Two-Dimensional Diamond-Shaped Filter Banks from One-Dimensional Filters

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Abstract

Nonrectangular transformation is proposed for the design of multidimensional filter banks. The advantage of nonrectangular transformation is the abundance of transformation kernels and their efficient implementations by ladder structures. The design of two-dimensional two-channel filter banks from one-dimensional filters is discussed and design examples are presented.

1. Introduction

Designing multidimensional (MD) filter banks is a challenging problem because the perfect reconstruction constraints is much difficult to achieve. Various transformations are used to convert the MD problem to a similar but simpler one-dimensional (1D) problem. McClellan transformation is one of the most efficient and widely applied method to design MD filter banks. Since the MD filters designed by McClellan transform are FIR and linear-phase, all the analysis and synthesis filters are FIR and linear phase. Furthermore, with appropriate transformation kernel, most 1D properties will carry through. The design procedure of McClellan transformation uses simple substitution of variables and its success depends on the availability of proper transformation kernel. Furthermore, only MD zero-phase FIR filters can be obtained.

In some filter specifications, the desired responses are characterized by ideal frequency responses in which passbands and stopbands are separated by boundaries that are not necessarily parallel to the frequency axes, such as in the diamond-shaped filters. Such filters can be obtained by a series of manipulations on a separable prototype filter with a rectangular passband. The prototype filter is upsampled on a nonrectangular grid. The upsampling process produces a parallelogram by rotating and shrinking the frequency response of the prototype filter, together with a change in the periodicity. Depending on the desired response, cascading to eliminate unwanted portions of the passband in frequency response, along with possible shifts and additions, may be used. The nonrectangular upsampling is then followed by a rectangular decimation of the sequence to expand the passband to the desired size. This procedure is being described as nonrectangular transformation [3].

The advantage of nonrectangular transformation is the abundance of transformation kernel. Intuitively, nonrectangular transformation can yield any MD support with straight-line boundaries. Furthermore, the resulting filters using such algorithms produce efficient filter structures that can be implemented with essentially 1D techniques where the corresponding orientations of processing are not parallel to the sample coordinates.

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In this paper, two-dimensional (2D) diamond-shaped filter bank designed using nonrectangular transformation is considered. By combining the results in 2D polynomial theory [2], the PR properties of the 2D diamond-shaped filter banks are structurally imposed if the analysis lowpass filter is a linear-phase FIR half-band or the first polyphase component of the FIR filter is minimum-phase. Furthermore, the results from polynomial theory allow efficient implementation of the filter banks by ladder structure. The PR property of the proposed filter banks allows maximum control of the filter response, such as the number of zeros at π. Design examples of linear phase FIR diamond-shaped PR filter banks from half band filter and FIR filter with minimum phase first polyphase component are presented in the subsequence sections.

2. Nonrectangular Transformation of 2D Diamond-Shaped Filter

The diamond-shaped support is shown in Figure 1a. To obtain a diamond-shaped lowpass filter using nonrectangular transformation, we notice that the support region in Figure 1b is a rotated and upsampled version of the diamond-shaped support. If the filter of Figure 1b is $F_1(z_1, z_2)$, the diamond filter $D(z_1, z_2)$ is given by

$$D(z_1, z_2) = F_1(z_1^{\frac{1}{2}} z_2^{\frac{1}{2}}, z_1^{\frac{-1}{2}} z_2^{\frac{1}{2}}) \ . \qquad (1)$$

Notice that the filter in Figure 1b can be obtained as the sum of the two filters in Figure 1c and 1d. Furthermore, Figure 1d can be obtained by shifting Figure 1c. Consequently if $F_2(z_1, z_2)$ is the response of Figure 1c, and $F_3(z_1, z_2)$ is the response of Figure 1d, then

$$F_3(z_1, z_2) = F_2(-z_1, -z_2) \ . \qquad (2)$$

Filter in Figure 1c can be obtained by tensor product of 1D lowpass filters. Let $G(z)$ be the prototype 1D filter which can be expressed in polyphase as $G(z) = G_0(z^2) + zG_1(z^2)$. The transfer function $F_1(\mathbf{z})$ is given by

$$F_1(z_1, z_2) = G(z_1)G(z_2) + G(-z_1)G(-z_2) \ .$$

As a result, the diamond-shaped filter $D(\mathbf{z})$ is given by

$$D(z_1, z_2) = 2G_0(z_1 z_2)G_0(z_1^{-1} z_2) + 2z_2 G_1(z_1 z_2)G_1(z_1^{-1} z_2) \ . \qquad (3)$$

If $G(z)$ is chosen to be half band, $G_0(z)=\frac{1}{2}$, a half band $D(\mathbf{z})$ with diamond-shaped support is given by

$$D(z_1,z_2)=1+2z_2G_1(z_1z_2)G_1(z_1^{-1}z_2)\ . \qquad (4)$$

3. Perfect Reconstruction 2D Ladder Structure

Consider the filter bank in Figure 2 with decimation matrix $\mathbf{M}$. The output $\hat{X}(\mathbf{z})$ and input $X(\mathbf{z})$ of the system is related by

$$\hat{X}(\mathbf{z})=T(\mathbf{z})X(\mathbf{z})+A(\mathbf{z})X(-\mathbf{z})\ , \qquad (5)$$

where $A(\mathbf{z})=\frac{1}{2}(H_0(-\mathbf{z})F_0(\mathbf{z})+H_1(-\mathbf{z})F_1(\mathbf{z}))$,
and $T(\mathbf{z})=\frac{1}{2}(H_0(\mathbf{z})F_0(\mathbf{z})+H_1(\mathbf{z})F_1(\mathbf{z}))$,
are aliasing and distortion functions respectively. If the analysis and synthesis filters are chosen to be $F_0(\mathbf{z})=H_1(-\mathbf{z})$ and $F_1(\mathbf{z})=-H_0(-\mathbf{z})$, the aliasing component is cancelled. Furthermore, $F_0(\mathbf{z})$ and $F_1(\mathbf{z})$ will have the desired diamond-shape support, if $H_0(\mathbf{z})$ and $H_1(\mathbf{z})$ have the desired support. Consequently the filter bank is PR if

$$T(\mathbf{z})=\frac{1}{2}(H_0(\mathbf{z})H_1(-\mathbf{z})-H_1(\mathbf{z})H_0(-\mathbf{z}))=z_1^{k_1}z_2^{k_2}\ . \qquad (6)$$

Let $H_0(\mathbf{z})$ be a diamond-shaped half band filter constructed using eq.(4). The companion filter $H_1(\mathbf{z})$ is given by $H_1(\mathbf{z})=2z_2$. The filter bank can be constructed by ladder structure as in Figure 3a. Obviously the performance of this filter bank is poor since the high pass filter is a pure delay. To obtain better performance, we cascade the ladder structure as shown in Figure 3b, by Lemma 3 of [2]. Consequently, the new analysis high pass filter is given by

$$H_1(\mathbf{z})=2z_2-C(\mathbf{z})H_0(\mathbf{z})\ , \qquad (7)$$

where $C(\mathbf{z})$ is the cascaded filter with property $C(-\mathbf{z})=C(\mathbf{z})$. As proved in [2], the solution space of $H_1(\mathbf{z})$ is spanned by $C(\mathbf{z})$. Therefore, $H_1(\mathbf{z})$ with the desired characteristics can be obtained by choosing different $C(\mathbf{z})$. Figure 3b shows the resulting ladder structure where we consider the polyphase component of variable z_2.

Notice that if $H_0(\mathbf{z})$ is chosen to have the form of eq.(3), linear-phase or minimum phase solution can be still be obtained. This is because Noether condition [2] requires that the inverse of $G_0(z_1z_2)$ and $G_0(z_1^{-1}z_2)$ are stable filters which reduce to minimum phase $G_0(z)$. Notice that if $G(z)$ is linear-phase, the resulting $D(\mathbf{z})$ is also linear-phase. Similarly, according to Lemma 3 of [2], $H_1(\mathbf{z})$ can be chosen as eq.(7), and Figure 3c is the resulting ladder structure. Moreover, G_0 and G_1 can be chosen as allpass IIR filters [5], which results in an IIR filter banks. IIR solution using allpass filters are considered in Section 4.

Wavelet filter banks can be obtained by using Bernstein polynomials. As discussed in [2] a maximal regular lowpass 1D prototype filter can be obtained from halfband filter $H(z)$, which is a Bernstein polynomial in z. Thus, $H_0(\mathbf{z})$ has the form of eq.(6) and $G_1(\mathbf{z})$ is a Bernstein polynomial. It can be proved that the regularity of the 2-D wavelets obtained from nonrectangular transfomation increases linearly with the regularity of the 1D filter so that it can be arbitrarily high. Design example is presented in the Section 5.

4. Allpass Solution

A wide family of practical transfer functions can be represented as

$$G(z)=\frac{A_0(z)+A_1(z)}{2}\ , \qquad (8)$$

where $A_0(z)$ and $A_1(z)$ are stable allpass filters [7]. Rewrite eq.(8) in polyphase form,

$$G(z)=\beta_0(z^2)+z^{-1}\beta_1(z^2) \qquad (9)$$

It is obvious that $\beta_0(z^2)$ and $\beta_1(z^2)$ and $\beta_0(z)$ and $\beta_1(z)$ have to be allpass. As a result, a 2D diamond-shaped filter bank can be constructed using Figure 3c with

$$H_0(\mathbf{z})=\beta_0(z_1z_2)\beta_0(z^{-1}z_2)+2z_2\beta_1(z_1z_2)\beta_1(z_1^{-1}z_2)$$
$$H_1(\mathbf{z})=2z_2-C(z_1z_2)(\beta_0(z_1z_2)\beta_0(z^{-1}z_2)+2z_2\beta_1(z_1z_2)\beta_1(z_1^{-1}z_2))$$

where $C(-z_1z_2)=C(z_1z_2)$. In particular, a half-band IIR filter can be constructed as

$$G(z)=1+z^{-1}\beta_1(z^2) \qquad (10)$$

where $A_0(z)=1/2$, and $A_1(z)=z^{-1}\beta_1(z^2)/2$ is allpass filter. Following the nonrectangular transformation, the 2D diamond-shaped filter bank can be constructed using Figure 3b with

$$H_0(\mathbf{z})=1+2z_2\beta_1(z_1z_2)\beta_1(z_1^{-1}z_2)$$
and
$$H_1(\mathbf{z})=2z_2-C(z_1z_2)(1+2z_2\beta_1(z_1z_2)\beta_1(z_1^{-1}z_2))$$

Notice that the same structure is also proposed by [5] and the case of $C(z_1z_2)=\beta_1(z_1z_2)\beta_1(z_1^{-1}z_2)$ has been analyzed. The system, however, is based on intutative observation of the structure of 2D half-band filters, and does not generalize to other nonrectangular transformation. Furthermore, the connection with 2D polynomial theory has not been exploited.

5. Design Examples

We consider the FIR solutions in this section only. The design of FIR solution can be summarized as follows.

1. Design a linear phase lowpass filter $G(z)$ with even order and half band response or the first polyphase component is minimum phase.
2. Construct $H_0(z)$ by performing the nonrectangular transformation according to eq. (3) for minimum phase solution or eq(4) for half band solution.
4. Construct $H_1(z)$ according to eq.(7).

Since $C(\mathbf{z})$ affects the response of the resulting filter banks, two choice of $C(z)$ is used in this paper.

i. $C(z) = G_1(z)$ and $C(\mathbf{z}) = C(z_1 z_2)C(z_1^{-1} z_2)$
ii. $C(z) =$ maximally flat half-band diamond shape FIR filter.

It should be noticed that $G(z)$ itself is required to be wavelet filters for constructing wavelet filter banks.

The frequency response of the resulting 2D diamond-shaped filter bank using the half band maximally flat linear-phase $G(z)$ with 15 taps and $C(z) = G_1(z)$, is plotted at Figure 4, where the resulting lowpass and highpass subband filters have size 27x27 and 53x53 respectively.

Similarly, linear-phase 2D diamond-shaped filter bank constructed by 1D linear phase filter with minimum phase first polyphase component is plotted in Figure 5. $G(z)$ is constructed by remez algorithm with 15 taps and $C(z)$ is chosen to be maximally flat half-band filters with 15 taps. The resulting lowpass and highpass subband filters have size 27x27 and 53x53 respectively.

IIR solution can be constructed similarly by choosing half-band IIR filters, or IIR filters where its first polyphase component has stable inverse.

6. Comparison of Existing Method

Similar 2D diamond-shaped filter banks structures and design methods have been proposed by [3,5,6], where the maximum regular case is discussed in [6], IIR case in [5] and iterative improvement is discussed in [3]. These approaches, however do not describe the connection between polynomial method, non-rectangular transformation and 2D filter banks design. Consequently, their approaches do not generalize to other nonrectangular transformation which leads to different supports and different sampling lattice. They are limited to rotation of the rectangular support by polyphase transformation. In particular, the 2D transformation considered in [5] is polyphase transformation which is different from nonrectangular transformation although both leads to the same structure. For example, the current design technique can be easily generalize to filter banks with fan filter support which cannot be achieved by polyphase transformation. Moreover, the design of linear phase PR solution using 1D filters with minimum phase first polyphase component is largely overlooked. This is because normally $G_0(z)$ does not exist in the formulation of 2D half-band filters.

7. Conclusions

In this paper, we have considered the design of 2D filters through nonrectangular transformation. Using polynomial theory [2], ladder structure is proposed for the design and implementation of the filter banks. The ladder structure imposes structural PR property, and results in maximal freedom in designing the response of the subband filters. The proposed design method is applicable in designing linear phase FIR, and IIR filter banks. Furthermore, the design of maximally regular wavelet filter banks is also discussed.

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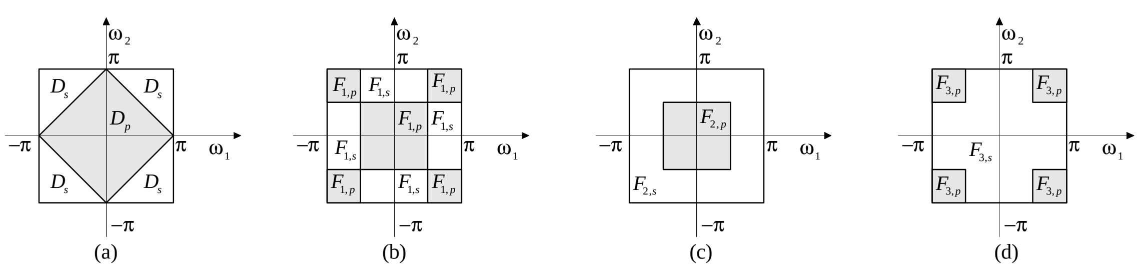


Figure 1. Ideal frequency response of a) diamond filters. b), c) and d) Frequency response of rectangular filters.

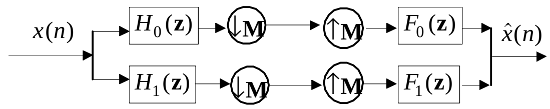


Figure 2. 2D filter banks.

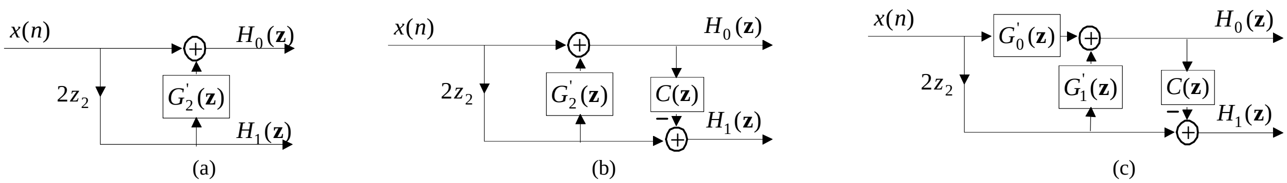
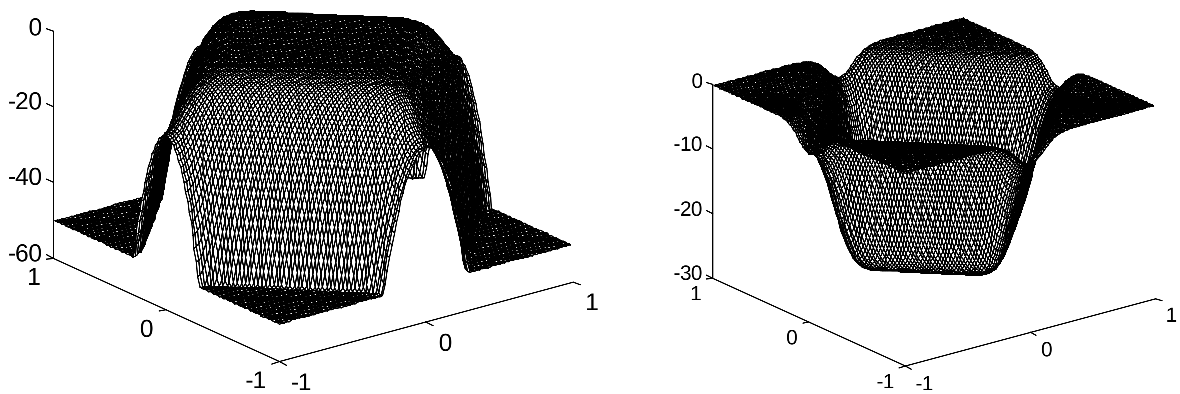


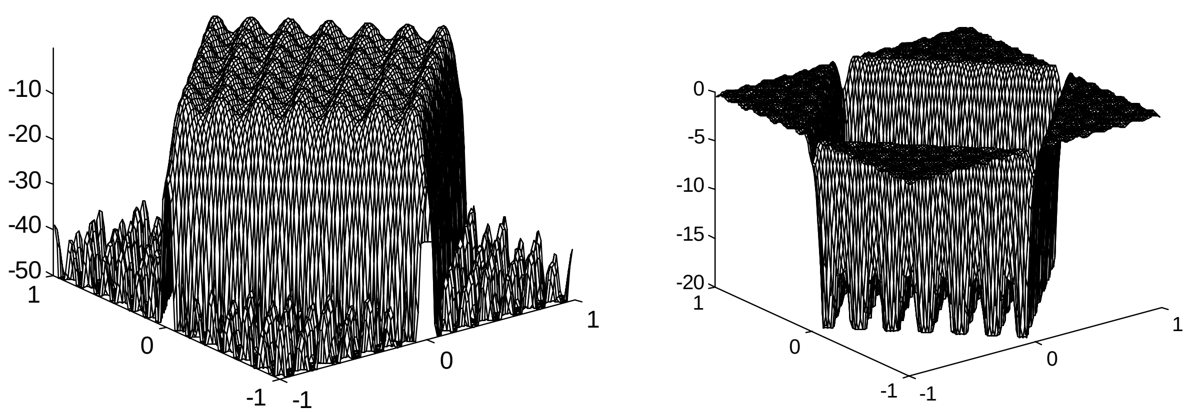
Figure 3. 2D analysis filter banks in ladder structure. a) High pass filter as pure delay. b) Cascade structure. c) G_0 minimal phase filter. ($G_2^{'}(\mathbf{z}) = G_2(z_1 z_2) G_2(z_1^{-1} z_2)$, $G_1^{'}(\mathbf{z}) = G_1(z_1 z_2) G_1(z_1^{-1} z_2)$ and $G_0^{'}(\mathbf{z}) = G_0(z_1 z_2) G_0(z_1^{-1} z_2)$).



Lowpass filter

Highpass filter

Figure 4. Diamond-shaped 2D PR filter banks with 27x27 taps lowpass filter and 53x53 taps highpass filter from 1D maximally flat half band FIR filter with 15 taps and $C(z) = G_1(z)$.



Lowpass filter

Highpass filter

Figure 5. Diamond-shaped 2D PR filter banks with 27x27 taps lowpass filter and 53x53 taps highpass filter from 1D FIR filter with 15 taps and minimum phase first polyphase components and $C(z)$ is maxmially flat half band FIR filter with 15 taps.