SIGNAL PROCESSING VIA SYNCHRONIZED CHAOTIC SYSTEMS WITH FEEDBACK CONTROL

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ABSTRACT

In this paper we focus on the transmission of information signals via chaotic oscillations. To this end, we consider systems which contain generators with additional control loop and could behave chaotically and which dynamics may be controlled using feedback or directional coupling. Below we discuss three schemes of signal transmission and detection using 1) phase or frequency controlled generators, 2) coupled Chua's circuits with the adaptive parameter control, and 3) directionally coupled generators extracting binary signal from chaos in a presence of noise. New capabilities of conventional control systems for producing and processing chaotic signals are very promising — both for individual use and for implementation in networks [Shalfeev *et al.*, 94].

1 PHASE AND FREQUENCY CONTROL OF CHAOTIC OSCILLATIONS

It is specified that phase or frequency control applied to a generator enables it not only to stabilize its quasiharmonic oscillations but, under some conditions, to produce chaotic frequency modulation of the output. In the last case, the characteristics of the output appear to be controllable since they are determined by parameters of the feedback loop. Complex dynamics of generators with closed phase or frequency control loop depends on the inertial properties of linear filters of the loop. Nonlinear signal processing using generators with closed frequency or delayed phase control loop is considered in this section. As a theoretical basis for computer experiments the bifurcation analysis of mathematical models has been carried out.

Dynamics of the frequency controlled generator is determined by the following equations:

$$\mu x''' + x'' + \lambda x' + x + \Phi(x) = \gamma, \qquad (1)$$
$$\Phi(x) = \frac{2\beta x}{1+\beta^2 x^2},$$

where x is frequency mismatch. We show that this control system has complex behavior and some regions in the (λ, μ)-parameter space correspond to the chaotic oscillations [Alexeyev & Shalfeev, 93].

The mathematical model of phase-locked loop with time delay (PLLD) is given by:

$$\varphi''(t) + \lambda\varphi'(t) + \sin\varphi(t-\tau) = \gamma, \qquad (2)$$

where φ is phase difference. Some specific features of PLLD in chaotic regimes have been established [Kozlov & Shalfeev, 94]: the possibility of producing chaotically phase-modulated signals with stabilized carrier; controlling correlation dimension (i.e. signal complexity) of the output; controlling the bandwidth of the output with fixed correlation dimension.

This properties may be useful for transmission of information using signals with chaotic frequency modulation [Kozlov & Shalfeev, 94], [Alexeyev, Kozlov & Shalfeev, 94], [Kozlov & Shalfeev, 93]. We show on Fig. 1 that analog message may be transmitted using chaotic oscillations of the systems (1) or (2) for a 1% mismatch between parameters of transmitting and receiving subsystems. The information message is detected using synchronous signal processing. The receiving subsystem is designed according to circuitry of the transmitter and provides "inverse" transformation of chaotically modulated signal to extract an information one. The solution corresponding to synchronization of chaotic oscillations in transmitting and receiving subsystems is proved to be globally asymptotically stable under some conditions. This means that initial phase difference doesn't matter for synchronizing of subsystems.

2 ADAPTIVE SYNCHRONIZATION

A method of chaos synchronization in self-oscillating systems using parameter control is considered on an example of Chua's circuits [Madan(Ed.), 93]. We use closed feedback loop control — a loop of phase variable feedback and a loop of adaptive parameter control in the following form [Kozlov & Shalfeev, 95]:

$$\begin{aligned} Y' &= f(Y,\beta), \\ Y_s' &= f(Y_s,b) + h(Y_s - Y), \end{aligned} \qquad (3)$$

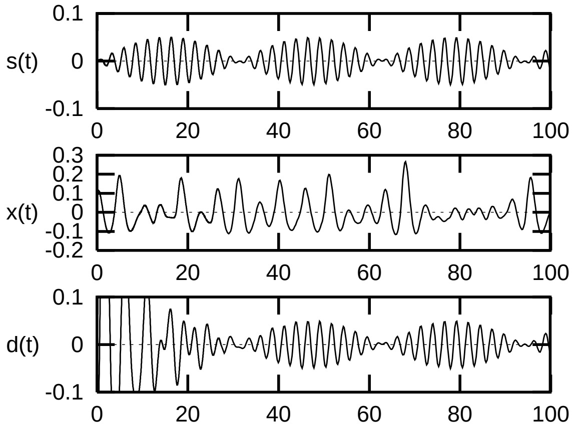

Figure 1: Analog encoding and decoding of the information signal $s(t)$ using mathematical model (1) of chaotic frequency modulation. $x(t)$ is time-varying instant frequency of the transmitted signal, $d(t)$ is the decoded message; the beginning of the message is lost due to the transient time needed for synchronization.

$$b' \quad = \quad g(Y_s - Y)\,,$$

where $Y = (x, y, z)^T, Y_s = (x_s, y_s, z_s)^T$ are the phase variables, h is a nonlinearity of coupling and g means a nonlinear control of parameter b. It is shown in Fig. 2 that under some conditions parameter b of the slave system tunes to the parameter β of the master system so it leads to the complete synchronization of chaotic oscillations $Y(t)$ and $Y_s(t)$. Let $x(t)$ be a transmitted component of chaotic oscillations of the master system. Parameter β of the master system varies along the time according to some information signal and parameter b of the slave system follows it — so this may be considered as a detection of the message. Note that initially parameters β and b does not equal each other so the master and slave subsystems are not synchronized. Complete synchronization is possible due to the local stability of the solution $Y_s(t) = Y(t),\ b(t) = \beta$ of (3). Automatic matching of the parameter of the slave system to the parameter of the master system which becomes possible with the use of adaptive parameter control means that one can measure the parameters of the system by its time series. Of course, a mathematical model of the master system must be known *a priori*. This principle may be used for designing devices for measurement, transmission and detection of signals.

The dependence of the recovery time τ (which characterize the speed of the transient processes of the controlled parameter to its initial value after small shock changes) from values of the stiffness of control $\varepsilon = g'(0)$ and the coupling coefficient $\delta = h'(0)$ has been investigated numerically. One can see from Fig. 3 that for specified δ the dependence $\tau \sim 1/\varepsilon$ is fulfilled in a rather broad range of variations of ε. It is also specified that

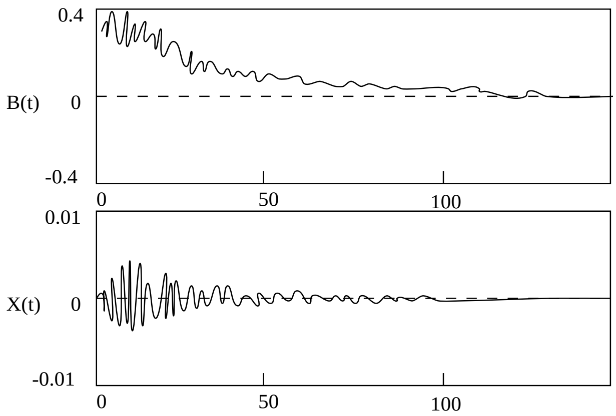

Figure 2: Synchronization of chaotic oscillations of mismatched systems (4) and (5) using adaptive control. $B(t) = b(t) - \beta$ is parameter mismatch, $X(t) = x_s(t) - x(t)$ is synchronization error.

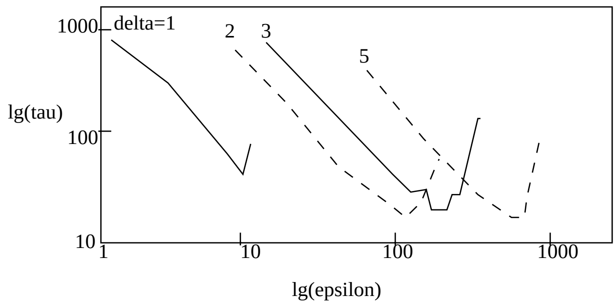

Figure 3: The recovery time τ versus stiffness of control ε for different values of the coupling coefficient $\delta = 1, 2, 3, 5$.

the increase of coupling coefficient δ at fixed stiffness of control ε leads to significant slowing down of the transition processes. But at the same time it leads to increase of capture range of controlled parameter b.

3 ROBUST SIGNAL TRANSMISSION

One of the principal problem of signal transmission is robustness of signal processing algorithms with respect to external noise. In our numerical experiments with mathematical model of the tunnel diode generator [Bazhenov, Kiyashko & Rabinovich, 94] we use some fruitful ideas proposed by [Cuomo, Oppenheim & Strogatz, 93] for the way of binary communications via chaotic oscillations and by [Lozi & Chua, 93] for reduction of noise using cascaded identical receivers. Both [Cuomo, Oppenheim & Strogatz, 93] and [Lozi & Chua, 93]'s solutions are based on the phenomenon of the synchronous response of stable system for external chaotic drive [Pecora & Carroll, 90].

The communication scheme suggested is displayed on

Fig. 4 where E is encoder transforming input signal s to chaotic oscillations y, D_1 and D_2 are the first- and second-cascade receivers, and d is the difference between their oscillations $d(t) = \tilde{y}_1(t) - \tilde{y}_2(t)$. Since the signal

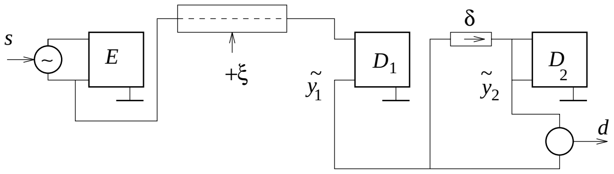

Figure 4: The robust communication scheme exploiting chaos. E is analog encoder, ξ channel noise, D_1 and D_2 are two identical receiving cascades synchronized with the directional coupling δ; s is input binary signal and d is detected one.

transmitted y is corrupted with the channel noise ξ both cascades D_1 and D_2 could not be synchronized. However, the output difference signal d corresponds to the input information signal s: if there is no input signal the difference dispersion is minimal and it significantly grows up when $s \neq 0$. Time series corresponding to this binary communication scheme are shown in Fig. 5 where $s(t)$ is input information signal, $y_{tr}(t)$ is the transmitted chaotic signal, $y_r(t)$ is the received signal corrupted with white noise (with signal-to-noise ratio 19dB), and $d(t)$ is the output difference signal.

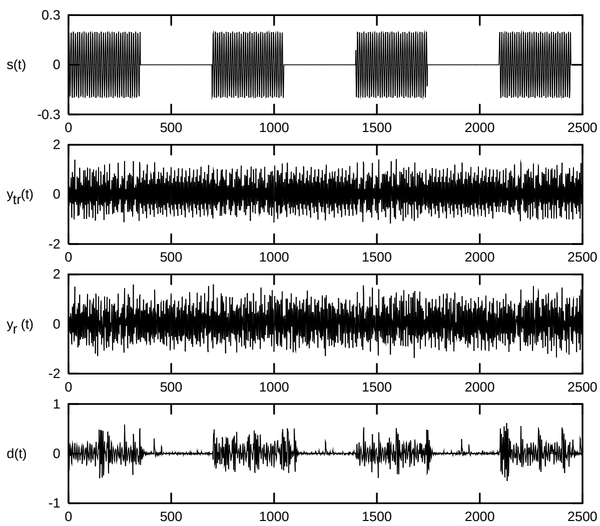

Figure 5: Detecting binary signal in chaos shift keying scheme in a presence of noise using coupled receivers.

We show how to decrease the sensitivity to noise of the receiver using directionally coupled self-oscillating systems. Varying the coupling parameter one can achieve alternatively either noise immunity or rapidity of the system.

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