PARTIALLY ADAPTIVE GENERALIZED SIDELOBE CANCELLER WITH PRESCRIBED ZEROS

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ABSTRACT

Linear constraints in adaptive beamformer are often used to control its transfer function. In this paper we utilized these constraints to reduce computational cost of the adaptive algorithm. For this aim, two types of constraints were proposed. The first one is that all zeros of the transfer function appear as conjugate-complex pairs lying on the unit circle. The second one is that some zeros have prescribed positions and the adaptation is realized by the rest of zeros. Developed constraints are applied to the generalized sidelobe canceller and used to blocking matrix design. Experiments proved that degradation in performance of the partially adaptive algorithm is a little compared to the full adaptive algorithm.

1. INTRODUCTION

To obtain a good interference cancellation, array must be composed of large number of sensors. In full adaptive algorithm, without additional transversal filters in sensors' lines, degree of the freedom is N-1, where N is number of sensors. The great number of the sensors rises up the computational complexity of the algorithm. It can be problem in real-time implementation of the algorithm. To overcome this problem, some authors propose partially adaptive algorithm [1], [2], [3] in which especially constraints are used to reduce degree of the freedom and hence computational complexity with minimum degradation of the performance.

In this paper we propose two types of constraints. The first set of constraints is that all spatial zeros must lie on the unit circle in the complex plane. This condition is motivated by the fact that the zeros on the unit circle produce maximal sidelobes attenuation. This set of constraints reduces the initial degree of freedom two times without significant degradation of the performance.

The second set of constraints is based on a priory knowledge of some directions of arrival and fixing some zeros of the transfer function. If all directions have the same probability, or if there is no a priory knowledge about them, a number of zeros may be equidistantly placed on the unit circle. The error between true directions and our guess can be corrected by the adaptive positioning of remaining zeros. This can be done by factorizing characteristic polynomial on two polynomials. The first one, with prescribed zeros is nonadaptive, and the second one is adaptive to the present interferences.

Applying generalized sidelobe canceller, the designed constraints take place in the blocking matrix and the optimal weightings can be estimated by the ordinary unconstrained LS or LMS algorithms. The blocking matrix design is also presented in this paper.

2. PROBLEM FORMULATION

Frost's adaptive beamformer [4] is displayed on fig.1. Desired signal arrives from the direction orthogonal to sensors' lines. Sensors' data vector $\mathbf{x}_t=[x_{0,t},\ldots, x_{N-1,t}]^T$ is weighted by the vector $\mathbf{w}=[w_0,\ldots,w_{N-1}]^T$ and form output signal $\mathbf{y}$ vector

$$\mathbf{y}^T=\mathbf{w}^H\mathbf{X} \quad , \qquad \mathbf{w}^H = [w_0,\ldots,w_{N-1}] \quad , \qquad (1)$$

where columns of N by n+1 data matrix $\mathbf{X}$ represent array data at times 0 to n. $(.)^H$ denotes complex conjugate transpose and $(.)^T$ real transpose. Upper case and lower case boldface symbols represent matrices and vectors, respectively.

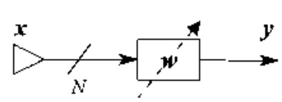

Fig.1 Frost's adaptive beamformer.

For the plane wave with wavelength λ, arriving from angle θ, spatial filtering characteristics of the array can be expressed as [5]

$$W(\theta,\lambda)=\sum_{n=0}^{N-1} w_n z^{-n},\ z=e^{jnkd\sin(\theta)},\ k=\frac{2\pi}{\lambda}, \quad (2)$$

where d is distance of the sensors. From the (2), $W(\theta,\lambda)$ can be viewed as z-transform of array weights on the unit circle.

The linearly constrained least squares beamforming problem is expressed as

$$\min_{W} y^{\mathrm{H}} y \quad \text{subject to} \quad \mathbf{C}^{\mathrm{H}} w = f \quad (3)$$

where $\mathbf{C}$ is N by L dimensional constraint matrix, and f is the L-dimensional response vector.

It is desirable to transform the constrained least squares problem (3) to unconstrained form using the method in [6] known as Generalized Sidelobe Canceller (GSC). The GSC represents a decomposition of w into two components: a nonadaptive beamformer w_0 satisfying $\mathbf{C}^{\mathrm{H}} w_0 = f$ and a product of a signal blocking matrix $\overline{\mathbf{C}}$ and an unconstrained adaptive weight vector $\overline{w}$. $\overline{\mathbf{C}}$ is N by N-L . rank N-L matrix satisfying following equations

$$\mathbf{C}^{\mathrm{H}}\overline{\mathbf{C}}=0 \quad (4)$$

$$w = w_0 - \overline{\mathbf{C}}\,\overline{w} \quad (5)$$

From (5), it is evident that $\mathbf{C}^{\mathrm{H}} w = f$ is satisfied for all $\overline{w}$.

3. NEW SETS OF CONSTRAINTS

The new set of constraints is based on the following assumptions:

A1. Zeros of the spatial characteristics had to be as deep as possible, i.g. to lie on the unit circle.

A2. Spatial filtering characteristics of the array $W(\theta,\lambda)$ had to be symmetric.

A3. As the number of sensors is $N=2n+2m+1$, $2n$ zeros have prescribed positions.

Corollary:

From the A1 and A2 it follows that weighting coefficients should be real and symmetric. This produce $(n+m)$ linear constraints. Finally, from the assumption A3 it follows that polynome $W(\theta,\lambda)$ can be factorized on two parts. The first one have prescribed m conjugate-complex zero pairs and the second one that depend upon the adaptation of the algorithm.

The power of the polynome $W(\theta,\lambda)$ is $2(n+m)$. Let break $W(\theta,\lambda)$ on two factors

$$\begin{aligned} W(\theta,\lambda) &= w_0+w_1 z^{-1}+\ldots+w_{2(n+m)} z^{-2(n+m)}= \\ &=(1+\alpha_1 z^{-1}+\ldots+\alpha_n z^{-n}+\ldots+\alpha_1 z^{-2n+1}+z^{-2k}) \\ &\quad(\beta_0+\beta_1 z^{-1}+\ldots+\beta_m z^{-m}+\ldots+\beta_1 z^{-2m+1}+\beta_0 z^{-2m}) \quad (6) \end{aligned}$$

The first one with fixed coefficients α_i is determined by the prescribed positions of zeros. The second one with coefficients β_j is adjustable by the adaptive algorithm. Because of the assumptions (A1) and (A2) both of the polynomes have symmetric coefficients. From the (6) we can write system of linear equations

$$\begin{aligned} w_0 &= \beta_0 \\ w_1 &= \alpha_1\beta_0+\beta_1 \\ &\ \vdots \\ w_{n+m} &= \ldots+\alpha_{n-1}\beta_{m-1}+\alpha_n\beta_m+\alpha_{n-1}\beta_{m-1}+\ \ldots \quad (7) \\ &\ \vdots \\ w_{2(n+m)-1} &= \alpha_1\beta_0+\beta_1 \\ w_{2(n+m)} &= \beta_0 \end{aligned}$$

From the system (7) it can be created $(2n+2m+1)$ by $(2n+m+1)$ constraint matrix $\mathbf{C}^{\mathrm{H}}$ with $(2n+m+1)$ constraints (Appendix 1, relation (A8))

In blocking matrix design we have freedom to choose last m rows, to be independent, and the rest ones to evaluate by the constraint (4) as it is explained in appendix A2. Weightings w_0 for the conventional beamformer part, can be chosen with m degree of freedom. This degree of freedom can be spent for minimum least squares solution of the weightings by the relation

$$w_0 = \mathbf{C}\,(\mathbf{C}^{\mathrm{H}}\mathbf{C})^{-1} f \quad (8)$$

4. BLOCKING MATRIX DECOMPOSITION

As the columns of the blocking matrix $\overline{\mathbf{C}}$ are symmetric, it can be displayed in block matrix form

$$\overline{\mathbf{C}}=\begin{bmatrix}\mathbf{B}_1\\ \mathbf{b}_S\\ \overline{\mathbf{I}}\mathbf{B}_1\end{bmatrix},\qquad \overline{\mathbf{I}}=\begin{bmatrix}0 & \cdots & 1\\ \vdots & 1 & \vdots\\ 1 & \cdots & 0\end{bmatrix} \quad (9)$$

From the (9), $\overline{\mathbf{C}}$ can be factorized on two matrix

$$\overline{\mathbf{C}}=\overline{\mathbf{Q}}\,\overline{\mathbf{C}}_\lambda\ ,\quad \overline{\mathbf{Q}}=\begin{bmatrix}\mathbf{I} & 0\\ 0\cdots0 & 1\\ \overline{\mathbf{I}} & 0\end{bmatrix},\quad \overline{\mathbf{C}}_\lambda=\begin{bmatrix}\mathbf{B}_1\\ \mathbf{b}_S\end{bmatrix} \quad (10)$$

The coeficients of the matrix $\overline{\mathbf{Q}}$ are independent on interference frequency, while the matrix $\overline{\mathbf{C}}_\lambda$ is frequency dependent. On the same manner we can factorize weighting vector w_0. It can be represented by three elements $w_0=[w_1^{\mathrm{T}}\quad w_S\quad \overline{\mathbf{I}}w_1^{\mathrm{T}}]^{\mathrm{T}}$ and then expressed in the form

$$w_0=\overline{\mathbf{Q}}\,w_\lambda\ ,\quad w_\lambda=[w_1^{\mathrm{T}}\ w_S]^{\mathrm{T}} \quad (11)$$

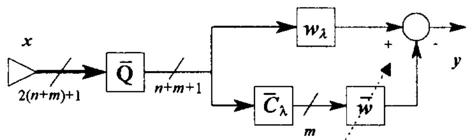

Fig.2 The partially adaptive beamformer.

5. EXPERIMENTAL RESULTS

Computer simulations were done to prove validity of proposed sets of constraints respectively to the degradation of the performance. In all experiments the signals were sampled with 10000Hz. The broadband desired signal arrives from direction orthogonal to sensors' line. All algorithms worked in time domain and the block processing mode. Experiments were done with following algorithms:

(*i*) Full adaptive algorithm with 15 sensors, (14 degree of freedom).

(*ii*) Partially adaptive algorithm with 15 sensors, 11 constraints and 4 degree of freedom. Polynome with fixed zeros was.

$$P_1(z) = 1 + 2.400943z^{-1} + 3.391285z^{-2} + 3.801121z^{-3} + 3.391285z^{-4} + 2.400943z^{-5} + z^{-6}$$

Blocking matrix was

$$\overline{C} = \begin{bmatrix} 1 & 0 & 0 & 0 & -.175 & -.832 & -.150 & .315 & -.150 & -.832 & -.175 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -.588 & .018 & -.018 & -.824 & -.018 & .018 & -.588 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & .429 & -.015 & -998 & -.832 & -.998 & -.015 & .429 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & .401 & -.411 & -.581 & -.820 & -.581 & -.411 & .401 & 1 & 0 & 0 & 0 \end{bmatrix}$$

Conventional beamformer weighting vector was

$$w_0 = .1\text{x}[.566\ .773\ .770\ .750\ .652\ .604\ .649\ .472\ .649\ .604\ .652\ .750\ .770\ .773\ .566]'$$

(*iii*) Full adaptive algorithm with 5 sensors, (4 degree of freedom) which has the same computational complexity as partially adaptive algorithm *ii*).

In the first experiment algorithm (*ii*) was tested. There were 12 narrowband interferences arriving from directions of 23.0^0, -23.0^0, 42.3^0, -42.3^0, 67.1^0, -67.1^0, 29.8^0, -29.8^0, 51.2^0, -51.2^0, 73.0^0, -73.0^0, with central frequencies respectively 3025Hz, 3025Hz, 2802Hz, 2802Hz, 2816Hz, 2816Hz, 2800Hz, 2800Hz, 2800Hz, 2800Hz, 2800Hz, 2800Hz. The result of the interference cancellation is displayed on the fig.3. As it can be seen from the fig.3., the interferences were strongly canceled.

In the next experiment interference was white noise signal band limited from 800Hz to 2400Hz, arriving from the angle of 45^0. As the beamformer works with fixed blocking matrix, and uses weightings without frequency correction, its performance is equivalent as there are lot of narrow band interferences arriving from different angles. Algorithms (*i*), (*ii*) and (*iii*) were tested and their performances were compared. The input and output Signal Interference Ration (SIR) and Gain of the tested algorithms are presented in the table 1. Time diagrams are shown on fig.4. Algorithms (*i*) and (*ii*) gives approximately same results although algorithm (*ii*) has lower computational cost. Algorithms (*ii*) and (*iii*) have the same computational coast but the partially adaptive algorithm (algorithm (*ii*)) has much better performances.

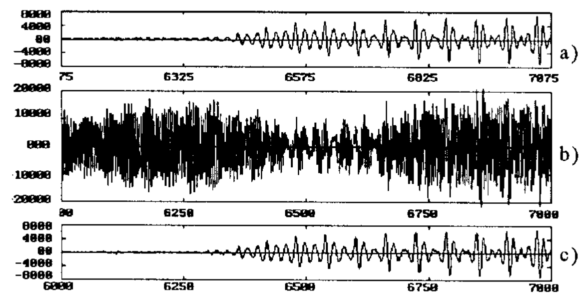

Fig.3. a) Original desired signal, b) first sensor's signal, c) signal on the output.

Table 1

Algorithm	Input SIR	Output SIR	Gain
Full adapt. 15/14 (*i*)	-13.2 dB	16.78 dB	29.98 dB
Part. adapt. 15/4 (*ii*)	-13.2 dB	16.57 db	29.77 dB
Full adapt. 5/4 (*iii*)	-13.2 dB	-5.95 dB	7.25 dB

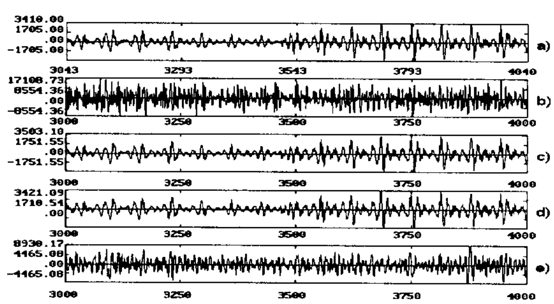

Fig. 2. a) original signal, b) signal+interferences on the sensor 1, c) Output of algorithm (*i*), d) Output of algorithm (*ii*), e) Output of algorithm (*iii*).

6. CONCLUSION

In order to reduce computational cost of the adaptive algorithm, we propose two new sets of constraints. First one is that all zeros of the transfer function appear as conjugate-complex pairs lying on the unit circle. The second one is that some of the zeros have fixed positions based of the a priory knowledge of the interferences' arriving angles. The first set of constraints reduces computationally cost approximately two times, while the second one additionally reduces it depending on the number of the fixed zeros. Designed blocking matrix can be factorized on two matrixes from which first one is frequency independent. This matrix operation can be realized by analog electronics components (fig.2). Applying A/D conversion after matrix $\bar{\mathbf{Q}}$, the number of analog signals that have to be digitized, can be reduced two times. Experimental results show that the performance of the proposed partially adaptive algorithm is approximately same as the full adaptation algorithm.

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Appendix A1. Constraint matrix calculation

Let represent first $n+m+1$ equations of the linear equation system (7) in matrix form

$$\begin{bmatrix} \mathbf{w}_1 \\ --- \\ \mathbf{w}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{0}_1 \\ --- & \\ \mathbf{A}_2 & \end{bmatrix} \begin{bmatrix} \mathbf{b}_1 \\ -- \\ \mathbf{b}_2 \end{bmatrix} \qquad \text{(A1)}$$

where $\mathbf{w}_1$, $\mathbf{w}_2$, $\mathbf{b}_1$ and $\mathbf{b}_2$ are column vectors

$\mathbf{w}_1=[w_0 \ldots w_m]^T$ $\quad$ $\mathbf{w}_2=[w_{m+1} \ldots w_{m+n}]^T$

$\mathbf{b}_1=[\beta_0 \ldots \beta_m]^T$ $\quad$ $\mathbf{b}_2=[\beta_{m-1} \ldots \beta_0]^T$

$\mathbf{A}_1$ is $m+1$ by $m+1$ matrix, and $\mathbf{A}_2$ n by $(n+m+1)$ matrix of form

$$\mathbf{A}_1 = \begin{bmatrix} 1 & 0 & \ldots & 0 \\ \alpha_1 & 1 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots \\ \ldots 0 \;\; 1 \;\; \alpha_1 & \ldots & \alpha_1 & 1 \end{bmatrix} \quad \mathbf{A}_2 = \begin{bmatrix} \ldots & \ldots & \ldots \\ \ldots \;\; \alpha_1 \ldots \alpha_2 \; \alpha_1 \; 1 \;\; 0 \\ \ldots 0 \; 1 \; \alpha_1 \ldots \; \alpha_2 \; \alpha_1 \; 1 \end{bmatrix}$$

and $\mathbf{0}_1$ is $m-1$ by n zero matrix. There is relation between $\mathbf{b}_1$ and $\mathbf{b}_2$

$$\mathbf{b}_2 = \bar{\mathbf{I}}\,\mathbf{b}_1 \;, \quad \bar{\mathbf{I}} = \begin{bmatrix} \ldots & 0 & 1 & 0 \\ \ldots & 1 & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots \end{bmatrix} \qquad \text{(A2)}$$

First $m-1$ equations from the system (A1) are linearly independent and we can solve it by the vector $\mathbf{b}_1$

$$\mathbf{b}_1 = \mathbf{A}_1^{-1}\mathbf{w}_1 \qquad \text{(A3)}$$

By the relations (A1), A2) and (A3) we can find relationship between $\mathbf{w}_1$ and $\mathbf{w}_2$

$$\mathbf{w}_2 = \mathbf{A}_2 \begin{bmatrix} \mathbf{b}_1 \\ \mathbf{b}_2 \end{bmatrix} = \mathbf{A}_2 \begin{bmatrix} \mathbf{I} \\ \bar{\mathbf{I}} \end{bmatrix} \mathbf{A}_1^{-1}\mathbf{w}_1 \qquad \text{(A4)}$$

From the (A4) we can find first set of constraints placed in the matrix $\mathbf{C}_1$ dimension n by $(2n+2m+1)$

$$\mathbf{C}_1 = \left[\mathbf{A}_2 \begin{bmatrix} \mathbf{I} \\ \bar{\mathbf{I}} \end{bmatrix} \mathbf{A}_1^{-1}, \; -\mathbf{I}_1, \; \mathbf{0}_1 \right] \qquad \text{(A5)}$$

where $\mathbf{I}_1$ is unit matrix dimension n by n and $\mathbf{0}_1$ is zero matrix dimension n by $(n+m)$.

From the unit gain constraint for the desired signal

$$\sum_{i=0}^{N-1} w_i = 1 \qquad \text{(A6)}$$

and symmetry of the weighting coefficients, the constraint matrix $\mathbf{C}_2$ takes a form

$$\mathbf{C}_2 = \begin{bmatrix} \frac{1}{N} & \frac{1}{N} & \cdots & \frac{1}{N} & \frac{1}{N} \\ 1 & 0 & \ldots & 0 & -1 \\ 0 & 1 & \ldots & -1 & 0 \\ \ldots & \ldots & & \ldots & \ldots \\ \ldots & 1 & 0 \; -1 & & \ldots \end{bmatrix} \qquad \text{(A7)}$$

Complete constraint matrix $\mathbf{C}$ is union of the matrixes $\mathbf{C}_2$ and $\mathbf{C}_1$

$$\mathbf{C}^H = \begin{bmatrix} \mathbf{C}_2 \\ \mathbf{C}_1 \end{bmatrix} \qquad \text{(A8)}$$

Appendix A2. Blocking matrix determination

Constraint matrix $\mathbf{C}^H$ can be partitioned on full rank matrix $\mathbf{C}_F$ dimension $2n+m+1$ by $2n+m+1$ and matrix $\mathbf{C}_L$ dimension $2n-m+1$ by m. Blocking matrix $\bar{\mathbf{C}}$ can be also partitioned on two matrixes $\bar{\mathbf{C}}_1$ and $\bar{\mathbf{C}}_2$ so that constraint equation (4) can be expressed in form

$$\mathbf{C}^H\bar{\mathbf{C}} = [\mathbf{C}_F \quad \mathbf{C}_L] \begin{bmatrix} \bar{\mathbf{C}}_1 \\ \bar{\mathbf{C}}_2 \end{bmatrix} = \mathbf{0} \qquad \text{(A9)}$$

Submatrix $\bar{\mathbf{C}}_2$ can be any matrix range of the m. Submatrix $\bar{\mathbf{C}}_1$ can be expressed by the relation

$$\bar{\mathbf{C}}_1 = -\mathbf{C}_F^{-1}\mathbf{C}_L\bar{\mathbf{C}}_2 \qquad \text{(A10)}$$