

PASSIVE IDENTIFICATION OF MULTIPATH CHANNEL

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ABSTRACT

RF transmissions are often done along multipath channel, due to reflections. A physical model of propagating along such a channel is available, and takes into account few parameters as angles of incidence of waves on the array, group delay for each path, Doppler shift, polarization. In order to compensate Rayleigh fading, a spatio-temporal separation of multipaths is proposed. Usually, this is done by transmitting a training sequence (known), which reduces the data rate. We show in this paper that a passive identification can be performed, using only received signals. Proposed algorithm proceeds in two steps: the first step is a blind deconvolution, and then a parametric estimation of the channel is performed. Many simulations exhibit performances of proposed algorithms.

1. INTRODUCTION

Radiocommunications through a multipath channel are mainly limited by Rayleigh fading and intersymbol interference. This is a cause of strong failure in the transmissions (measured by error probability for instance). This can be overcome by identification of propagation channel. The classical way is the sending of a (known) training sequence, whose corresponding response through the channel leads, after a parametric identification by spatio-temporal high resolution methods [1], to the propagation conditions (this is an *active* identification). Unfortunately, propagation channels are almost always non-stationary. Channels parameters have to be updated by sending periodically the training sequence, reducing thus the transmission rate.

The purpose of this paper is to propose a *passive* identification algorithm, which needs no training sequence, allowing then a higher transmission rate and requiring no specific device to send and receive the training sequence. This algorithm performs in two steps:

- ① first, a blind deconvolution provides an estimation of the impulse response of the channel.
- ② secondly, a spatio-temporal high resolution method is applied on previous results, giving characteristic parameters

of the channel. This requires a parametric model of the impulse response.

The organization of this paper is as follows. In section 2, we recall a parametric model of a multipath channel. In section 3 are presented three recent blind deconvolution algorithms, which are furthermore relied to the maximum likelihood estimation. A parametric estimation of the channel is suggested in section 4. Some simulations illustrate properties of proposed algorithms in section 5.

2. A PARAMETRIC MODEL OF PROPAGATION

First let's introduce some notations and definitions:

- $s(\cdot)$: signal emitted by the source;
- M : number of paths;
- N : number of sensors on the receiving array;
- $\tau_m, \theta_m, \Delta_m$: group delay, azimuth and elevation

angles of the signal transmitted through the m^{th} path;

- α_m : attenuation on the m^{th} path. It is Rayleigh distributed if line of sight (LOS) path is absent, and follows a Rice distribution in the opposite case [2];
- ϕ_m : random phase uniformly distributed on $[0, 2\pi]$.

The impulse response between the emitter and the i^{th} sensor, noted $h_i(t)$, is given by [2]:

$$h_i(t) = \sum_{m=1}^M \alpha_m a_i(\theta_m, \Delta_m) \delta(t - \tau_m) e^{j\phi_m}, \text{ for } i = 1, \dots, N, (1)$$

where $a_i(\theta, \Delta)$ is the response of the i^{th} sensor in the bearing defined by θ and Δ , relatively to a sensor of reference. For simulation convenience, we shall consider, in the sequel, that only one angle, noted θ , is unknown.

The signal reaching each sensor is perturbed by an additive noise, assumed to be gaussian, zeromean and of variance σ^2 , white both spatially and temporally:

$$x_i(t) = (h_i * s)(t) + n_i(t), \text{ for } i = 1, \dots, N. (2)$$

The numerical impulse response involves not only the one of the channel, previously described, but takes into account the additional devices used to send and receive the signal, as shown on figure 1.

Figure 1

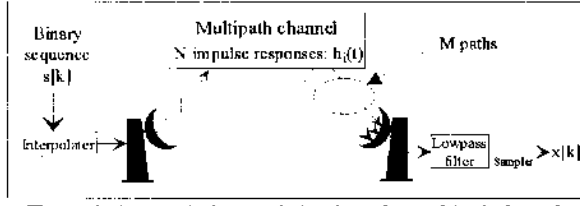


Figure 1: A numerical transmission through a multipath channel.

In this paper, we assume that the lowpass filter is ideal, that is its transfer function is equal to 1 in the bandpass and 0 elsewhere. Its impulse response is the *sinc* function. Unfortunately, this is a non causal, IIR filter. In order to work with a causal, FIR filter, we must make the former causal, and take only a finite number ($L+1$) of coefficients. The numerical impulse response is then:

$$\begin{cases} h_i[k] = I_s \sum_{m=1}^M \alpha_m a_i(\theta_m) e^{j\theta_m k} \text{sinc}(k - I_s \tau_m) & k \in \{0, 1, \dots, L\} \\ h_i[k] = 0 & \text{for } k < 0 \text{ or } k > L \end{cases} \quad (3)$$

A more general model, including Doppler shift and polarization for example, can be used. Estimation of all these parameters may complicate explanations but poses no theoretical problem.

3. BLIND DETERMINATION OF THE IMPULSE RESPONSE

3.1 Feasibility study

The blind deconvolution aims at estimating the N impulse responses $h_i[k]$, without knowing the emitted sequence. The question is whether it is possible to find N filters $g_i[k]$ allowing to invert the input/output relationship, as explained on figure 2, in the noiseless case:

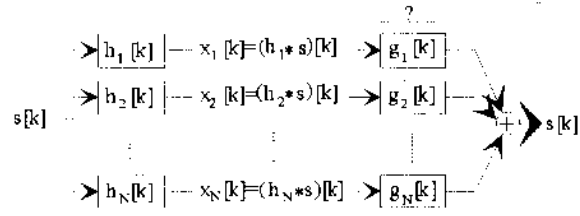


Figure 2: Framework of the multichannel blind deconvolution.

By taking the z transform, it comes:

$$X_i(z) = H_i(z)S(z), \quad \text{for } i = 1, \dots, N. \quad (4)$$

If $S(z)$ is a polynomial, and if $H_1(z), \dots, H_N(z)$ (which are polynomials of degree L since they correspond to FIR filters) are coprime, $S(z)$ is the highest common factor of $X_1(z), \dots, X_N(z)$. According to the generalized Bezout theorem, there are N polynomials $G_1(z), \dots, G_N(z)$ such that:

$$\sum_{i=1}^N G_i(z)H_i(z) = 1. \quad (5)$$

The blind deconvolution of FIR multichannel is realized by:

$$S(z) = \sum_{i=1}^N G_i(z)X_i(z). \quad (6)$$

3.2 Representation of spatio-temporal data

The output of the i^{th} sensor at the time k is given by the convolution product:

$$x_i[k] = \sum_{l=0}^{L_s} h_i[l]s[k-l] + n_i[k]. \quad (7)$$

There are two ways of writing the data under vectorial shape:

• as the set of the outputs of the whole array at time k :

$$\mathbf{x}[k] = [x_1[k] \quad \dots \quad x_N[k]]^T. \quad (8)$$

This becomes, according to the emitted signal:

$$\mathbf{x}[k] = \mathbf{H}\mathbf{s}[k, L+1] + \mathbf{n}[k], \quad (9)$$

with

$$\mathbf{H} = \begin{bmatrix} h_1[0] & \dots & h_1[L] \\ \vdots & & \vdots \\ h_N[0] & \dots & h_N[L] \end{bmatrix} \quad (10)$$

and

$$\mathbf{s}[k, L+1] = [s[k] \quad \dots \quad s[k-L]]^T. \quad (11)$$

The noise vector $\mathbf{n}[k]$ has the same shape as $\mathbf{x}[k]$.

Finally, the outputs are gathered in a spatio-temporal vector according to the following framework:

$$\mathbf{x} = [\mathbf{x}[1]^T \quad \dots \quad \mathbf{x}[K]^T]^T. \quad (12)$$

• as the set of the outputs of the i^{th} sensor for K successive snapshots:

$$\mathbf{x}_i = [x_i[K] \quad \dots \quad x_i[1]]^T. \quad (13)$$

It can be written, according to the emitted signal values:

$$\mathbf{x}_i = \mathfrak{M}_i \mathbf{s}[K, K+L] + \mathbf{n}_i, \quad (14)$$

with \mathfrak{M}_i being a $K \times (K+L)$ Sylvester matrix:

$$\mathfrak{M}_i = \begin{bmatrix} h_i[0] & \dots & h_i[L] & 0 & \dots & 0 \\ 0 & h_i[0] & \dots & h_i[L] & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \dots & 0 & h_i[0] & \dots & h_i[L] \end{bmatrix}, \quad (15)$$

and

$$\mathbf{s}[K, K+L] = [s[K] \quad \dots \quad s[1-L]]^T. \quad (16)$$

Finally, the whole set of data can be gathered in a spatio-temporal vector:

$$\mathbf{x} = [\mathbf{x}_1^T \quad \dots \quad \mathbf{x}_N^T]^T, \quad (17)$$

which may be written this way:

$$\mathbf{x} = \mathfrak{M} \mathbf{s}[K, K+L] + \mathbf{n}, \quad (18)$$

with

$$\mathfrak{M} = [\mathfrak{M}_1^T \quad \dots \quad \mathfrak{M}_N^T]^T. \quad (19)$$

The aim of the section 3.3 is to estimate the coefficients of the impulse responses, ordered in a single vector:

$$\mathbf{h} = [h_1[0] \ \dots \ h_1[L] \ \dots \ h_N[0] \ \dots \ h_N[L]]^T. \quad (20)$$

3.3 Presentation of the three studied methods

We shall compare in the sequel three algorithms for the blind deconvolution of the multichannel. They all need the assumption that the N transfer functions $H_i(z)$ are coprime (see § 3.1).

3.3.1 Subspace method [3]

The input/output relationship, in the noiseless case, can be written as (18):

$$\mathbf{y} = \mathfrak{H} \mathbf{s}. \quad (21)$$

Subject to $K+L < KN$, the observation \mathbf{y} belongs to the subspace spanned by the columns of \mathfrak{H} . The determination of a base of the orthogonal subspace gives, for each vector \mathbf{u}_i of this base an equation $\mathfrak{H}\mathbf{u}_i = 0$, which is linear relatively to the coefficients of the impulse response.

The excessive number of equations respect to number of unknowns coefficients leads to a least squares solution.

3.3.2 Least squares approach [4]

The N equations (4), which can also be written:

$$S(z) = \frac{X_1(z)}{H_1(z)} = \dots = \frac{X_N(z)}{H_N(z)}, \quad (22)$$

give $N(N-1)/2$ polynomial equalities:

$$H_j(z)X_i(z) = H_i(z)X_j(z). \quad (23)$$

This system being overdetermined, admits a least squares solution.

3.3.3 Prediction error method [5]

Provided that the symbols $s[k]$ stem from a zero-mean white process, the deconvolution amounts to find, from the writing (12), the innovation of an autoregressive process by a linear prediction method.

3.4 Connexion with conditionnal maximum likelihood method (CML)

The input $s[k]$ is considered as an unknown data, the parameters to be estimated being the $K(L+1)$ coefficients of the impulse responses. The only random data is the additive noise.

Under the assumption of white additive noise, the likelihood of observations on sensors is:

$$p(\mathbf{x}|\mathbf{h}, \mathbf{s}) = \frac{1}{\left(\sqrt{2\pi\sigma^2}\right)^{NK}} \exp\left\{-\frac{1}{2\sigma^2}\|\mathbf{x} - \mathfrak{H}\mathbf{s}\|^2\right\}. \quad (24)$$

The CML criterion leads to the minimization of:

$$Q(\mathbf{s}, \mathfrak{H}) = \|\mathbf{x} - \mathfrak{H}\mathbf{s}\|^2 = \|\mathbf{x} - \hat{\mathbf{x}}\|^2. \quad (25)$$

In order to eliminate \mathbf{s} , we must search for the minimum of Q relatively to \mathbf{s} , which will only depend on \mathbf{h} .

Let's notice that (25) can be rewritten:

$$Q(\mathbf{s}, \mathbf{h}) = \text{tr}\left((\mathbf{x} - \hat{\mathbf{x}})(\mathbf{x} - \hat{\mathbf{x}})^H\right), \quad (26)$$

which leads to:

$$dQ = \text{tr}\left(\mathbf{G} \cdot d\mathbf{s}^H + \mathbf{G}^H \cdot d\mathbf{s}\right), \quad (27)$$

with

$$\mathbf{G} = \mathfrak{H}^H \mathfrak{H} \mathbf{s} - \mathfrak{H}^H \mathbf{x}. \quad (28)$$

Subject to \mathfrak{H} being full column rank, we can easily deduce, from (27) and (28) the CML estimation of \mathbf{s} :

$$\hat{\mathbf{s}} = (\mathfrak{H}^H \mathfrak{H})^{-1} \mathfrak{H}^H \mathbf{x}, \quad (29)$$

and of \mathbf{x} :

$$\hat{\mathbf{x}} = \mathfrak{H}(\mathfrak{H}^H \mathfrak{H})^{-1} \mathfrak{H}^H \mathbf{x}. \quad (30)$$

The criterion to be minimized becomes:

$$Q_b(\mathfrak{H}) = \|\mathbf{x} - \Pi_{\mathbf{h}} \mathbf{x}\|^2 = \|\Pi_{\mathbf{h}} \mathbf{x}\|^2, \quad (31)$$

where $\Pi_{\mathbf{h}} = \mathfrak{H}(\mathfrak{H}^H \mathfrak{H})^{-1} \mathfrak{H}^H$ is the projector on the subspace spanned by the columns of \mathfrak{H} (signal subspace), and $\Pi_{\mathbf{h}}^\perp$ is the projector on the orthogonal subspace (noise subspace). \mathfrak{H} is linear in \mathbf{h} while $\Pi_{\mathbf{h}}$ is strongly non-linear.

Let's notice that these results can be naturally connected to the methods proposed in § 3.3.

• The first is based on a pre-estimation of signal subspace. It requires several spatio-temporal observations as defined by (17), for signal vectors \mathbf{s} assumed to be independent. This step of signal subspace pre-estimation may be inaccurate if only few measures are available.

• Concerning the second method, let's simply notice that CML method (see (31)) amounts to maximize the following quantity:

$$Q'_b = \|\Pi_{\mathbf{h}} \mathbf{x}\|^2 = \text{tr}\left((\mathfrak{H}^H \mathfrak{H})^{-1} \mathfrak{H}^H \mathbf{x} \mathbf{x}^H \mathfrak{H}\right), \quad (32)$$

where the factor $\mathfrak{H}^H \mathbf{x}$ seems to be the convolution of the output by the candidate filter. The influence of $(\mathfrak{H}^H \mathfrak{H})^{-1}$ can be minimized by a constraint on \mathbf{h} .

• The third method (error prediction), based on strong hypothesis on s , could be connected to stochastic maximum likelihood (SML), under the hypothesis $E[\mathbf{s}\mathbf{s}^H] = \mathbf{I}_{K+L}$.

Let's finally note that the three algorithms presented are only approximations of maximum likelihood method. Actually, this one is too complex to be applied and one prefers to use suboptimal but more simple algorithms.

4. PARAMETRIC ESTIMATION OF THE CHANNEL

Once impulse response has been estimated in the shape defined by (20), we wish to extract the values of the spatio-temporal parameters (τ, θ) .

From (3), we can write:

$$\mathbf{h} = \sum_{m=1}^M \alpha_m e^{j\theta_m} \mathbf{c}(\theta_m, \tau_m), \quad (33)$$

$\mathbf{c}(\theta_m, \tau_m)$ being a vector of length $N(L+1)$ which is characteristic of the propagation on the m^{th} path, and defined by:

$$\mathbf{c}(\theta, \tau) = \begin{bmatrix} a_1(\theta) \text{sinc}(-\tau F_s) & \dots & a_1(\theta) \text{sinc}(L - \tau F_s) & \dots \\ \dots & a_N(\theta) \text{sinc}(-\tau F_s) & \dots & a_N(\theta) \text{sinc}(L - \tau F_s) \end{bmatrix}^T \quad (34)$$

Theoretical impulse response is then a linear combination of M vectors $\mathbf{c}(\theta, \tau)$, named spatio-temporal steering-vectors. It is no longer the case for its estimate $\hat{\mathbf{h}}$, because of estimation and modelization noises. We shall modify equation (33) by adding a noise \mathbf{w} which takes into account these uncertainties:

$$\hat{\mathbf{h}} = \sum_{m=1}^M \alpha_m e^{i\phi_m} \mathbf{c}(\theta_m, \tau_m) + \mathbf{w}. \quad (35)$$

The set of parameters $\{(\theta_m, \tau_m); m=1, \dots, M\}$ can be estimated by using the spatio-temporal high resolution method MUSIC [1]. It requires a number P of estimations and the computation of their covariance matrix:

$$\mathbf{R}_h = \frac{1}{P} \sum_{p=1}^P \hat{\mathbf{h}}_p \hat{\mathbf{h}}_p^H. \quad (36)$$

Finally, one have to compute its eigen-decomposition and then to maximize the MUSIC pseudo-spectrum.

5. SIMULATIONS

Computer simulations have been conducted to evaluate performances of the proposed algorithms. The receiver est a uniform linear array with $N=5$ sensors. Data are sampled the baud rate. The emitted signal is a FSK modulation. Propagation splits the channel into $M=3$ paths, characterized respectively by angle of incidence and group delay, expressed in symbol rate, $(30^\circ; 0)$, $(55^\circ; 3.4)$ and $(69^\circ; 4.1)$. Impulse response, for the first sensor, has been drawn on figure 3.

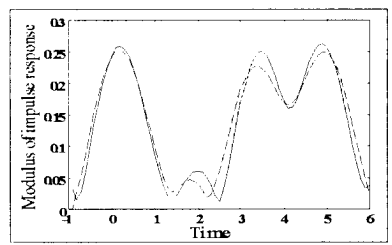


Figure 3: Real impulse response (—) and estimated one (---).

We have used the three algorithms presented in section 3 to estimate the impulse responses. An example of result is drawn on figure 3. In order to measure their performances, we have computed (figure 4) the root mean square error of each estimation, defined by:

$$RMSE(\mathbf{h}) = \sqrt{\frac{1}{N_h} \frac{1}{N(L+1)} \sum_{i=1}^{N_h} \|\mathbf{h}_i - \hat{\mathbf{h}}\|^2}, \quad (37)$$

where $N_h = 100$ is the number of trials. These algorithms estimate impulse responses of length 7 ($L=6$). $K=12$ are used to build the spatio-temporal observation (for subspace method). The three algorithms perform with the same number of snapshots (420).

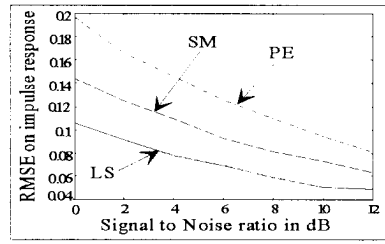


Figure 4: Root mean square error on estimation of impulse responses. (LS: least squares - SM: subspace method - PE: prediction error)

The spatio-temporal method MUSIC is then applied to previous results in order to estimate angles of incidence and group delay. We have drawn on figure 5 RMS errors on estimations of parameters, defined as in (37).

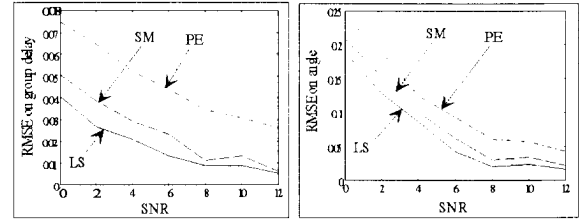


Figure 5: RMS error on parametric estimation.

LS approach presents best performances. This is probably due to the fact that SM works, in these simulations, with short data. PE method is known to be robust to an overdetermination of the order of impulse response, but it is not the case here since the real IR is infinite.

6. CONCLUSION

We have presented an algorithm for the passive identification of a multipath channel. This one uses recent studies on blind deconvolution and we have compared three algorithms. Best results are provided by the least squares approach, which is furthermore the simplest one. We think that making this method adaptive is promising in non stationary context.

REFERENCES

[1] P. Gounon and S. Bozinoski, "High Resolution Spatio-temporal Analysis by an Active Array", Proceedings of ICASSP, Vol. 5, 1995.
 [2] H. Hashemi, "The Indoor Radio Propagation Channel", Proceedings of the IEEE, Vol. 81, No. 7, July 1993.
 [3] E. Moulines, P. Duhamel, J.F. Cardoso and S. Mayrargue, "Subspace Methods for the Blind Identification of Multichannel FIR Filters", IEEE Transactions on Signal Processing, Vol. 43, No. 2, February 1995.
 [4] G. Xu, H. Liu, L. Tong and T. Kailath, "A Least Squares Approach to Blind Channel Identification", IEEE Transactions on Signal Processing, Vol. 43, No. 12, pp. 2982-2993, December 1995.
 [5] K. Abed-Meraim, P. Duhamel, D. Gesber, P. Loubaton, S. Mayrargue, E. Moulines, D. Slock, "Prediction error Methods for Time-Domain Blind Identification of Multichannel FIR Filters", Proceedings of ICASSP, 1995.