quad (6)$$

The quantities μ_Δ and μ_α are positive scalars that control convergence rate and system stability while $f(v) = (\cos(\pi v) - sinc(v))/v$. To significantly reduce the computation load of the AMC algorithm, look-up tables of the *sinc* and cosine function are used [3]. As a result, $(3M+1)$ additions and $(3M+7)$ multiplications are required at each iteration.

Assuming white signal source and $r'(k) \to s(k)$ and for highly resolvable multipath, the expected value of (4) is derived as follows,

$$\begin{aligned} &E\{\hat{\Delta}(k+1)\} - E\{\hat{\Delta}(k)\} \\ &\approx -2\mu_\Delta E\{\sum_{i=1}^{M} f(i - \hat{\Delta}(k)) s(k-i) \cdot (s(k) \\ &\quad + \alpha s(k-\Delta) - \hat{\alpha}(k) \sum_{j=1}^{M} sinc(j - \hat{\Delta}(k)) s(k-j))\} \\ &= -2\mu_\Delta E\{\sum_{i=1}^{M} f(i - \hat{\Delta}(k)) s(k-i) \\ &\quad \cdot (\alpha \sum_{j=-\infty}^{\infty} sinc(j-\Delta) s(k-j) \\ &\quad - \hat{\alpha}(k) \sum_{j=1}^{M} sinc(j - \hat{\Delta}(k)) s(k-j)) \\ &\approx -2\mu_\Delta \alpha \sigma_s^2 E\{\sum_{i=1}^{M} sinc(i-\Delta) f(i - \hat{\Delta}(k))\} \\ &\quad + 2\mu_\Delta \sigma_s^2 E\{\hat{\alpha}(k) \cdot \sum_{i=1}^{M} sinc(i - \hat{\Delta}(k)) f(i - \hat{\Delta}(k))\} \\ &\approx -2\mu_\Delta \alpha \sigma_s^2 E\{f(\Delta - \hat{\Delta}(k))\} + 2\mu_\Delta \sigma_s^2 E\{\hat{\alpha}(k) \cdot f(0)\} \end{aligned} \quad (7)$$

Since $f(0) = 0$, by using the first order approximation for $f(\Delta - \hat{\Delta}(k))$, (7) becomes

$$E\{\hat{\Delta}(k+1)\} \approx E\{\hat{\Delta}(k)\} - \frac{2}{3}\mu_\Delta \alpha \sigma_s^2 \pi^2 (E\{\hat{\Delta}(k)\} - \Delta) \quad (8)$$

If $0 < \mu_\Delta < 3/(\alpha \sigma_s^2 \pi^2)$ is satisfied, solving (8) will give the learning trajectory of the multipath delay estimate which is of the form

$$E\{\hat{\Delta}(k)\} \approx \Delta + (\hat{\Delta}(0) - \Delta)(1 - \frac{2}{3}\mu_\Delta \alpha \sigma_s^2 \pi^2)^k \quad (9)$$

In a similar manner, the convergence behaviour of $\hat{\alpha}(k)$ can be obtained and is approximated by

$$E\{\hat{\alpha}(k)\} \approx \alpha + (\hat{\alpha}(0) - \alpha)(1 - 2\mu_\alpha \sigma_s^2)^k \quad (10)$$

Here, we assume that $\hat{\Delta}(k) \to \Delta$ and $0 < \mu_\alpha < 1/\sigma_s^2$.

In addition, the multipath delay variance can be shown to be

$$var(\hat{\Delta}) = \frac{\mu_\Delta \sigma_s^2}{\alpha} \quad (11)$$

Notice that $var(\hat{\Delta})$ increases with decreasing α and this agrees with the autocorrelation method in which the second dominant peak that corresponds to the multipath delay, is difficult to locate if the multipath gain is small. The variance of $\hat{\alpha}(k)$ is given by

$$var(\hat{\alpha}) = \mu_\alpha \sigma_s^2 \quad (12)$$

3 COMPARISON WITH THE CRLB

From (6), an optimal realization of the AMC is to search the minimum point of the cost function $C(a,b)$, where

$$C(a,b) = \sum_{k=1}^{N} \left(r(k) - a \sum_{i=1}^{M} sinc(i-b) r'(k-i) \right)^2 \quad (13)$$

and N is the observation time. The co-ordinates of this global minimum, $(\hat{\alpha}^o, \hat{\Delta}^o)$, essentially represent the multipath gain and delay estimate. When a and b are at the neighborhood of α and Δ respectively, the delay variance, $var(\hat{\Delta}^o)$, is given by [4]

$$var(\hat{\Delta}^o) = \left. \frac{E\left\{ \left(\frac{\partial C(\alpha,b)}{\partial b} \right)^2 \right\}}{\left(E\left\{ \frac{\partial^2 C(\alpha,b)}{\partial^2 b} \right\} \right)^2} \right|_{b=\hat{\Delta}^o} \quad (14)$$

For white signal and noise case, (14) can be simplified to

$$var(\hat{\Delta}^o) = \frac{3}{N\pi^2\alpha^2} \quad (15)$$

If $\Delta >> 1$, $\alpha << 1$ and the SNR is high, this variance is identical to the CRLB for estimation of a single multipath delay [5].

4 SIMULATION RESULTS & DISCUSSIONS

Extensive computer simulations have been conducted to corroborate the theoretical derivations and to evaluate the performance of the AMC for multipath time delay estimation. The source signal $s(k)$ and the corrupting noise $n(k)$ were white Gaussian random variables and they were produced by a pseudorandom noise generator. The power of $s(k)$ was fixed to unity and different SNRs were obtained by proper scaling of the random noise sequence. The delayed signal $s(k-\Delta)$ was produced by passing $s(k)$ through an FIR filter whose transfer function was $\sum_{i=-30}^{30} sinc(i-\Delta)z^{-i}$. We fixed $\Delta \in (0, 14)$ and M was chosen to be 15 to cover all possible delays. In our studies, $A(z)$ was freely updated according to the recursive LMS algorithm proposed by Feintuch [6] at the beginning of the adaptation for 400 iterations to deduce an initial estimate of Δ from the peak of the filter coefficients. The AMC was then adjusted using (4) and (5). The initial value of $\hat{\alpha}(k)$ was arbitrarily set as 0.5 while the step sizes μ_α and μ_Δ were assigned a value of 0.004. All simulation results provided were averages of 200 independent runs.

The learning trajectory of the multipath gain and delay estimates under a noise-free condition is shown in Figure 2. In this test, the system parameters were given by $\alpha = 0.8$ and $\Delta = 5.4$. After initialization, $A(z)$ was adjusted iteratively and it can be observed that $\hat{\alpha}(k) \to 0.81$ and $\hat{\Delta}(k) \to 5.41$ at approximately the 1000th iteration. The convergence rate of $\hat{\Delta}(k)$ was close to the predicted value whereas that of the gain estimate was slightly slower than the analytical calculation. It is because we have assumed that the multipath delay estimate has already approached its optimal value when deriving (10). Upon convergence, the measured variance of $\hat{\alpha}(k)$ and $\hat{\Delta}(k)$ were 0.0039 and 0.0042 respectively, which agreed with their theoretical values of 0.004 and 0.005.

Figure 3 and Figure 4 demonstrate the ability of the AMC to estimate nonstationary system parameters. The actual multipath gain and delay were given step offsets after every 2000 iterations. It can be seen that the AMC tracked all these step changes in less than a thousand iterations at noise-free condition and when SNR = 10dB. In Figure 4, we see that the trajectory of $\hat{\Delta}(k)$ was almost unaffected by the corrupting noise and the delay estimates were very accurate. However, the learning rate of $\hat{\Delta}(k)$ slowed down noticeably as the multipath gain decreased, and it can be explained easily using (9). While Figure 3 shows that the gain estimates in the noise-free condition were accurate but they were less satisfactory when under noisy environment.

The mean square delay errors for different α and Δ in a noise-free condition are shown in Figure 5. In this test, Δ was varied from 1.0 to 14.0 and two values of α, namely, 0.2 and 0.8, were tried. It is noted that the AMC provided a smaller mean square error for a larger gain and this is verified by (11). Due to improper inverse modeling, the mean square delay error had a relatively large value when the multipath delay was close to 1. Nevertheless, when Δ is larger than 5, the delay variances were comparable to the predicted values of 0.02 and 0.005, although there are some peaks with smaller amplitudes.

To conclude, a simple adaptive system (AMC) for multipath time delay estimation has been proposed. The AMC is configured as an adaptive IIR filter which aims to remove the multipath component in the received signal. Using an LMS-style algorithm, the estimate of the multipath parameters are adjusted explicitly and iteratively. Theoretical analysis of the parameter estimates is derived and it is proved that the variance of the estimator can achieve the CRLB for high SNR conditions. Computer simulations show that it can track time-varying delays accurately if the multipath is resolvable.

Appendix I

We shall show that $A(z)$ is a stable system by applying Rouché's theorem. Two functions, $u(z)$ and $v(z)$, which are analytic on the contour $|z| \leq 1$, are defined as follows,

$$u(z) = z^M$$

and

$$v(z) = \hat{\alpha} \sum_{i=1}^{M} sinc(i - \hat{\Delta}) z^{M-i}$$

such that $(u(z) + v(z))z^{-M}$ equals the demoninator of $A(z)$. We then evaluate the magnitudes of $u(z)$ and $v(z)$ at $|z| = 1$,

$$|u(z)| = |z|^M = 1$$

and

$$|v(z)| \leq \hat{\alpha} \cdot |\sum_{i=1}^{M} sinc(i - \hat{\Delta})|$$

From [2], the discrete Fourier series representation of $e^{j\omega\hat{\Delta}}$ is given by

$$e^{j\omega\hat{\Delta}} = \sum_{i=-\infty}^{\infty} sinc(\hat{\Delta} + i) e^{-j\omega i} \qquad \text{(A.1)}$$

Setting $\omega = 0$ in (A.1), we obtain $\sum_{i=-\infty}^{\infty} sinc(\hat{\Delta} + i) = 1$. Thus for sufficiently large M and for resolvable multipath, the magnitude of $v(z)$ on the contour $|z| = 1$ is bounded by $|v(z)| \leq \hat{\alpha}$. It can be seen that $|u(z)| > |v(z)|$ for $|z| = 1$. By Rouché's theorem, the functions $u(z)$ and $u(z) + v(z)$ have M zeros interior to the unit circle. Consequently, $A(z)$ has all poles lying inside the unit circle and hence its stability is proved.

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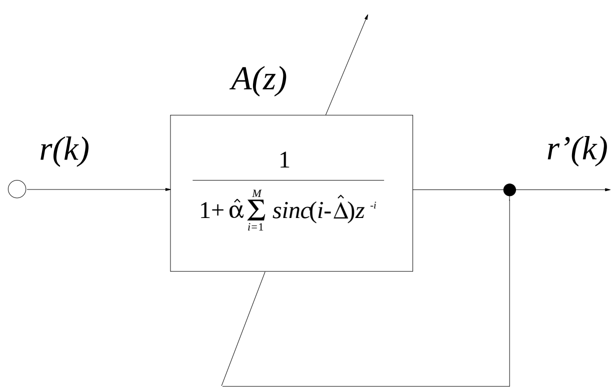

Figure 1: System block diagram of the AMC

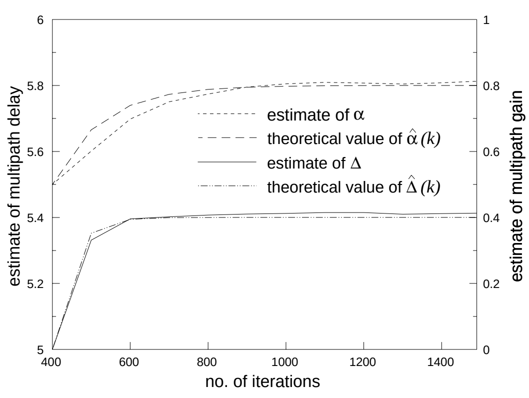

Figure 2: Theoretical and experimental values of $\hat{\alpha}$ and $\hat{\Delta}$ in a noise-free condition

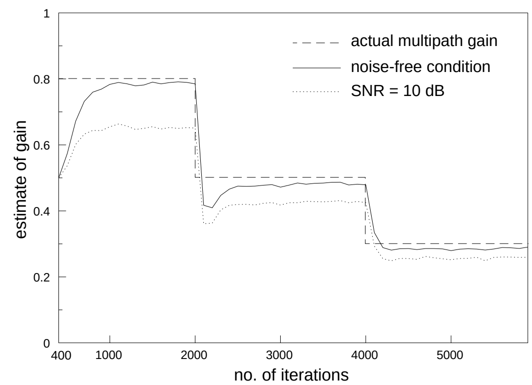

Figure 3: Tracking behaviour of $\hat{\alpha}(k)$

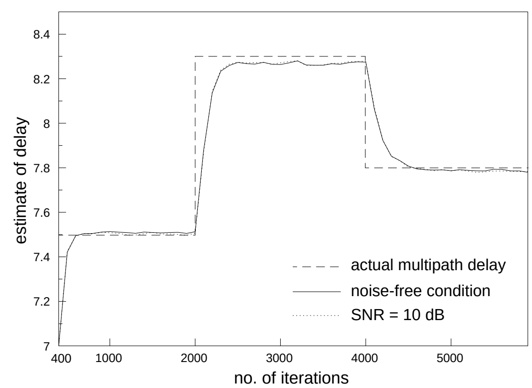

Figure 4: Tracking behaviour of $\hat{\Delta}(k)$

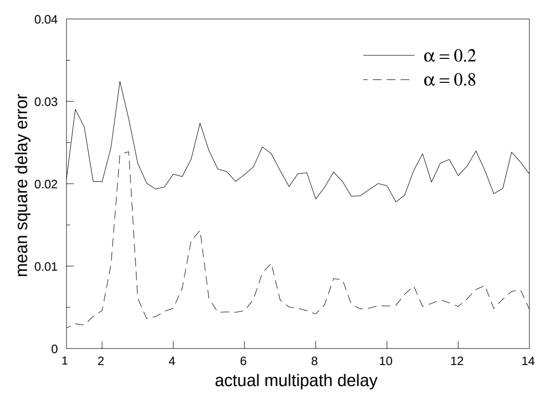

Figure 5: Mean square delay error versus α and Δ