CROSSTALK CANCELLATION AND MEMORY TRUNCATION IN TRANSMULTIPLEXER FILTER BANKS — TRANSMISSION OVER NON-IDEAL CHANNELS

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ABSTRACT

When transmultiplexers with overlapping frequency bands are used for the transmission of data over non-ideal channels, intersymbol interference and crosstalk between different data channels will arise. This paper addresses the design of optimal linear networks that reduce the effects mentioned above. A receiver structure based on a combination of crosstalk reduction, memory truncation and Viterbi detection will be proposed. The filter design method presented here is based on the maximization of a signal-to-noise ratio (SNR) at the detector input. The SNR will be defined for channel memories being truncated to arbitrary lengths. Thus, low-complexity Viterbi detectors working independently for all data channels can be used. The design of minimum mean squares error (MMSE) equalizer networks is included in the framework.

1 INTRODUCTION

Transmultiplexers are systems that convert time-division multiplexed signals into frequency-division multiplexed signals and vice versa. Essentially, these systems are filter banks of the type shown in Figure 1. The transmission from input i to output k is described by the impulse responses

$$t_{i,k}(m) = q_{i,k}(mN), \tag{1}$$

where

$$q_{i,k}(n) = g_i(n) * c(n) * h_k(n), \quad i,k = 0,1,\ldots,M-1. \tag{2}$$

Herein, the asterisk denotes convolution.

In the noise-free case, perfect reconstruction (PR) of the input data with a delay of m_0 samples can be obtained when the following condition holds:

$$t_{i,k}(m) = \delta_{i,k}\ \delta_{m,m_0}, \qquad i,k = 0,1,\ldots,M-1, \tag{3}$$

where $\delta_{i,k}$ denotes the Kronecker symbol. For an ideal channel with impulse response $c(m) = \delta_{m,0}$, (3) is the time domain version of the following condition on the polyphase matrices $\boldsymbol{H}_p(z)$ and $\boldsymbol{G}_p(z)$ of the analysis and synthesis filter bank [1]:

$$\boldsymbol{H}_p(z)\ \boldsymbol{G}_p(z) = z^{-m_0}\ \boldsymbol{I}. \tag{4}$$

When PR holds for an ideal channel and when we have a non-ideal channel in practice, at least intersymbol interference will arise. In addition, when a critically sampled system ($N = M$) is used, the frequency bands have to be overlapping and we will have crosstalk between different data channels. In other words, the input data cannot be recovered perfectly. Solutions to this problem based on various types of equalization have been proposed [2]. However, it is well known that receivers based on maximum likelihood detection via the Viterbi algorithm are superior to those based on linear equalizers [3]. The only drawback is that Viterbi detectors can become very complex: In the transmultiplexer case, all data sequences $d_k(m)$ and all impulse responses

$$t_{i,k}(m),\ i,k = 0,1,\ldots,M-1,$$

have to be considered simultaneously.

In this paper, a receiver structure is proposed where linear filters are used for crosstalk cancellation and for the equalization of uncritical parts of the transmission channel while the critical part of the channel (i.e.: the part with zeros close to the unit circle) is considered in the Viterbi detector. Two filter design methods which can be viewed as extensions of the memory-truncation algorithms in [4] and [5], respectively, will be presented. As will be shown in the next section, the extension of the method in [5] to the transmultiplexer case leads to a linear set of equations. The extension of the method in [4] leads to an eigenvalue problem.

2 NETWORK DESIGN

2.1 Optimality Criterion

For the following derivation let us assume that crosstalk only appears between adjacent channels (this assumption holds for most systems for practical use). The equalizer network for the kth data channel, which also takes the signals in channels $k-1$ and $k+1$ into account, is shown in Figure 2. The systems $h_{k-1,k}(m)$ and $h_{k+1,k}(m)$ are used for crosstalk cancellation while the system $h_{k,k}(m)$ can be regarded as some kind of equalizer. The system with impulse response $p_k(m)$ in Figure 2 is the residual system being considered in the Viterbi detector. It can be regarded as the critical (hardly equalizable) part of $t_{k,k}(m)$.

Note that the network can be easily extended to the case where crosstalk appears between all channels. In this case, the original analysis filter bank can be omitted since the filter network becomes a complete analysis filter bank.

All systems, $h_{k-1,k}(m)$, $h_{k,k}(m)$, $h_{k+1,k}(m)$, and $p_k(m)$ have to be designed simultaneously. The optimality criterion is

$$E\left\{|e_k(m)|^2\right\} \stackrel{!}{=} \min, \tag{5}$$

where $E\{\ \}$ denotes the expectation operation.

The lengths L_{p_k} of the residual impulse responses $p_k(m)$, $k=0,1,\ldots,M-1$, and the lengths of the prefilters $h_{j,k}(m)$ are arbitrary. Note that for the choice $L_{p_k}=1$ the memory truncation approach proposed here reduces to an MMSE approach that takes adjacent channels into account.

The error signal in the kth channel can be written in the following form[1]

$$e_k(n) = \boldsymbol{r}_k^T(n)\ \boldsymbol{h}_k - \boldsymbol{d}_k^T(n-j_0)\ \boldsymbol{p}_k, \tag{6}$$

where

$$\begin{aligned}
\boldsymbol{r}_k(n) &= \begin{bmatrix} \boldsymbol{r}_{p_k}(n) \\ \vdots \\ \boldsymbol{r}_{p_k}(n-L_h+1) \end{bmatrix}, \\
\boldsymbol{h}_k &= \begin{bmatrix} \boldsymbol{h}_{p_k}(0) \\ \vdots \\ \boldsymbol{h}_{p_k}(L_h-1) \end{bmatrix}, \\
\boldsymbol{d}_k(n) &= \begin{bmatrix} d_k(n) \\ \vdots \\ d_k(n-L_p+1) \end{bmatrix}, \\
\boldsymbol{p}_k &= \begin{bmatrix} p_k(0) \\ \vdots \\ p_k(L_p-1) \end{bmatrix},
\end{aligned}$$

and

$$\begin{aligned}
\boldsymbol{r}_{p_k}(n) &= \begin{bmatrix} r'_{k-1}(n) \\ r'_k(n) \\ r'_{k+1}(n) \end{bmatrix}, \\
\boldsymbol{h}_{p_k}(n) &= \begin{bmatrix} h_{k-1,k}(n) \\ h_{k,k}(n) \\ h_{k+1,k}(n) \end{bmatrix}.
\end{aligned}$$

Assuming stationary data processes $d_k(m)$, $k=0,1,\ldots,M-1$, a time invariant channel $c(n)$, and a stationary additive noise process $\eta(n)$, the criterion (5) becomes

$$\boldsymbol{h}_k^H \boldsymbol{R}_{rr}^{(k)} \boldsymbol{h}_k - \boldsymbol{h}_k^H \boldsymbol{R}_{rd}^{(k)} \boldsymbol{p}_k - \boldsymbol{p}_k^H \boldsymbol{R}_{dr}^{(k)} \boldsymbol{h}_k + \boldsymbol{p}_k^H \boldsymbol{R}_{dd}^{(k)} \boldsymbol{p}_k \stackrel{!}{=} \min, \tag{7}$$

where

$$\begin{aligned}
\boldsymbol{R}_{rr}^{(k)} &= E\left\{\boldsymbol{r}_k^*(n)\ \boldsymbol{r}_k^T(n)\right\}, \\
\boldsymbol{R}_{rd}^{(k)} &= E\left\{\boldsymbol{r}_k^*(n)\ \boldsymbol{d}_k^T(n-j_0)\right\}, \\
\boldsymbol{R}_{dr}^{(k)} &= E\left\{\boldsymbol{d}_k^*(n-j_0)\ \boldsymbol{r}_k^T(n)\right\} = [\boldsymbol{R}_{rd}^{(k)}]^H, \\
\boldsymbol{R}_{dd}^{(k)} &= E\left\{\boldsymbol{d}_k^*(n-j_0)\ \boldsymbol{d}_k^T(n-j_0)\right\}.
\end{aligned}$$

2.2 Eigenfilter Method

According to [4] let us first find the optimal vector $\boldsymbol{h}_k$ for a fixed vector $\boldsymbol{p}_k$ in the sense of (7). We get[2]

$$\boldsymbol{h}_{\text{opt}} = \boldsymbol{R}_{rr}^{-1}\ \boldsymbol{R}_{rd}\ \boldsymbol{p}. \tag{8}$$

Substitution of $\boldsymbol{h}_{\text{opt}}$ into (7) gives

$$-\boldsymbol{p}^H\ \boldsymbol{R}_{rd}^H\ \boldsymbol{R}_{rr}^{-1}\ \boldsymbol{R}_{rd}\ \boldsymbol{p} + \boldsymbol{p}^H\ \boldsymbol{R}_{dd}\ \boldsymbol{p} \stackrel{!}{=} \min. \tag{9}$$

In order to avoid the trivial solution, we solve (9) under the condition $\boldsymbol{p}^H\boldsymbol{p}=1$. For uncorrelated data with correlation matrix $\boldsymbol{R}_{dd}=\sigma_d^2\boldsymbol{I}$ (9) leads to the following eigenvalue problem:

$$\left[\sigma_d^2\boldsymbol{I} - \ \boldsymbol{R}_{dr}^H\ \boldsymbol{R}_{rr}^{-1}\ \boldsymbol{R}_{rd}\right]\ \boldsymbol{p} = \lambda\ \boldsymbol{p}. \tag{10}$$

The optimal vector $\boldsymbol{p}$ is the eigenvector that belongs to the smallest eigenvalue λ.

2.3 Linear Set of Equations

An interesting alternative to the eigenvector solution shown above is the extension of the filter design method in [5] to the transmultiplexer case. Here we solve (7)

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[1] The superscript ^{T} denotes transposition of a vector or matrix. Moreover, the superscripts * and ^{H} denote complex conjugation and conjugate transposition ($\boldsymbol{r}^H = [\boldsymbol{r}^*]^T$), respectively.

[2] In order to simplify the notation, the subscript $_k$ and the superscript $^{(k)}$ are omitted in the following formulae.

under the condition $p(0) = 1$. In matrix notation our criterion (7) becomes

$$[1, \tilde{\boldsymbol{p}}^H, \boldsymbol{h}^H] \begin{bmatrix} \boldsymbol{R}_{dd} & -\boldsymbol{R}_{rd}^H \\ -\boldsymbol{R}_{rd} & \boldsymbol{R}_{rr} \end{bmatrix} \begin{bmatrix} 1 \\ \tilde{\boldsymbol{p}} \\ \boldsymbol{h} \end{bmatrix} \stackrel{!}{=} \min, \qquad (11)$$

where

$$\boldsymbol{p}^T = [1, \tilde{\boldsymbol{p}}^T]. \qquad (12)$$

For uncorrelated data with $\boldsymbol{R}_{dd} = \sigma_d^2 \boldsymbol{I}$, (11) leads to the following linear set of equations for $\tilde{\boldsymbol{p}}$ and $\boldsymbol{h}$:

$$\begin{bmatrix} \tilde{\boldsymbol{R}}_{dd} & -\tilde{\boldsymbol{R}}_{rd}^H \\ -\tilde{\boldsymbol{R}}_{rd} & \boldsymbol{R}_{rr} \end{bmatrix} \begin{bmatrix} \tilde{\boldsymbol{p}} \\ \boldsymbol{h} \end{bmatrix} = \begin{bmatrix} \boldsymbol{0} \\ \boldsymbol{r}_{rd} \end{bmatrix}, \qquad (13)$$

where

$$\boldsymbol{R}_{dd} = \begin{bmatrix} \sigma_d^2 & \boldsymbol{0}^T \\ \boldsymbol{0} & \tilde{\boldsymbol{R}}_{dd} \end{bmatrix} \qquad (14)$$

and

$$\boldsymbol{R}_{rd} = \left[\boldsymbol{r}_{rd}, \tilde{\boldsymbol{R}}_{rd} \right]. \qquad (15)$$

Note that (13) can be solved efficiently via partitioned inversion or simply by adaptation.

2.4 Linear Set of Equations with Modifications

Although the solution to (13) in combination with $p(0) = 1$ gives optimal filters in the sense of (7), a slight modification may be useful. In order to explain this, let us first observe that for uncorrelated data ($\boldsymbol{R}_{dd} = \sigma_d^2 \boldsymbol{I}$) the upper part of (13) gives

$$\sigma_d^2 \, \tilde{\boldsymbol{p}}_k = [\tilde{\boldsymbol{R}}_{rd}^{(k)}]^H \, \boldsymbol{h}_k. \qquad (16)$$

This means that the coefficients in $\tilde{\boldsymbol{p}}_k$ are equal to $L_{p_k} - 1$ of the L_{p_k} dominant subsequent values of the overall impulse response $\tilde{t}_{k,k}(n)$ which describes the transmission from input k to output k when the network is in use.

$$\begin{aligned} \tilde{t}_{k,k}(m) &= t_{k,k-1}(m) * h_{k-1,k}(m) + t_{k,k}(m) * h_{k,k}(m) \\ &\quad + t_{k,k+1}(m) * h_{k+1,k}(m) \end{aligned} \qquad (17)$$

If we want the impulse response $p_k(m)$ to be equal to all L_{p_k} dominant values of $\tilde{t}_{k,k}(n)$, we have to re-compute $p_k(0)$. From the restriction

$$\sigma_d^2 \, \boldsymbol{p}_k = [\boldsymbol{R}_{rd}^{(k)}]^H \, \boldsymbol{h}_k \qquad (18)$$

with

$$\boldsymbol{p}_k^T = [p_k(0), \tilde{\boldsymbol{p}}_k^T], \qquad (19)$$

we get

$$p_k(0) = \frac{1}{\sigma_d^2} \, [\boldsymbol{r}_{rd}^{(k)}]^H \, \boldsymbol{h}_k. \qquad (20)$$

Advantages of this re-computation of $p_k(0)$ are discussed in [5] for the single channel case.

3 EXAMPLES

In the following example a critically sampled system with $N = M = 8$ was used. Without transmission channel $c(n)$ the filter bank allows almost perfect reconstruction.

Figure 3 shows an example for the impulse responses $t_{2,3}(m)$, $t_{3,3}(m)$, and $t_{4,3}(m)$ in the presence of a non-ideal transmission channel $c(n)$ and for the remaining impulse responses $\tilde{t}_{2,3}(m)$, $\tilde{t}_{3,3}(m)$, and $\tilde{t}_{4,3}(m)$ after prefiltering, where

$$\begin{aligned} \tilde{t}_{2,3}(m) &= t_{2,2}(m) * h_{2,3}(m) + t_{2,3}(m) * h_{3,3}(m) \\ \tilde{t}_{3,3}(m) &= t_{3,2}(m) * h_{2,3}(m) + t_{3,3}(m) * h_{3,3}(m) \\ &\quad + t_{3,4}(m) * h_{4,3}(m) \\ \tilde{t}_{3,2}(m) &= t_{4,4}(m) * h_{4,3}(m) + t_{4,3}(m) * h_{3,3}(m) \end{aligned}$$

Note that $\tilde{t}_{3,3}(m)$ has much higher energy than $t_{3,3}(m)$. Therefore, the relative reduction of crosstalk is much higher than could be expected from the plots.

Figure 4 shows the SNRs in the 3rd and 5th data channel for different lengths L_p and L_h. Especially the results for channel 3 show that the SNR can be dramatically increased when going from MMSE equalization ($L_p = 1$) to memory truncation ($L_p > 1$).

4 CONCLUSION

In this paper, new receiver concepts for data transmission with transmultiplexers based on memory truncation were presented. The results show that memory truncation leads to remarkably higher SNRs than MMSE equalization. Therefore, a very good performance (even in the case of critical channels) can be expected from this method.

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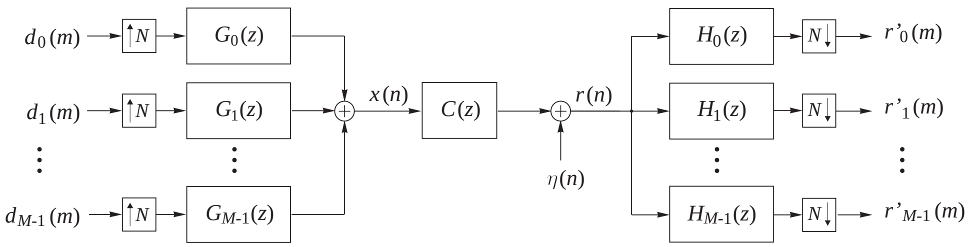

Figure 1: Transmultiplexer with transmission channel and additive noise

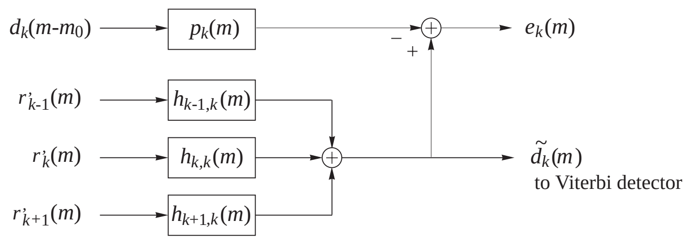

Figure 2: Filter network for memory truncation and crosstalk cancellation in the kth data channel

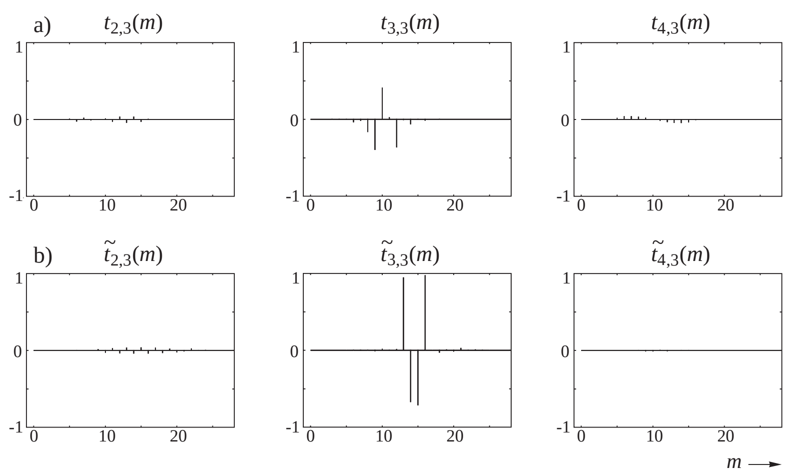

Figure 3: Transmission and crosstalk impulse responses (SNR at the receiver input: $\mathrm{SNR}_0 = 30$ dB; pre-filter lengths: $L_h = 11$): a) initial impulse responses; b) with pre-filtering ($L_p = 4$).

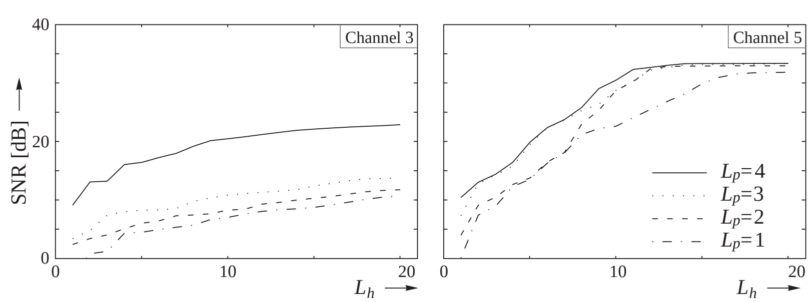

Figure 4: SNRs in 3rd and 5th data channel ($\mathrm{SNR}_0 = 30$ dB)