TWO-STAGE POLYPHASE INTERPOLATORS AND DECIMATORS FOR SAMPLE RATE CONVERSIONS WITH PRIME NUMBERS

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Abstract

In this paper we demonstrate that it can be advantageous from a computational point of view to use a two-stage realization instead of a single-stage realization for sample rate conversions with prime numbers. One of the stages performs a conversion by a factor of two using linear-phase, or approximately linear-phase, half-band filters. The other stage changes the sample rate by the rational factor $N/2$ using a linear-phase FIR filter. The actual filtering can be performed at the lowest of the two sample frequencies involved. It is also possible to exploit the coefficient symmetry of the linear-phase FIR filter in the stage that changes the rate by a rational factor. The overall workload of the two-stage realization can therefore be reduced compared with the corresponding single-stage realization.

1 INTRODUCTION

Interpolation and decimation can be performed efficiently using a polyphase structure. This structure is based on a decomposition of the overall transfer function, $H(z)$, into N subfilters $H_n(z^N)$, such that $H(z)$ can be rewritten in the form

$$H(z) = \sum_{n=0}^{N-1} z^{-n} H_n(z^N) \qquad (1)$$

where N denotes the sample rate conversion ratio. When the polyphase structure is used for sample rate conversions by a factor N, the filters $H_n(z^N)$ can be realized to work at the lower sample frequency, as illustrated in Figs. 1 and 2. The arithmetic workload is thereby reduced by a factor N.

The filter $H(z)$ can always be restated in the form of (1) if it corresponds to an FIR filter. In the IIR case, the most efficient filters are obtained by letting the subfilters be allpass filters [1]. These filters are referred to as Nth-band IIR filters due to the close relationship to Nth-band FIR filters. If the phase behaviour is of less importance, the Nth-band IIR filters are known to be the most efficient structures for sample rate conversions with factors of N.

The Nth-band IIR filters can also be realized to have approximately linear-phase, by letting one of the allpass branches consist of pure delays. However, the workload of these filters is significantly higher than for the corresponding non-linear phase filters. Compared with conventional FIR filters, they are in many cases still in favour though.

From a computational point of view, it is often favourable to use a multi-stage realization of interpolation and decimation instead of using a single-stage realization. This holds for both FIR filters and Nth-band IIR filters [2, 3]. One drawback of this approach is that sample rate conversions with prime numbers can not be performed in a straightforward manner. However, by including stages that convert the sample rate by rational factors, it is possible to exploit the benefits of the multi-stage realization even in the case of conversions with prime numbers.

In this paper we show that sample rate conversions with prime numbers for applications where the phase behaviour is of importance, can be more efficiently performed using a two-stage realization instead of using a single-stage realization. The two-stage realization uses a linear-phase FIR filter and a half-band filter. The half-band filter can be either a linear-phase half-band FIR filter or an approximately linear-phase half-band IIR filter. The latter is a special case of Nth-band IIR filters for $N = 2$. The overall workload of the two-stage realization can be lower than for the corresponding single-stage realization.

For $N > 2$, the Nth-band IIR filters exhibit a number of extra don't care bands, as shown in Fig.

3. In some cases it is necessary to remove these extra bands to prevent aliasing into the transition band in the decimation case, and to avoid the corresponding imaging errors in the interpolation case. By using a two-stage structure, the problem of the extra don't care bands are eliminated. It should be mentioned however that aliasing into the transition band occurs even for the two-stage realization, but in this case the aliased components originates only from the upper part of the transition band. This is due to the fact that a half-band filter is used in one of the stages. Half-band filters do not have extra don't care bands, but their transition bands always include a quarter of the sample frequency.

Following this introduction, section two presents the two-stage realization. In section three we demonstrate by means of examples that the two-stage structures can be more efficient, in terms of arithmetic operations, than the corresponding single-stage structures.

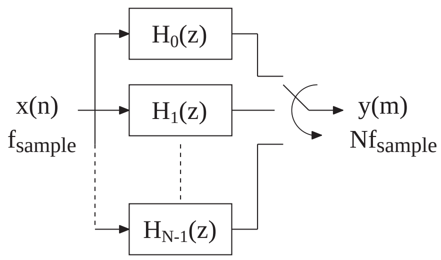

Figure 1. Polyphase interpolator.

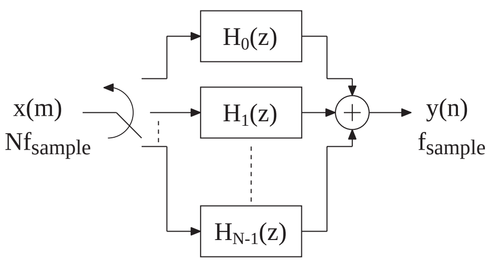

Figure 2. Polyphase decimator.

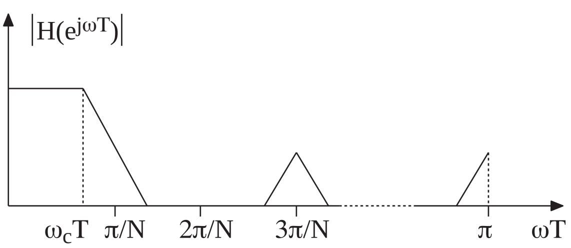

Figure 3. Magnitude response for an *N*th-band IIR filter.

2 TWO-STAGE REALIZATION

To reduce the workload for interpolation and decimation with prime numbers we use the structures shown in Figs. 4 and 5.

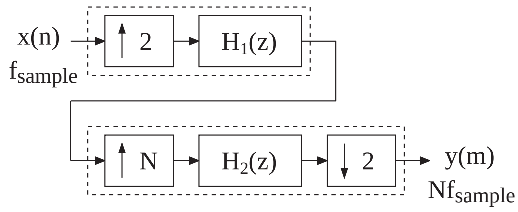

Figure 4. Two-stage structure for interpolation with a factor *N*.

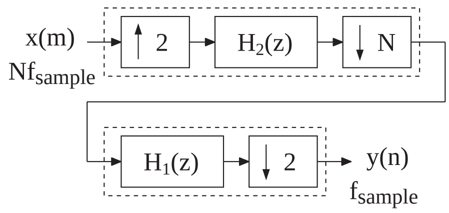

Figure 5. Two-stage structure for decimation with a factor *N*.

Due to the duality between interpolation and decimation, we restrict the discussion in this paper to interpolation. Assume that the desired interpolation factor is the prime number N. The conversion is performed in two stages using the structure shown in Fig. 4. In the first stage, interpolation with the factor 2 is performed. In the second stage, a sample rate conversion with the rational factor $N/2$ follows. The overall converter thus performs interpolation with the desired factor N.

The first filter can be a half-band filter working at the input sample frequency. Half-band filters are also very efficient since these have a small number of nonzero-valued coefficients. If the second filter, $H_2(z)$, is an FIR filter, not only the first filter but also the second filter can be realized to work at the input sample frequency. This is due to the fact that in the case of sample rate conversions with rational factors using FIR filters, it is always possible to move the downsamplers to the input of the converter, and the upsamplers to the output of the converter [4]. This is valid as long as the upsampling and downsampling factors are relatively prime. The actual filtering in the two-stage structure is thus performed at the lowest of the two sample rates involved.

In Fig. 6 the structure for sample rate conversion with the factor $N/2$ is shown. The downsamplers are placed at the input of the converter. The factors m, n, and N are related according to

$$2n - mN = -1 \tag{2}$$

The filters $G_0(z)$ and $G_1(z)$ corresponds to the polyphase subfilters of the filter $H_2(z)$ in Figs. 4 and 5, i.e.

$$H_2(z) = G_0(z^2) + z^{-1}G_1(z^2) \tag{3}$$

If the filter $H_2(z)$ is an FIR filter, then the filters $G_0(z)$ and $G_1(z)$ can also be expressed in the polyphase forms

$$G_0(z) = \sum_{n=0}^{N-1} z^{-n} G_{0n}(z^N) \tag{4}$$

and

$$G_1(z) = \sum_{n=0}^{N-1} z^{-n} G_{1n}(z^N) \tag{5}$$

By using polyphase structures for these two filters, the final structure for conversion with the rational factor $N/2$ is derived. The final structure is shown in Fig. 7. The filtering in this structure is performed at half the input sample frequency, which is equal to the original input sample frequency. All filtering in the two-stage structure is thus performed at the lowest of the two sample frequencies involved. The corresponding structure for conversion with the factor $2/N$ is easily obtained using the transposition theorem.

Since the downsampling factor for the converter shown in Figs. 6 and 7 always is two, it is possible to exploit the symmetry of the coefficients of linear-phase FIR filters, as long as the order of the filter is even. This is due to the fact that in this case both the multiplier coefficients, corresponding to $g(n)$ and $g(M-n)$, where M is the filter order, belong to one and the same filter, which is either $G_0(z)$ or $G_1(z)$.

In terms of number of multiplications and additions per sample, the two-stage structure has potential advantages for applications where the phase response is of importance. The order of a linear-phase FIR filter or an approximately linear phase Nth-band filter depends heavily on the relative transition band width of the filter. In the two-stage structure, the band width of the individual filters are wider than for the filter in the corresponding single-stage structure, which means that the overall complexity in some cases can be reduced.

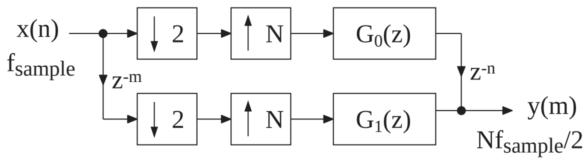

Figure 6. Sample rate conversion by $N/2$.

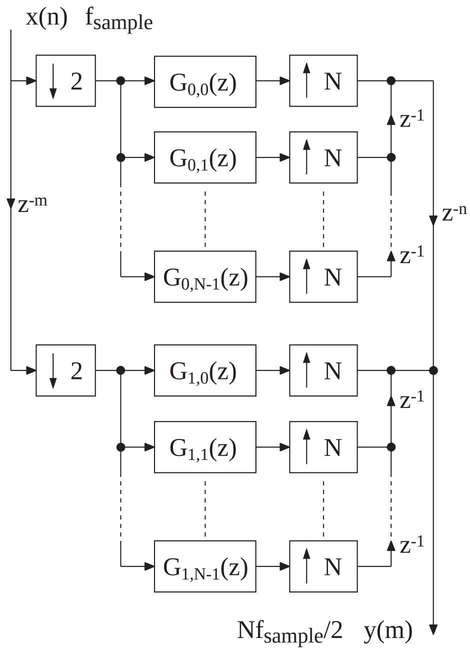

Figure 7. Structure for sample rate conversion by $N/2$ derived from the structure in Fig. 6 using polyphase decomposition.

3 EXAMPLES

We have considered two of the specifications in [1]. These are as follows:

Ex 1: $N = 3$, $\omega_c T = 0.8\pi/3$, $\omega_s T = 1.2\pi/3$, $A_{min} = 48$ dB

Ex 2: $N = 7$, $\omega_c T = 0.8\pi/7$, $\omega_s T = 1.2\pi/7$, $A_{min} = 70$ dB

For the filters used in the conversion by the factor two, we have used approximately linear-phase half-band IIR filters, and linear-phase half-band FIR filters. For the filters that are used in the stages performing the sample rate changes by rational factors, linear-phase FIR filters were used. The maximal passband deviations of these filters were chosen to 0.2 dB. The passband ripples of the half-band filters are uniquely determined by their stopband attenuations and need not be specified.

The multiplication rates and group delay deviations for the two examples are summarized in Table 1 and Table 2, respectively. For a comparizon, the table also includes the results for the corresponding linear-phase *N*th-band filters, and the corresponding linear-phase FIR filters, meeting the same requirements.

From the tables we can see that for the first case where the conversion ratio is 3, the two-stage structure requires a larger number of multiplications compared with the corresponding single-stage 3rd-band structure. However, in many cases it is necessary to remove the extra don't care bands that *N*th-band IIR filters possess, by adding an extra masking filter. If this is taken into account, the complexity of the two structures will be about the same. Compared with the single-stage FIR realization, the two-stage structure needs fewer operations, even when only FIR filters are used.

For the second case where the conversion ratio is 7, the two-stage realization has a lower multiplication rate than the corresponding single-stage realizations. This holds for both the case where the half-band filter is realized using an IIR filter, and the case where it is realized using an FIR filter. Thus, with the two-stage approach, it is in this case possible to realize sample rate converters having exact linear phase, at a lower multiplier cost compared with the corresponding single-stage approximately linear-phase *N*th-band IIR converters.

Example 1, N=3	Multiplication rate	Group delay $\Delta\tau_g$
Two-stage H1: Half-band IIR H2: FIR	4	0.32
Two-stage H1: Half-band FIR H2: FIR	5	0
One-stage FIR	6	0
One-stage 3rd-band IIR [1] (without correction filter)	2.7	0.66

Table 1. Results from example 1.

Example 2, N=7	Multiplication rate	Group delay $\Delta\tau_g$
Two-stage H1: Half-band IIR H2: FIR	4.1	0.15
Two-stage H1: Half-band FIR H2: FIR	4.7	0
One-stage FIR	7.6	0
One-stage 7th-band IIR [1] (without correction filter)	6	0.38

Table 2. Results from example 2.

4 CONCLUSIONS

In this paper it has been demonstrated that it can be favourable from a computational point of view to use a two-stage realization instead of a single-stage realization for sample rate conversions with prime numbers. Comparizons showed that by realizing the converters in two stages using only linear-phase FIR filters, or a combination of approximately linear-phase half-band IIR filters and linear-phase FIR filters, the workload can be reduced compared with realizing the converters in one stage using approximately linear-phase *N*th-band IIR filters.

The two-stage realization also eliminates the problem of the extra don't care bands that *N*th-band IIR filters exhibit. Another advantage of a two-stage realization is that the number of coefficients of each of its two filters is reduced in comparizon with the filter that is used in the corresponding single-stage realization. The search for optimal filter coefficients for these two filters can be performed independently. The search time can thereby be reduced dramatically.

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