SPATIO-TEMPORAL WAVELET TRANSFORMS FOR IMAGE SEQUENCE ANALYSIS. *

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ABSTRACT

This paper intends to present an integrated approach of constructing new spatio-temporal wavelets for discrete signal analysis. The main illustrative field of applications considered here stands as the analysis of digital image sequences. Nevertheless, this can be readily extended to any kind of spatio-temporal signals. Continuous wavelet transforms, continuous series, discretized series and discrete transforms are considered here in an unified way. The analysis to be developed relies only on dynamic parameters like uniform translation and rotation, on kinematic parameters like velocity and speed and on structural parameters as scale and orientation. This digital processing intends to cover the detection and the focalization on motion-based regions of interest in order to perform tracking, classification, segmentation, multiscale trajectory construction and eventually a selective reconstruction of the useful content.

Key Words: continuous wavelet transform, motion analysis, spatio-temporal filtering, image sequence processing.

1. INTRODUCTION

The primary goal of the present work is to thoroughly investigate all the families of spatio-temporal wavelet transforms in order to bring together the continuous, the frame and the discrete versions and to construct new families of tools for the analysis of spatio-temporal signals. Though digital image sequences ($2D + T$ signals) are presented as the main illustrative application in this paper, the theory extends to any kind of ($nD+T$) signals. The main goal of this research work is to develop specific digital tools for motion analysis, multiscale trajectory constructions, selective motion-based feature extraction, focalization and reconstruction. On one side, the discrete wavelet transform has already proved its usefulness as a powerful tool for signal filtering with numerous properties (orthogonality, bi-orthogonality and linear-phase response, selectivity, regularity, statistical adaptivity and criteria-based optimum design) amenable to applications like coding, interpolation, restoration, reconstruction and synthesis. In that field, motion-compensated wavelet filtering [8-9] (i.e. temporal filtering applied along the assumed motion trajectories) has also been demonstrated as a highly efficient tool. Modeling motion with affine models allows several generalizations; among them, this provides connexions with the continuous wavelet transforms. Indeed, the affine transformation can be exploited to construct groups like the affine-Galilei group and to leads us to new admissible spatio-temporal wavelets. On the other side, the continuous wavelet transform stands as a powerful tool of signal analysis introducing numerous sets of physical parameters to put in operation [5]. In the field of image sequence analysis, the current spatio-temporal phenomena require to take simultaneously into account a vast choice of analyzing parameters the most important of which are the translation, the rotation and the deformation, the orientation (analysis direction), the scale, the speed and eventually the acceleration. The informations of interest embedded in the image sequences are intended to be detected, analyzed and selectively reconstructed by the discretized inverse wavelet transform.

2. DEFINITIONS

In whole analogy to continuous and discrete Fourier transforms, several wavelet transforms may be defined. These are namely the *continuous wavelet transform*, the *wavelet series or wavelet frame*, the *discretized wavelet series* and the *discrete wavelet transform*. Under conditions of sufficient regularity, any discrete wavelet filter can be associated to an existing continuous wavelet Ψ (not necessarily with analytical formulation) such that filtering a sampled signal with the discrete wavelet yields exactly or, at least closely, the sampled version of the signal filtered with the associated continuous wavelet Ψ [3]. In fact, the discretization of continuous wavelet transforms is numerically stable and invertible on admissible frames [1-4].

Let us first consider the continuous spatio-temporal spatio-temporal wavelet transform of a signal $s(\vec{x},t)$ defined in the Hilbert space $L^2(\mathbf{R}^n \times \mathbf{R}, d^n\vec{x}dt)$ and define a doubly-indexed family of wavelets constructed by dilating and translating in the affine group. In this case, the spatio-temporal wavelet transform $W[s(\vec{x},t);\ a,\vec{b},\tau]$ is defined as an inner product expressed in bracket notations

$$\begin{aligned} & c_\Psi^{-1/2}\ \Psi_{a,\vec{b},\tau}\ .\ s(\vec{x},t) \\ = & \ c_\Psi^{-1/2} \int_{\mathbf{R}^n\times\mathbf{R}} d^n\vec{x}\ dt\ \bar{\Psi}\left[\frac{\vec{x}-\vec{b}}{a}, \frac{t-\tau}{a}\right] s(\vec{x},t) \\ = & \ c_\Psi^{-1/2} \int_{\mathbf{R}^n\times\mathbf{R}} d^n\vec{k}d\omega\ e^{i(<\vec{k}|\vec{b}>+\omega\tau)}\bar{\hat{\Psi}}\left(a\vec{k},a\omega\right)\ \hat{s}\left(\vec{k},\omega\right) \end{aligned}$$

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where $\vec{k}$ and ω are the spatial and temporal frequencies respectively and the symbols $\hat{}$ and $\bar{}$ stand for the Fourier transform and the complex conjugate. The wavelet Ψ is the *mother wavelet* which must verify the condition of square-integrability meaning that there exits a constant c_Ψ such that

$$c_\Psi \ = \ (2\pi)^{n+1} \int_{\mathbf{R}^n \times \mathbf{R}} d^n\vec{k} \ d\omega \ \frac{|\hat{\Psi}(\vec{k},\omega)|^2}{|\vec{k}|^n \ |\omega|} \ < \ \infty$$

The wavelets $\Psi_{a,\vec{b},\tau}$ are the members of the *of spatio-temporal wavelet family* generated by the affine group and defined in the space of the parameters a, $\vec{b}$, τ. Continuous and discrete wavelet transforms have each reconstruction formulae allowing numerical reconstruction, practically processed in a frame which defines a family of functions of $L^2(R^n \times R, d^n\vec{x}dt)$ $\{g_{a,\vec{b},\tau}(\vec{x},t)\}_{a,\tau \in Z, \vec{b} \in Z^n}$. This leads to

$$s(\vec{x},t) \ = \ \sum_{a,\vec{b},\tau} < g_{a,\vec{b},\tau} | s(\vec{x},t) > \ g_{a,\vec{b},\tau}(\vec{x},t)$$

A numerical efficient way of performing the wavelet transform consists of working in the spectral domain by means of (2D+T) FFT. Fast continuous wavelet transform is also a current topic of research [2].

3. BUILDING SPATIO-TEMPORAL WAVELETS

The construction of spatio-temporal wavelet families and the interconnections are both depicted in Figure 1. Discrete spatio-temporal wavelets have already been studied as motion-compensated wavelet filters i.e. as subband filter applied along the motion trajectories [8-9]. Using motion-compensated filters requires at each point, where the filter is applied, to build motion trajectories and to compute the signal intensities along them. One convenient way of constructing trajectories consists on estimating motion and performing a joint motion-based segmentation of the signal. Affine transformations are an efficient way to model linear motions for mobile segments in deformation. It takes into account translation, rotation, scaling and shear. One idea developed in this work is to express all these elementary transformations as unitary operators in the spatio-temporal domain $nD + T$, and to write useful generalizations, namely the *translation*, the *dilation*, the *rotation*, the *solid-deformation* and eventually the *acceleration*.

Starting from the particular case of motion-compensated discrete wavelets, families of continuous spatio-temporal wavelets may be constructed by exploiting the idea of associating groups to sets of unitary operators defined in the previous section. The set of the operator parameters defines a group characterized by one law of composition, the identity and then the inverse element. If the group representation in the Hilbert space $L^2(\mathbf{R}^n \times \mathbf{R}, d^n\vec{x}dt)$ posesses the properties of *unitarity*, *irreducibility* and *square-integrability* [6], then admissible mother wavelets may be defined in the spatio-temporal space and all the operations defined in the group can be applied on the mother wavelet to generate all the wavelets in the family. One member of the wavelet family can be associated to any point of the parameter space. The analysis process consists in representing the signal in the parameter space or, at least, in some plane (sub-space) of interest.

For example, let us consider the operator $\{\mathbf{\Omega} \ : \ L^2(\mathbf{R}^n \times \mathbf{R})\} \ \rightarrow \ L^2(\mathbf{R}^n \times \mathbf{R})\}$ defined on the set of parameters $(a, c, r, \vec{b}, \tau, \epsilon)$ of unitary transformations (a the spatio-temporal dilation, c the factor tuning the speed module, r the spatial rotation, $\vec{b}$ and τ the spatial and temporal translations, and, ϵ the time reversal parameter); they lead to unitary irreducible and square-integrable representations. This defines the *kinematical wavelets* described by Duval-Destin and Murenzi [5]. Going to a more general construction, another group is worth being studied. That is the affine-Galilei group derived from the Galilei group by adding independent time and space dilations, a_0 and a respectively. The generic set of parameters is referred to as $(\tau, \vec{b}, \vec{v}, a_0, a, r)$ where τ and $\vec{b}$ are the time and space translations, $\vec{v}$ the vector of speed, r the spatial rotation in $\mathbf{R}^n$. The representations of the affine-Galilei group in the spatio-temporal domain fails to be square-integrable and requires quotients [10] to retrieve the property. Among all the sections (quotients), we consider that obtained by letting $a_0 = a$ as the most interesting one leading to square-integrable representations in space $\mathbf{R}^n \times \mathbf{R}$). The action of the parameters in the space can be symbolically written in a matrix formulation as

$$\begin{pmatrix} \vec{x}' \\ t' \\ 1 \end{pmatrix} \ = \ \begin{pmatrix} aR(\theta) & \vec{v} & \vec{b} \\ 0 & a_0 & \tau \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \vec{x} \\ t \\ 1 \end{pmatrix}$$

with $R(\theta)$ the rotation matrix. Therefore, if g is element of the group $g = (\tau, \vec{b}; \vec{v}; a_0, a, r)$, the law of composition derives from the matrix multiplication $gg' = (a_0\tau' + \tau,$ $ar\vec{b}' + \vec{v}\tau' + \vec{b}; ar\vec{v}' + a_0'\vec{v}; a_0a_0', aa', rr')$. A similar case of study can be carried out on the Weyl-Poincaré group (where a dilation parameter added to the Poincaré group) to derive admissible *relativistic wavelets* without requesting in this case any quotient.

The discretization issue consists on condensing in the most efficient way all the information of the wavelet transform on a discrete lattice of parameters. The reconstruction is exact if the frame of wavelets $g_{a,\vec{b},\tau,c,r}$ generates a basis of $L^2(\mathbf{R}^n \times \mathbf{R}, d^n\vec{x}dt)$. Admissible frames or series of wavelets may be defined in each family of continuous wavelets presented in the previous chapter to tile the parameter space. In fact, these series and their discretized versions aim at supporting the numerical computations and the discrete signal analyses or syntheses (signal reconstructions). Complete or selective analyses may be performed respectively in the whole parameter space or in some sub-spaces. *The most interesting application is here to selectively reconstruct objects with a specific and given velocity. This object has to be considered as being the only useful information to be observed.* The lattice density of the frame defines the level of the reconstruction quality performing in that sense *some quantization* of the reconstructed signal. Selective reconstructions consist in providing only sufficient frame density in the plane of interest.

4. APPLICATIONS

Applications for the digital image sequence analysis are here essentially proposed with Galilean wavelets to analyze the signal content according to scale, orientation and velocity. Motions like uniform translations and accelerations can be efficiently analyzed and tracked. Eventually, motion-based selective reconstructions may be presented. The anisotropic *Morlet wavelet* is mainly considered in a non-separable spatio-temporal form

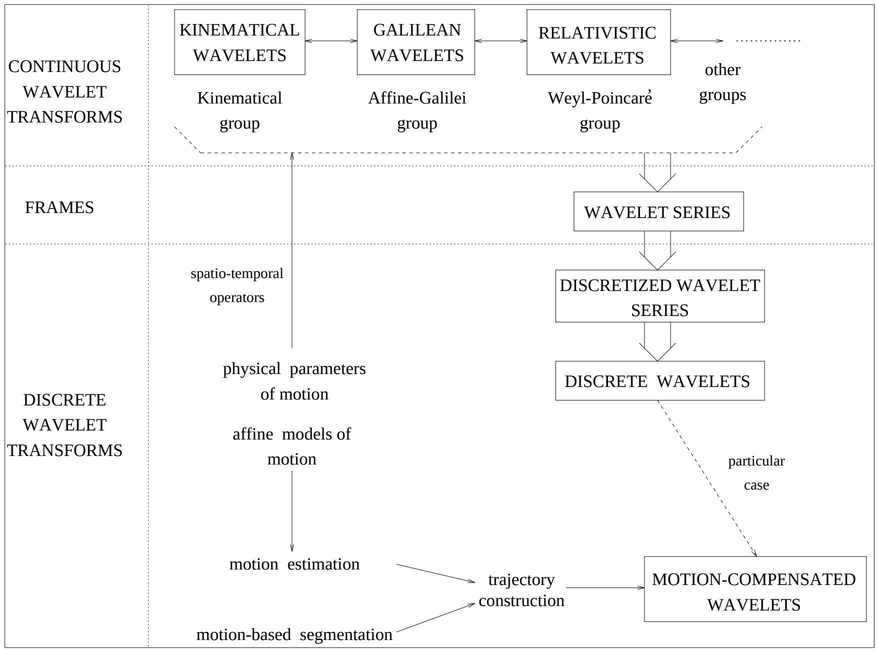

Figure 1. The families of spatio-temporal wavelet transforms.

$$\Phi(\vec{x}, t) = e^{i\vec{k}_0 \vec{X}} e^{-\frac{1}{2}<\vec{X} \mid C\vec{X}>} - e^{-\frac{1}{2}<\vec{k}_0 \mid D\vec{k}_0>} e^{-\frac{1}{2}<\vec{X}|C\vec{X}>}$$

where $\vec{X} = (\vec{x}, t)^T \in \mathbf{R}^n \times \mathbf{R}$, C is a $n \times n$ positive definite matrix $< \vec{X}|C\vec{X} >> 0$. For $2D + T$ signals, $C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/\epsilon \end{pmatrix}$ where ϵ is the factor of temporal anisotropy of the wavelet. Figures 2 and 3 show the Galilean Morlet wavelet at a velocity $(1, 0)$ and the synthetic image sequence to be analyzed at the 25th image (sequence with four objects in affine transformation: translation, rotation and shear). Figures 4 and 5 present the Fourier energy localization resulting from motions and the wavelet analysis in the velocity plane. Figure 6 presents the wavelet analysis in the spatial parameter domain at the 25th image working as a motion-based segmentation algorithm. Figure 5 derives some frame bound ratios $\frac{B/A}{}$ for the Galilean Morlet wavelet for *reference lattices* made of translations $b_{0,x}, b_{0,y}, b_{0,\tau}$, rotation L_0, scale a_0 and velocity $(\gamma_{0,x}, \gamma_{0,y})$.

5. CONCLUSIONS

In this paper, one new family of spatio-temporal wavelet transforms has been presented as a tool to analyze spatio-temporal signals according to physical parameters of translation motion. Other kind of motion can be considered in the same way. The signal analysis has been first developed in the continuous realm by means of spatio-temporal operators, group theory and representations properties. Any group or quotients leading to square-integrable representations in the spatio-temporal domain $(nD + T)$ leads to admissible wavelets and generates a new family. Wavelet and signal discretizations through admissible frames allows discrete analyses and selective reconstructions. Interestingly, the Fourier domain turns out to be an efficient realm where to study motion and tracking as well as to develop efficient numerical algorithms (FFT and parallelization).

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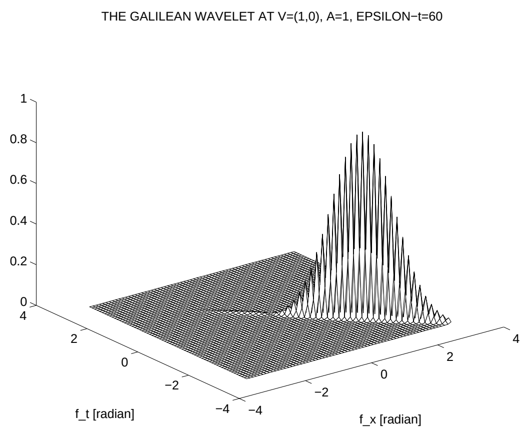

Figure 2. Galilean Morlet wavelet.

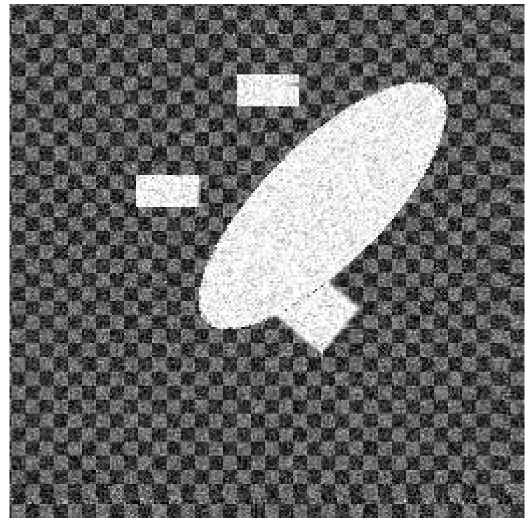

Figure 3. Synthetic sequence: 25th image.

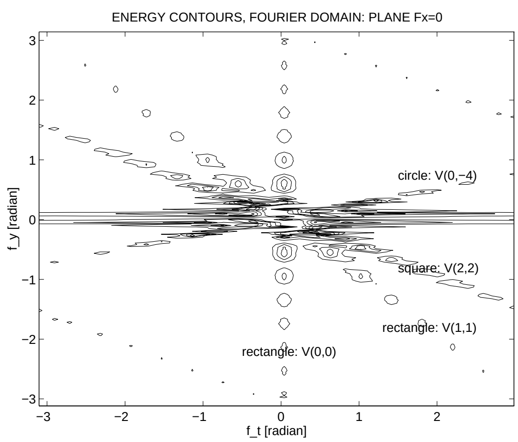

Figure 4. Component velocity planes embedded in the Fourier transform of a synthetic sequence.

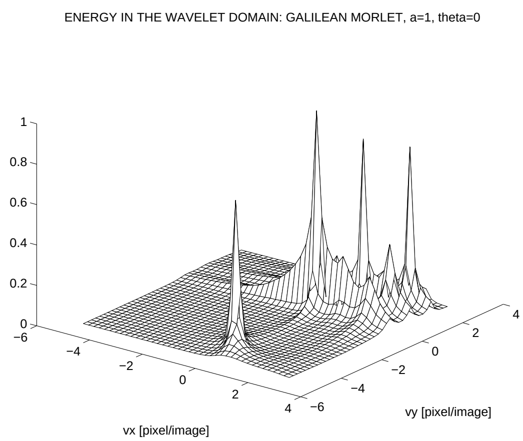

Figure 5. Motion analysis in the synthetic sequence: energy contours.

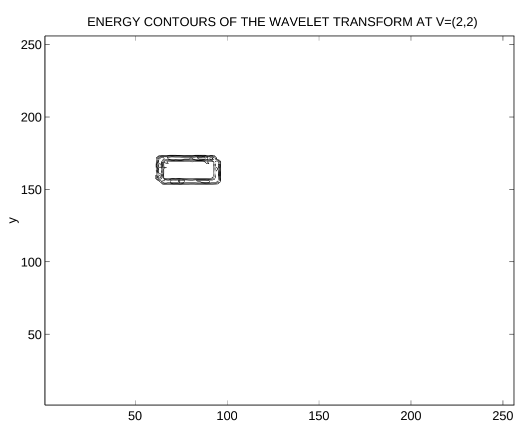

Figure 6. Synthetic sequence: wavelet analysis on the 25th image (Galilean Morlet wavelet), $\vec{v} = (2, 2)$.

$b_{0,x}$	$b_{0,y}$	$b_{0,\tau}$	L_0	a_0	$\gamma_{0,x}$	$\gamma_{0,y}$	B/A
1	1	1	20	2	1	1	1.1427
1	1	1	20	2	1.5	1.5	1.3141
1	1	1	20	2	2	2	1.9145
1	1	1	20	2	2.5	2.5	3.1001
1	1	1	20	2	0.75	0.75	1.0632
0.75	0.75	0.75	20	2	1	1	1.1327
1.5	1.5	1.5	20	2	1	1	1.1590

Table 1: Frame bounds for the Morlet wavelet $\vec{k}_0 = (\pi[2/ln2]^{1/2}, 0, 0)$ and $\epsilon_t = 60$.