PERIODICITY RETRIEVAL FROM NONSTATIONARY SIGNALS USING JOINT ORDER STATISTICS

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ABSTRACT

Composite signals are nonstationary processes consisting of trend, noise and cyclic components. A cyclic component consists of periodic or almost periodic data. In this paper we present a method based on nonlinear order statistics that evaluates the fundamental period of a cyclic component. This information can be used for decomposition of composite signals.

1 INTRODUCTION

Composite signals are nonstationary signals decomposable into two correlated components, which are the trend and the cyclic data, and an uncorrelated residual (noise). By cyclic data, we refer to periodic or almost periodic[1], constant mean value components of the composite signals. Alternatively, the terms "cyclic component" or even "periodicity" will be used to denote the same thing. By trend, we refer to a moving average of the composite signal, which can be represented as a slow and consistent monotonic change, with a limited number of possibly abrupt changes[2]. The abrupt changes of the trend can be modelled as superposed step functions[2].

A need for separating periodicities from trends in composite signals is encountered in a wide range of areas, such as climatology, seismology, oceanography, economics, social statistics or biomedical monitoring. In some applications the main interest is put on trends (e.g., tracking slow changes of microvascular perfusion with a laser Doppler flowmeter), in others on cyclic components (e.g., taking the pulse from finger with the same sensor).

Some methods used for extraction and forecasting of economic trends, such as the Holt-Winters algorithm or Brown's smoothing techniques[2], are based on an a priori knowledge of the fundamental period of the cyclic component (seasonal data). In most applications, however, this period has to be determined from the composite signal.

Spectral analysis tools, either based on Fourier transform or on autoregressive models (Prony, Pisarenko or Kumaresan & Tufts methods), often fail to distinguish between the spectral content of the cyclic component and the one of the underlying trend. Filter banks encounter similar problems. This happens whenever significant parts of the two components spectra overlap.

The parametric models of composite signals, such as autoregressive integrated moving average (ARIMA), or stepwise autoregression[2], meet difficulties in ascertaining the order of the implied polynomials. Adaptive updating of their coefficients is not fast enough to allow tracking of step changes. Difficulties with abrupt changes occur when using polynomial regression, as well.

The nonparametric methods, such as nonlinear order statitics (NLOS), i.e., rank filters, are increasingly used in robust estimation[3]. Recently, rank filters have been applied to frequency analysis[4]. A combination of two order statistics (joint order statistic envelopes) gave satisfactory results in tracking discrete periodicities superimposed on noisy, possibly steplike trends. In [4], the slope of the underlying trends had to be different from zero. In this paper, a way to overcome this limitation will be investigated.

2 JOINT ORDER STATISTICS

The order statistics of a set of data

$$\{x\}_k = \{x_{k-M+1}, x_{k-M+2}, \ldots x_k\}$$

are defined as the set of the same data rearranged in ascending order:

$$x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(M)}$$

The indexes without parentheses correspond to the temporal order within the window, while the indexes between parentheses correspond to the magnitude order.

The most popular NLOS estimator is the median, $x(\frac{M+1}{2})$ for odd M. The median smoothes out noisy signals and is reputed to be robust to large-amplitude uncorrelated impulsive noise. However, when such a noise corrupts a signal consisting of a periodicity superimposed on a mean-value drift (i.e., a trend, or a ramplike signal), its notorious "edge preserving" property makes the extraction of the periodicity features (extrema, increasing and decreasing half-period trends, mean-value crossings) unreliable.

The envelope of a signal may be formed by considering the signal extrema, $x_{(1)}$ and $x_{(M)}$, taken over a sliding window of length M. The main drawback of a single extremum is its high sensitivity to outliers. For instance, a spurious peak due to the noise may be misinterpreted as a periodicity extremum. The robustness to outliers can be improved if both extrema are used simultaneously. If we denote by $\Delta\chi$ the difference between successive maxima:

$$\Delta\chi_k = \max_{k-M+1\le i\le k}(x_i) \quad - \max_{k-M\le i\le k-1}(x_i) \qquad (1)$$

and by $\Delta\mu$ the difference between successive minima:

$$\Delta\chi_k = \min_{k-M+1\le i\le k}(x_i) \quad - \min_{k-M\le i\le k-1}(x_i) \qquad (2)$$

taken over two successive windows of length M, the sign of a half-period trend may be described by:

$$\begin{aligned}\Sigma_k &= \delta_{\mathrm{sgn}(\Delta\chi_k),\mathrm{sgn}(\Delta\mu_k)}\cdot \mathrm{sgn}(\Delta\chi_k)\\ &= \begin{cases} 1 & \text{for ascending trend, } \uparrow \\ 0 & \text{no trend, } \updownarrow \\ -1 & \text{descending trend, } \downarrow \end{cases} \qquad (3)\end{aligned}$$

where δ stands for the Kronecker Delta symbol.

Two successive outputs of the local trend detector Σ_i create pairs whose values have different interpretations:

$$\Sigma_k\Sigma_{k-1} = \begin{cases} 1, & \text{the local trend does not change} \\ 0, & \text{either no trend or gradual change of the trend sign,} \\ -1, & \text{the local trends have opposite signs.} \end{cases} \qquad (4)$$

An extremum is detected as a succession of two opposite half-period trends,

$$\Xi_k = \delta_{\Sigma_k\Sigma_{k-1},-1} + \frac{1}{2}\delta_{\Sigma_k\Sigma_{k-1},0}\,\delta_{|\Sigma_k+\Sigma_{k-1}|,1} \qquad (5)$$

Note that gradual changes of the trend sign are taken into consideration in the second term of the right-hand side.

The extrema count Z_M is the sum of all detected extrema,

$$Z_M = \sum_{i=1}^{N}\Xi_{i,M} \qquad (6)$$

Consequently, the period of a periodicity observed on a N-sample interval is:

$$T_M = \frac{2N}{Z_M} = \frac{2N}{\sum_{i=1}^{N}\Xi_{i,M}} \qquad (7)$$

where the subscript M indicates that the above estimates are derived from the local extrema of M-sample long windows.

Equation (7) is valid for all periodic signals whose period consists of one 'hill' and one 'valley', and can be modified for more complex periodic signals.

3 SCALE-ORDER-COUNT SPACE

Equations (6) and (7) would keep the same form if the $\Delta\chi$ and $\Delta\mu$ in (1) and (2) were differences of order statistics other than the extrema. Similar envelopes can be formed by quasiextrema, for example. If an integer q is the order of extremicity, such that $q = 1$ for extrema, $q = 2$ for quasiextrema, etc., than the corresponding envelopes are formed from OS pairs $(x_{(q)}, x_{(M-q+1)})$. Equation (6) becomes:

$$Z_{q,M} = \sum_{i=1}^{N}\Xi_{i,q,M} \qquad (8)$$

A nonlinear parametric space is obtained by evaluating equation (8) for different extremicity orders q and window sizes M. This space will be refered to as Scale-Order-Count space or SOC space. The shape of the surface corresponding to the number of detected hills and valleys $Z_{q,M}$ on a signal interval will vary in accordance with the content of the signal.

3.1 Noiseless Signal

Figure 1 represents the SOC space for a noiseless sinusoid. The surface $Z_{q,M}$ is flat between two sharp edges. The left edge is determined by the maximum defined value of q for a given window length M. Evidently, q has an upper limit determined by the window length M, i.e., $Z_{q,M}$ has physical sense for $q < \frac{M+1}{2}$ when M is odd, or $q \le \frac{M}{2}$ when M is even. In Figure 1, for $q > \frac{M+1}{2}$, $Z_{q,M}$ is set to zero by definition, while for $q \le \frac{M+1}{2}$ it has a constant value computed according to equation (8). For $q = \frac{M+1}{2}$, the OS envelopes turn into a single median value. As it can be seen in Figure 1, the extrema count of the moving median value of a noiseless periodicity (the left edge) does not differ from the extrema counts taken by OS envelopes whose $q < \frac{M+1}{2}$. The right edge occurs when the window

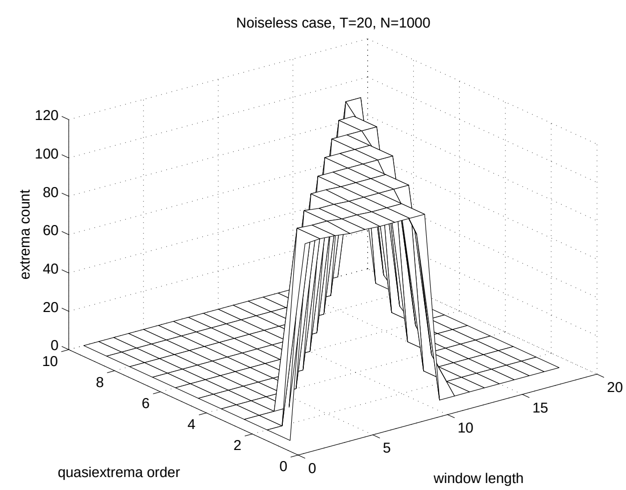

Figure 1: *The SOC space in the noisy case.*

length M becomes larger or equal to one half of the period of the cyclic component, $M \geq \frac{T}{2}$. For higher q, the right edge occurs at window lengths $M_{r.e.} = \lfloor\frac{T+(2q-1)}{2}\rfloor$, i.e., M increases in order to compensate for samples that have been trimmed off. Consequently, the period of a noiseless periodicity can be computed either from the extrema count $Z_{q,M}$, with $q \leq \frac{M+1}{2}$, as in equation (7), or, with lower precision, from the right edge coordinates $(M_{r.e.}, q_{r.e.})$, using $T \approx 2(M_{r.e.} - q_{r.e.}) + 1$.

3.2 Noisy Signal

In the case of a periodicity mixed with noise, the extrema count $Z_{q,M}$ is no more constant in the region $\{1 \leq q \leq \frac{M+1}{2}\} \bigcap \{M \leq M_{r.e.}\}$, as shown in Figure 2. The surface edges are not abrupt anymore, and it

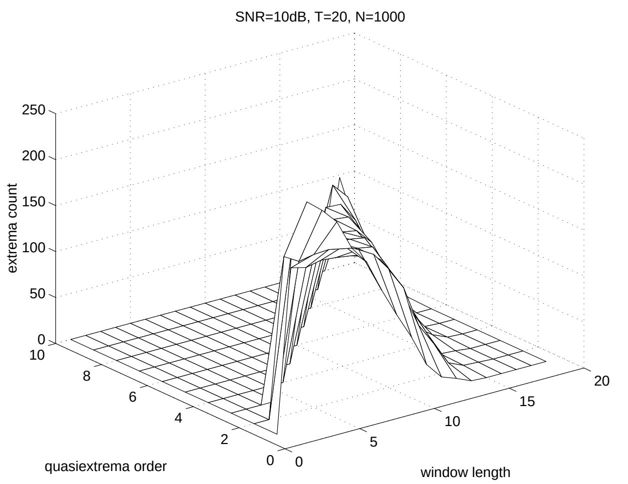

Figure 2: *The SOC space in the case of a noisy periodicity.*

becomes more difficult to estimate the period from equation (7) and from its relationship with the right edge coordinates. Note that the left edge, corresponding to the median, is more sensible to noise than the SOC estimates near the right edge. $Z_{q,M}$ and $(q_{r.e.}, M_{r.e.})$ give a redundant number of period estimates which cluster around the true value T.

This redundancy can be used to estimate the period of a cyclic component severely corrupted by noise. The coordinates of several important points on the SOC surface are related to the period value.

4 PERIOD ESTIMATION

In order to limit the amount of obtained extrema count estimates, we shall consider only the first two orders of quasiextrema, i.e., $q = 1$ and $q = 2$. This assures that for a wide range of window lengths $M \geq 5$, the observed portion of the scale-order surface is far from the rippled left edge (corresponding to the number of extrema of the median filtered signal) of the surface. The remaining quantity of quasiextrema counts is still abundant. Of special interest are the counts close to the right edge $(q, M_{r.e.})$ of the surface, as well as the corresponding window lengths.

The simplest way to find the right edge is to look for large changes in extrema counts for successive window lengths. If the extrema count $Z_{q=\text{const},M}$ falls by more than one third at some length M, the point $(q, M-1, Z_{q,M})$ is considered to be on the right edge. This gives one period estimate, namely $T_{q,M} = 2M - q$. The validity of this estimate may be ascertained by its consistency with the extrema count based period estimate, $\frac{2N}{Z_{q,M-1}}$, at and around the point $(q, M-1)$. The area of interest spreads on the left side from $(q, M-1, Z_{q,M})$, within some range ΔM, i.e., on the interval $(M - \Delta M, M-1)$. Larger M are not considered, since the SOC surface is expected to decrease abruptly at corresponding window lengths.

Suppose the new SOC estimate corresponds to the point $Z_{q+\Delta q, M-\epsilon}$. If the two estimates are not close enough, the procedure is repeated for another window length giving rise to a significant slope.

The error of these two estimated values with respect to the true period will decrease with the number of samples N, as shown in Figure 3.

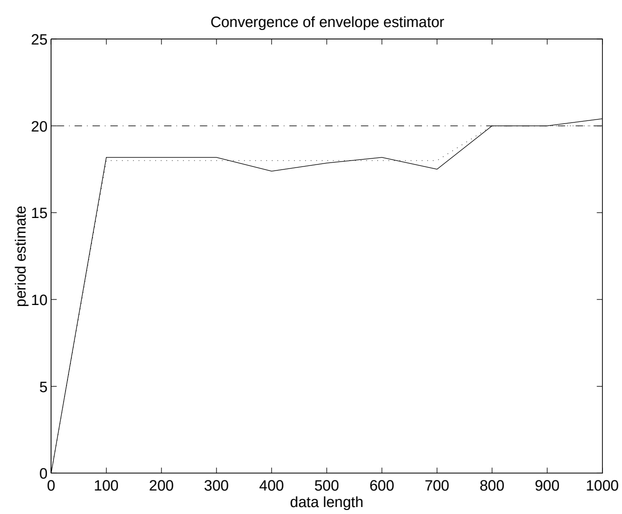

Figure 3: *The JNLOS period estimator applied to a sinusoid mixed with gaussian noise (signal-to-noise ratio is 2 dB) and superimposed on an elliptic trend. The estimate is reevaluated after each hundred samples. The precision increases with data length.* $\frac{2N}{Z_{q,M-1}}$ *is represented in solid line, and* $2(M-1)$ *in dotted line. The true period is represented by the dash-dot line.*

The deviation of estimates from the actual value of the period may be reduced if several estimates are made simultaneously. The procedure is as follows.

The quasiextrema counts of the two retained orders will have the largest slope in the neighbourhood of $M = \frac{T}{2} + q - \frac{1}{2}$. So the first two candidates $T_{Z\alpha}$ and $T_{Z\beta}$ will be derived from the extrema counts (Z_{q,M_q}) whose forward differences $Z_{q,M_q} - Z_{q,M_q+1}$ are the largest. The period candidates T_{Zq} are obtained easily by equation (7). Another two period candidates, T_A and T_B, are computed as $2M_q + 1 - q$.

Let us consider some neighbourhood of the maximum slope scales M_q. It should coincide in some extent with the right edge $M_{r.e.}$. In order to choose the best extrema counts $Z_{q,M}$ for $q = 1, 2$, we should consider the absolute difference of the corresponding period estimates obtained as $T_Z = \frac{2N}{Z_{q,M_{r.e.}}}$ and those chosen as $T_M \approx 2M_{r.e.} + 1 - q$. The minimum absolute difference should denote the quasiextrema counts and their respective window lengths with highest consistency. Six more candidates are obtained this way: T_{Z1} and T_{Z2}, from the best extrema count estimates for two quasiextrema orders, and T_{M1} and T_{M2} the periods derived from corresponding window lengths; $T_{Z\Omega}$ will be obtained from equation (7), by finding the extrema count $Z_{Q,\Omega} > 0$ such that the difference $|\frac{2N}{Z_{Q,\Omega}} - 2\Omega - 1 + Q|$ is minimal at (Q, Ω); T_Ω will be computed from the coresponding coordinates as $2\Omega + 1 - Q$.

Among these candidates, some have the same values or differ only slightly each from other. The simplest way to choose among them is to take their median value,

$$T_e = \text{median}(V) \qquad (9)$$

where V is a vector formed of the estimates:

$$V = [T_{Z\alpha}, T_{Z\beta}, T_A, T_B, T_{Z1}, T_{Z2}, T_{M1}, T_{M2}, T_{Z\Omega}, T_\Omega]^T.$$

In the case of a zero-mean periodicity $s_T(n)$, eventual baseline whose slope is less than $\frac{(\max s_T - \min s_T)}{\min \Delta_n(\max s_T, \min s_T)}$, where $\Delta_n(\max s_T, \min s_T)$ is the time distance between the periodicity extrema, does not affect the period estimation. Note that if the baseline presents step changes whose total number is much lower than the number of cycles on the observed data length, the period estimate will be insignificantly underestimated.

5 RESULTS

In Figures 4 and 5 we present the results of the period estimation for a discrete sinusoid mixed with additive gaussian noise. The signal-to-noise ratio of this signal is chosen to be very poor, 2 dB. A half-ellipse, with semi-axes $\frac{N}{2}$ and $\frac{N}{10}$, is added to the signal, and plays the role of an underlying trend.

Relatively short data records of $N = 100$ samples are considered. For signal periods between 6 and 30 samples, the relative errors under the chosen signal-to-noise ratio range from 0 to 25%. For the example given in Figure 4, the estimated period is equal to the true one, i.e., 10 samples. Similarly, im Figure 5, the estimated period is equal to 21.05, while the true period is 20 samples.

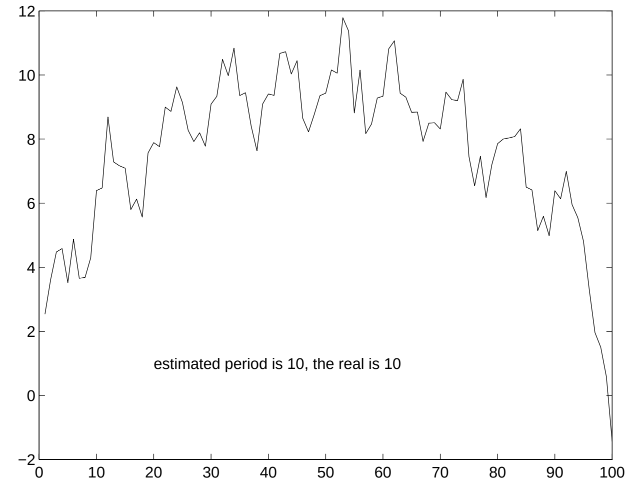

Figure 4: *A signal with T=10, the estimate is 10*

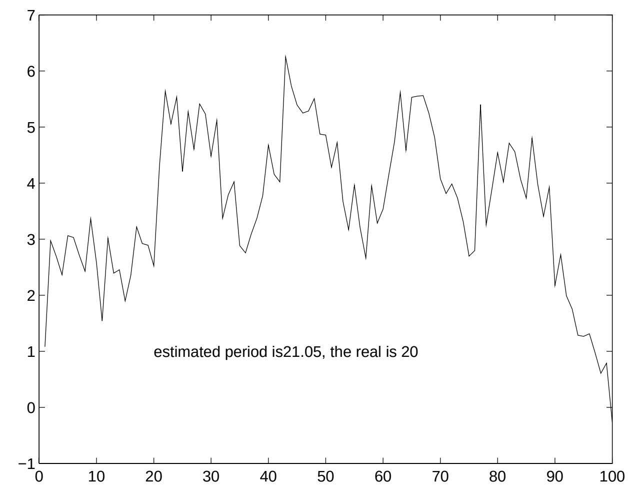

Figure 5: *A signal with T=20 the estimate is 21.*

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